

APPLICATION OF THE HILBERT METHOD TO FREE-SURFACE FLOW PROBLEMS

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ABSTRACT: This paper investigates the two-dimensional flow of a free surface of an inviscid and incompressible fluid over a non-smooth bottom. It considers the surface tension effect while neglecting gravity. The potential region in the W -plane is transformed onto the auxiliary λ -plane using the Schwarz-Christoffel transformation. The primary goal of this study is to offer an approximate solution to this problem employing the perturbation approach and the Hilbert transformation. The computations are conducted for high Weber numbers and small slopes of the bottom. Solutions are presented and illustrated up to the first-order approximation. Free-surface shapes for various Weber numbers and different small slopes are plotted.

KEY WORDS: Surface tension, conformal mapping, Weber number, Hilbert's transformation, perturbation technique.

1 INTRODUCTION

The phenomenon of fluid flow over irregular terrain with a free surface attracts considerable scholarly and practical interest. This category of phenomena, often known as Neumann-Kelvin problems, is characterized by the nonlinear boundary condition on the free-surface, as dictated by the Bernoulli equation. As the shape of the free-surface is undetermined, it must be considered when addressing the problem. For further information on this topic, refer to Bhimsen K. Shivamoggi [1] and Batchelor [2].

Several researchers have successfully applied the Hilbert method in combination with perturbation techniques to analyze free-surface flow problems. For example, Abd-el-Malek and Hanna [3] treated the flow over a ramp with gravity effects, Abd-el-Malek [4] provided an approximate solution for gravity-affected flow from a planar sluice gate, Masoud and Abd-el-Malek [5] investigated flows over irregular topography, Bounif and Gasmi [6, 7], and [8] determined the numerical solutions for free

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surface flow: over a step, over a trapezoidal obstacle and Past an Obstacle, as well as other configurations.

A related study by the authors Bounif and Gasmi [9] considered free-surface flow over a smooth sinusoidal obstacle. In contrast, the present work investigates flow over a non-smooth bottom configuration, leading to a different mathematical formulation and new analytical results.

In the present study, we investigate the two-dimensional flow of an inviscid, incompressible fluid over a non-smooth, assuming the flow is irrotational and steady. We consider the effects of surface tension while neglecting gravity. The aim of this study is to approximately solve the specified problem, characterized by a small slope and large Weber number. To this end, we employ the Hilbert method for a mixed boundary value problem within this transformed plane, thereby deriving a system of nonlinear integral equations. The resulting system is then solved using a perturbation technique. Our results demonstrate that the proposed methodology is efficient, mathematically tractable, and yields approximate solutions to these types of problems.

2 PROBLEM FORMULATION

Let us consider the two-dimensional flow of an incompressible and inviscid fluid over a descending step. The flow is uniform far upstream, with a constant velocity U_1 and depth h_1 . We take the point C as the origin in the (x', y') -plane, where the x' -axis extends from left to right and the y' -axis is vertical at point C (see Fig. 1).

We introduce the harmonic functions, the stream function ψ and the potential function ϕ . Then, for a two-dimensional, inviscid, irrotational, and incompressible

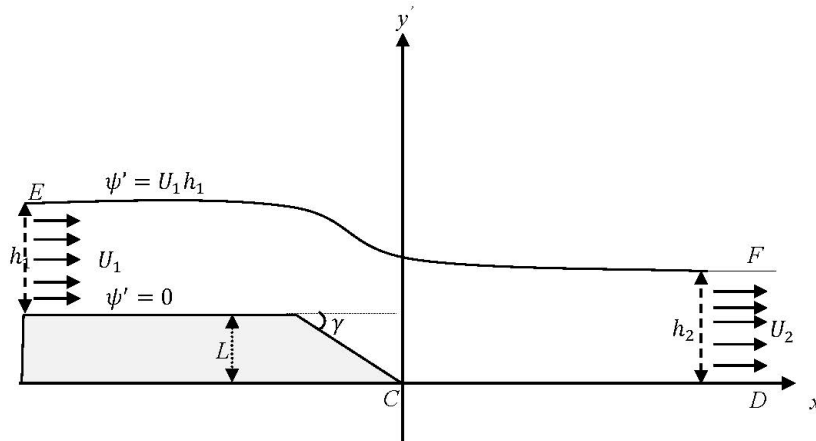


Figure 1. The physical plane of the flow $z' = x' + iy'$.

fluid, the velocity potential $\tilde{\phi}$ satisfies Laplace's equation:

$$(1) \quad \Delta \tilde{\phi} = 0.$$

Next, we introduce the complex potential function:

$$(2) \quad W' = \phi' + i\psi',$$

within the flow space, the function $W'(z')$ represents an analytical function of the complex variable $z' = x' + iy'$.

The x' and y' directions' velocity components are denoted by u' and v' , then, the complex conjugate velocity is:

$$(3) \quad \frac{dW'}{dz'} = u' - iv' = q'e^{-i\theta}.$$

When the pressure is uniform, Bernoulli's equation along the free surface EF has the following form:

$$(4) \quad \frac{1}{2}q'^2 + \frac{p'}{\rho} = \frac{1}{2}U_1^2 + \frac{p_0}{\rho},$$

where ρ is the fluid's density, p' is the fluid pressure and p_0 is the atmospheric pressure at the interior.

By applying the relationship between p' and p_0 , equation (4) becomes:

$$(5) \quad \frac{1}{2}q'^2 + \frac{T'}{\rho}\kappa' = \frac{1}{2}U_1^2,$$

$\kappa' = 1/R'$ denotes the free-surface's curvature.

Use the dimensionless analysis to retrieve this dimensionless equation.

3 CONFORMAL MAPPING

For simplicity, we non-dimensionalize the variables using h_1 and U_1 as the length unit and the velocity unit, respectively. The resulting dimensionless variables are defined as follows:

$$(6) \quad z = \frac{z'}{h_1} = \frac{x' + iy'}{h_1}; \quad W = \frac{W'}{U_1 h_1}; \quad q = \frac{q'}{U_1}; \quad R = \frac{R'}{h_1}; \quad \kappa = \kappa' h_1.$$

According to Gasmi [10], equation (5) can be written in dimensionless form:

$$(7) \quad q^2 + \frac{2}{\alpha} \left| \frac{\partial \theta}{\partial \phi} \right| q = 1,$$

where α is the Weber number defined by:

$$(8) \quad \alpha = \frac{\rho U_1^2 h_1}{T},$$

T is the surface tension.

By using (6), the complex conjugate velocity in (3) has the following form:

$$(9) \quad \zeta = \frac{dW}{dz} = \frac{1}{U_1} \frac{dW'}{dz'} = \frac{q'}{U_1} e^{-i\theta} = q e^{-i\theta}.$$

Moreover, when utilizing the logarithmic hodograph variable:

$$(10) \quad \omega = \ln \zeta = \ln q - i\theta,$$

from (9) and (10), we obtain:

$$(11) \quad z = \int e^{-\omega} dW.$$

We can set $\psi = 0$ on the streamline $ABCD$, $\psi = 1$ on the streamline EF can be inferred from the choice of dimensionless variables.

By definition, the domain of the flow in z -plane is transformed into an infinite band $0 \leq \psi \leq 1$ and $-\infty \leq \phi \leq +\infty$ by the potential function W (Fig. 2).

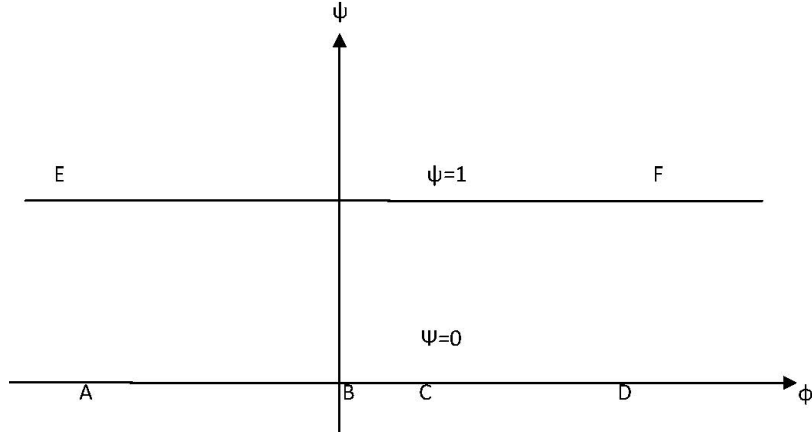
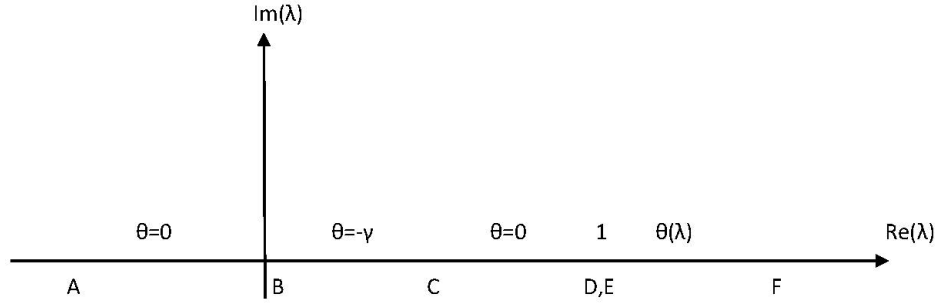


Figure 2. The potential W -plane.

Next, the flow space in the W -plane is mapped onto the upper half of the auxiliary λ -plane using the transformation of Schwarz-Christoffel (see Fig. 3).

Figure 3. The auxiliary λ -plane.

The obtained mapping function is

$$(12) \quad W(\lambda) = -\frac{1}{\pi} \ln(1 - \lambda).$$

This Table 1 shows the locations of the principal points of the flow domain in the W -plane and the λ -plane:

Table 1. The locations of the principal points of the flow domain in the W -plane and the λ -plane

Cartesian plane	W -plane	λ -plane
A	$W = -\infty$	$\lambda = -\infty$
B	$W = 0$	$\lambda = 0$
C	$W = \phi_C > 0$	$\lambda = \lambda_C > 0$
D	$W = +\infty$	$\lambda = 1$
E	$W = -\infty$	$\lambda = +\infty$
F	$W = +\infty$	$\lambda = 1$

4 NUMERICAL PROCEDURE

The Hilbert transformation is introduced for the mixed boundary-value problem defined on the λ -plane. Its general solution takes the form:

$$(13) \quad \chi(\lambda) = \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{Im[\chi(\tau)]}{\tau - \lambda} d\tau + \sum_{j=0}^{\infty} A_j \lambda^j,$$

where A_j and $p.v.$ represent real constants and the integral's principal value respectively.

To transform the function $\omega(\lambda)$ into $\chi(\lambda)$, we introduce an auxiliary function $H(\lambda)$. For the present problem, we define it as:

$$(14) \quad H(\lambda) = \begin{cases} \sqrt{1-\lambda}, & \lambda < 1, \\ -i\sqrt{\lambda-1}, & \lambda > 1, \end{cases}$$

with

$$(15) \quad \chi(\lambda) = \frac{\omega(\lambda)}{H(\lambda)} = \frac{\ln q(\lambda) - i\theta(\lambda)}{H(\lambda)} = R(\lambda) + iS(\lambda),$$

the values of θ are:

$$(16) \quad \theta(\lambda) = \begin{cases} 0, & \lambda < 0, \\ -\gamma, & 0 < \lambda < \lambda_c, \\ 0, & \lambda_c < \lambda < 1, \\ \theta(\lambda), & \lambda > 1. \end{cases}$$

Using (14), (15) and (16), we have:

$$(17) \quad Im\chi(\lambda) = \begin{cases} 0, & \lambda < 0, \\ \frac{\gamma}{\sqrt{1-\lambda}}, & 0 < \lambda < \lambda_c, \\ 0, & \lambda_c < \lambda < 1, \\ \frac{\ln q(\lambda)}{\sqrt{\lambda-1}}, & \lambda > 1. \end{cases}$$

Considering the upstream condition. Thus, (13) takes the form:

$$(18) \quad \chi(\lambda) = \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{Im[\chi(\tau)]}{\tau - \lambda} d\tau.$$

Using (15) and (18), the corresponding forms of $R(\lambda)$ and $S(\lambda)$ are:

$$(19) \quad R(\lambda) = \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{S(\tau)}{\tau - \lambda} d\tau,$$

and

$$(20) \quad S(\lambda) = \frac{-1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{R(\tau)}{\tau - \lambda} d\tau.$$

We obtain a system of integral equations by using (19) and (20). The equations below are all that we need for our study:

$$(21) \quad \theta(\lambda) = \frac{\sqrt{\lambda-1}}{\pi} \int_1^\infty \frac{\ln q(\tau)}{(\tau-\lambda)\sqrt{\tau-1}} d\tau + \frac{\gamma\sqrt{\lambda-1}}{\pi} \int_0^{\lambda_c} \frac{1}{(\tau-\lambda)\sqrt{1-\tau}} d\tau, \quad \lambda > 1,$$

and

$$(22) \quad \ln q_b(\lambda) = \frac{\sqrt{1-\lambda}}{\pi} \int_1^\infty \frac{\ln q(\tau)}{(\tau-\lambda)\sqrt{\tau-1}} d\tau + \frac{\gamma\sqrt{1-\lambda}}{\pi} \int_0^{\lambda_c} \frac{1}{(\tau-\lambda)\sqrt{1-\tau}} d\tau, \quad 0 < \lambda < \lambda_c.$$

By integration, (21) and (22) take the forms:

$$(23) \quad \theta(\lambda) = \frac{\sqrt{\lambda-1}}{\pi} \int_1^\infty \frac{\ln q(\tau)}{(\tau-\lambda)\sqrt{\tau-1}} d\tau + \frac{2\gamma}{\pi} \arctan \left(\frac{\sqrt{\lambda-1}(\sqrt{1-\lambda_c}-1)}{\lambda-1+\sqrt{1-\lambda_c}} \right), \quad \lambda > 1,$$

and

$$(24) \quad \ln q_b(\lambda) = \frac{\sqrt{1-\lambda}}{\pi} \int_1^\infty \frac{\ln q(\tau)}{(\tau-\lambda)\sqrt{\tau-1}} d\tau + \frac{\gamma}{\pi} \ln \left(\frac{(\sqrt{1-\lambda_c}-\sqrt{1-\lambda})(1+\sqrt{1-\lambda})}{(\sqrt{1-\lambda_c}+\sqrt{1-\lambda})(1-\sqrt{1-\lambda})} \right), \quad 0 < \lambda < \lambda_c.$$

Using (11) and (12) to find the coordinates (x, y) , we get:

$$(25) \quad x(\lambda) = x_\infty - \frac{1}{\pi} \int_\lambda^\infty \frac{\cos \theta(\tau)}{(1-\tau)q(\tau)} d\tau, \quad \lambda > 1,$$

$$(26) \quad y(\lambda) = 1 + L - \frac{1}{\pi} \int_\lambda^\infty \frac{\sin \theta(\tau)}{(1-\tau)q(\tau)} d\tau, \quad \lambda > 1.$$

Along BC , we have ($0 < \lambda < \lambda_c$, $q = q_b$ and $\theta = -\gamma$), then we obtain:

$$(27) \quad L = \frac{1}{\pi} \int_0^{\lambda_c} \frac{\sin \gamma}{(1-\tau)q_b(\tau)} d\tau.$$

Now, the problem is completely described by (7), (23), (25), (26) and (27), which will be solved in the following section.

5 NUMERICAL SOLUTION

5.1 APPROXIMATE FORMULATION OF THE PROBLEM

The solution of (7) is provided, when the Weber number is high and first-order Taylor expansion is applied, the result is:

$$(28) \quad q(\lambda) \approx 1 - \frac{1}{\alpha} \left| \frac{\partial \theta}{\partial \phi} \right|,$$

using (12), for $\lambda > 1$ we obtain:

$$(29) \quad q(\lambda) \approx 1 - \frac{\pi}{\alpha} (\lambda - 1) \left| \theta'(\lambda) \right|,$$

therefore, we approximate $\ln q(\lambda)$ and $\frac{1}{q(\lambda)}$ by:

$$(30) \quad \ln q(\lambda) \approx -\frac{\pi}{\alpha} (\lambda - 1) \left| \theta'(\lambda) \right|,$$

and

$$(31) \quad \frac{1}{q(\lambda)} \approx 1 + \frac{\pi}{\alpha} (\lambda - 1) \left| \theta'(\lambda) \right|,$$

substituting (30) in (23), we get the approximate equation of the slope angle of the free-surface relative to the horizontal:

$$(32) \quad \begin{aligned} \theta(\lambda) \approx & \frac{-\sqrt{\lambda-1}}{\alpha} \int_1^\infty \frac{\sqrt{\tau-1}}{\tau-\lambda} \theta'(\tau) d\tau \\ & + \frac{2\gamma}{\pi} \arctan \left(\frac{\sqrt{\lambda-1}(\sqrt{1-\lambda_c}-1)}{\lambda-1+\sqrt{1-\lambda_c}} \right), \quad \lambda > 1. \end{aligned}$$

Small inclination angles cause small variations in θ , As a result, θ approximates $\sin \theta$, while $\cos \theta = 1$.

By applying equation (31) and performing the necessary simplifications, the free-surface equations (25) and (26) can be written as:

$$(33) \quad x(\lambda) \approx x_\infty - \frac{1}{\pi} \int_\lambda^\infty \frac{1}{1-\tau} d\tau - \frac{\theta(\lambda)}{\alpha}, \quad \lambda > 1,$$

$$(34) \quad y(\lambda) \approx 1 + L - \frac{1}{\pi} \int_\lambda^\infty \frac{\theta(\tau)}{1-\tau} d\tau - \frac{\theta^2(\lambda)}{2\alpha}, \quad \lambda > 1.$$

By using (27), we obtain:

$$(35) \quad L \approx \frac{\gamma}{\pi} \int_0^{\lambda_c} \frac{1}{(1-\tau)q_b(\tau)} d\tau.$$

The perturbation method is used to solve this resulting system.

5.2 PERTURBATION TECHNIQUE

We express $q(\lambda)$, $\theta(\lambda)$, $x(\lambda)$ and $y(\lambda)$ as asymptotic expansions in terms of the perturbation parameter γ :

$$(36) \quad q(\lambda) = q_0(\lambda) + \gamma q_1(\lambda) + \gamma^2 q_2(\lambda) + \dots,$$

$$(37) \quad \theta(\lambda) = \theta_0(\lambda) + \gamma \theta_1(\lambda) + \gamma^2 \theta_2(\lambda) + \dots,$$

$$(38) \quad x(\lambda) = x_0(\lambda) + \gamma x_1(\lambda) + \gamma^2 x_2(\lambda) + \dots,$$

$$(39) \quad y(\lambda) = y_0(\lambda) + \gamma y_1(\lambda) + \gamma^2 y_2(\lambda) + \dots$$

Upon substitution of the expansions (36)-(39) into the equations (29), (32), (33) and (34) respectively, and by equating terms of equal powers of γ , the zeroth and first-order approximation for $\lambda > 1$ take the following form:

- **Zeroth-order approximation**

$$(40) \quad q_0(\lambda) \approx 1 - \frac{\pi}{\alpha}(\lambda - 1)\theta'_0(\lambda),$$

$$(41) \quad \theta_0(\lambda) \approx -\frac{\sqrt{\lambda-1}}{\alpha} \int_1^\infty \frac{\sqrt{\tau-1}}{\tau-\lambda} \theta'_0(\tau) d\tau,$$

$$(42) \quad x_0(\lambda) \approx x_\infty - \frac{1}{\pi} \int_\lambda^\infty \frac{1}{1-\tau} d\tau - \frac{\theta_0(\lambda)}{\alpha},$$

$$(43) \quad y_0(\lambda) \approx 1 + L - \frac{1}{\pi} \int_\lambda^\infty \frac{\theta_0(\tau)}{1-\tau} d\tau - \frac{\theta_0^2(\lambda)}{2\alpha},$$

- **First-order approximation**

$$(44) \quad q_1(\lambda) \approx -\frac{\pi}{\alpha}(\lambda - 1)\theta'_1(\lambda),$$

$$(45) \quad \theta_1(\lambda) \approx -\frac{\sqrt{\lambda-1}}{\alpha} \int_1^\infty \frac{\sqrt{\tau-1}}{\tau-\lambda} \theta'_1(\tau) d\tau + \frac{2}{\pi} \arctan \left(\frac{\sqrt{\lambda-1}(\sqrt{1-\lambda_c}-1)}{\lambda-1+\sqrt{1-\lambda_c}} \right),$$

$$(46) \quad x_1(\lambda) \approx -\frac{1}{\alpha} \theta_1(\lambda),$$

$$(47) \quad y_1(\lambda) \approx -\frac{1}{\pi} \int_\lambda^\infty \frac{\theta_1(\tau)}{1-\tau} d\tau - \frac{\theta_1^2(\lambda)}{2\alpha}.$$

5.3 SOLUTION OF THE PROBLEM

For $\lambda > 1$, the zeroth-order solution is given by:

Considering the direction of velocity the horizontal orientation:

$$(48) \quad \theta_0(\lambda) \approx 0,$$

substituting this value in (40), we obtain the flow velocity:

$$(49) \quad q_0(\lambda) \approx 1,$$

the free streamline equations:

$$(50) \quad x_0(\lambda) \approx x_\infty - \frac{1}{\pi} \int_\lambda^\infty \frac{1}{1-\tau} d\tau,$$

and we have the formula:

$$(51) \quad x_\infty = \frac{1}{\pi} \int_{\lambda_c}^\infty \frac{1}{1-\tau} d\tau,$$

implies that

$$(52) \quad x_0(\lambda) \approx \frac{-1}{\pi} \ln \left(\frac{\lambda-1}{1-\lambda_c} \right),$$

and

$$(53) \quad y_0(\lambda) \approx 1 + L.$$

For high Weber number α , the first term in (45) and the second term in (47) can be neglected. Therefore, for $\lambda > 1$, the first-order solution is obtained by using the zeroth-order solution:

$$(54) \quad q_1(\lambda) \approx \frac{1}{\pi} \frac{(\sqrt{1-\lambda_c}-1)(\sqrt{1-\lambda_c}+1-\lambda)}{\lambda\sqrt{\lambda-1}(\lambda-\lambda_c)},$$

$$(55) \quad \theta_1(\lambda) \approx \frac{2}{\pi} \arctan \left(\frac{\sqrt{\lambda-1}(\sqrt{1-\lambda_c}-1)}{\lambda-1+\sqrt{1-\lambda_c}} \right),$$

$$(56) \quad x_1(\lambda) \approx \frac{-2}{\pi\alpha} \arctan \left(\frac{\sqrt{\lambda-1}(\sqrt{1-\lambda_c}-1)}{\lambda-1+\sqrt{1-\lambda_c}} \right),$$

$$(57) \quad y_1(\lambda) \approx \frac{4(1-\sqrt{1-\lambda_c})}{\pi^2\sqrt[4]{1-\lambda_c}} \left(\arctan \left(\frac{\sqrt{\lambda-1}}{\sqrt[4]{1-\lambda_c}} \right) - \frac{\pi}{2} \right).$$

For large Weber number α , we get:

$$(58) \quad \ln q_b(\lambda) \approx \frac{\gamma}{\pi} \ln \frac{(\sqrt{1-\lambda_c} - \sqrt{1-\lambda})(1 + \sqrt{1-\lambda})}{(\sqrt{1-\lambda_c} + \sqrt{1-\lambda})(1 - \sqrt{1-\lambda})}, \quad 0 < \lambda < \lambda_c,$$

the height L is:

$$(59) \quad L \approx \frac{-\gamma}{\pi} \ln(1 - \lambda_c).$$

Using an approximate first-order solution, we arrive at:

$$(60) \quad q(\lambda) \approx 1 + \frac{\gamma}{\pi} \frac{(\sqrt{1-\lambda_c} - 1)(\sqrt{1-\lambda_c} + 1 - \lambda)}{\lambda\sqrt{\lambda-1}(\lambda - \lambda_c)},$$

$$(61) \quad \theta(\lambda) \approx \frac{2\gamma}{\pi} \arctan \left(\frac{\sqrt{\lambda-1}(\sqrt{1-\lambda_c} - 1)}{\lambda - 1 + \sqrt{1-\lambda_c}} \right),$$

$$(62) \quad x(\lambda) \approx \frac{-1}{\pi} \ln \left(\frac{\lambda - 1}{1 - \lambda_c} \right) - \frac{2\gamma}{\pi\alpha} \arctan \left(\frac{\sqrt{\lambda-1}(\sqrt{1-\lambda_c} - 1)}{\lambda - 1 + \sqrt{1-\lambda_c}} \right),$$

$$(63) \quad y(\lambda) \approx 1 + L + \frac{4\gamma(1 - \sqrt{1-\lambda_c})}{\pi^2 \sqrt[4]{1-\lambda_c}} \left(\arctan \left(\frac{\sqrt{\lambda-1}}{\sqrt[4]{1-\lambda_c}} \right) - \frac{\pi}{2} \right).$$

6 NUMERICAL RESULTS

The flow configuration, as shown in Fig. 1, models flow over a descending step. Using the previous method and after calculations, the shape of the free-surface shape for fixed values of the Weber number $\alpha = 1000$, the angle $\gamma = \pi/8$ and the height $L = 0.05$ is shown in Fig. 4.

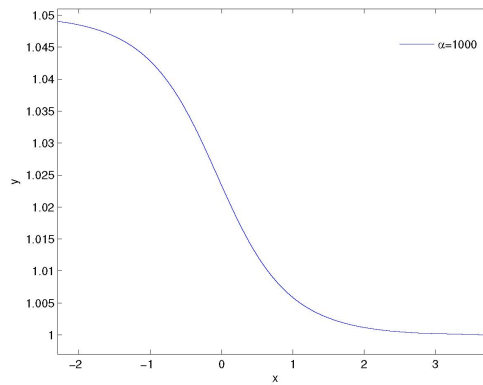


Figure 4. The free-surface shape for the values: $\alpha = 1000$, $\gamma = \pi/8$ and $L = 0.05$.

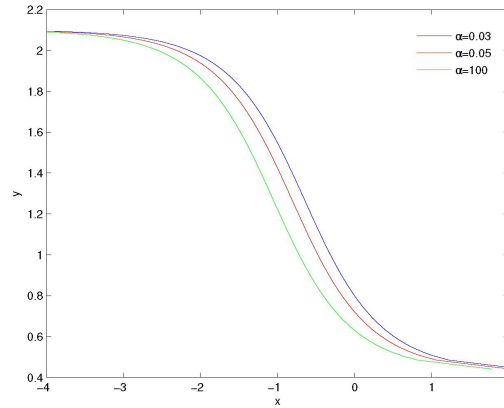


Figure 5. The effects of the Weber number on the shape of the free-surface for the values: $\gamma = \pi/6$ and $L = 1.1$.

The free-surface shapes of the flow are plotted in Fig. 5, for different values of the Weber number α , fixed values of $\gamma = \pi/6$ and $L = 1.1$.

Figure 5 illustrates the effect of different values of the Weber number α on the free-surface profiles. It can be noted that as surface tension increases (i.e., as the Weber number decreases), the curvature of the free-surface decreases, the curvature of the free-surface decreases. This behavior is physically expected, since surface tension tends to smooth the interface and reduce sharp variations

Figure 6 illustrates the influence of the inclination angle γ on the free-surface profile for $\alpha = 10000$ and $L = 0.2$. It is observed that variations in γ significantly

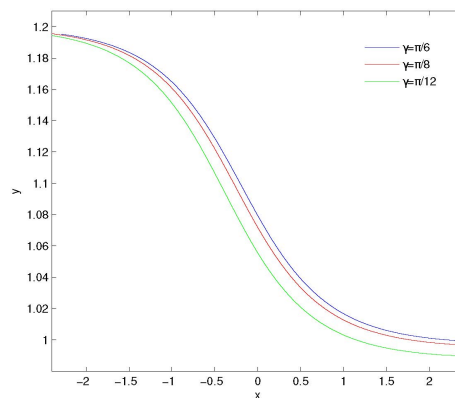


Figure 6. The effects of the angle γ on the shape of the free-surface for the values: $\alpha = 10000$ and $L = 0.2$.

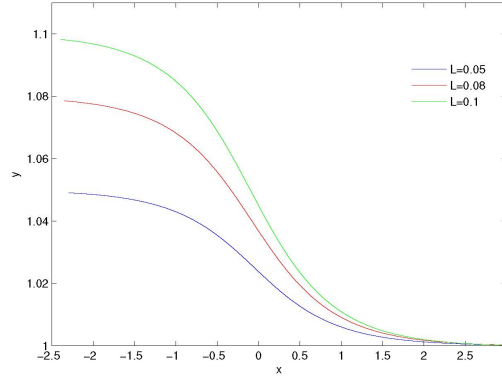


Figure 7. The effects of the depth L on the shape of the free-surface for the values: $\alpha = 10000$ and $\gamma = \pi/6$.

alter the curvature and shape of the free-surface, indicating the strong dependence of the flow structure on the bottom inclination.

Figure 7 illustrates the effect of different depths L on the free-surface profile. It demonstrates that increasing the depth modifies the amplitude and shape of the free surface, highlighting the influence of the step height on the overall flow behavior.

Overall, the numerical results demonstrate that the free-surface profile is strongly influenced by the Weber number, the inclination angle, and the depth parameter. The obtained solutions confirm the physical consistency of the model and illustrate the effectiveness of the proposed analytical approach in capturing the main features of free-surface flows over non-smooth bottoms.

7 CONCLUSION

The study formulates the flow over a descending step as a system of nonlinear integral equations and uses perturbation techniques to derive approximate solutions at high Weber numbers. The results illustrate the free-surface profiles and show the impact of parameters like inclination angle, height, and Weber number on the flow. The findings highlight the effectiveness of the proposed scheme and provide valuable insights into flow characteristics, demonstrating how these parameters influence the free-surface profiles.

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