

FREE VIBRATION OF BIO-INSPIRED HELICOIDAL COMPOSITE BEAMS WITH EXPONENTIALLY GRADED LAYERS

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ABSTRACT: This paper presents the free vibration behaviour of bio-inspired helicoidally symmetric laminated composite (BIHSLC) beams using higher-order shear deformation theory (HSDT) via the Ritz method. The stiffness and mass matrices are formulated through a variational approach, and the governing equations of motion are derived accordingly. The developed model is validated against existing results. A comprehensive study is conducted to investigate the influence of lamination schemes, number of layers, boundary conditions, grading parameters, aspect ratios, and orthotropy ratios. Notably, the fundamental frequencies exhibit significant variation depending on the specific lamination scheme, boundary conditions, grading parameters, number of layers, and aspect ratio. The novel insights presented in this study contribute to the understanding and future design of advanced composite beam structures.

KEY WORDS: Bio-inspired laminated composite, composite beams, Higher order theory, Ritz method; Grading parameter; free vibration analysis.

1 INTRODUCTION

Industries such as aerospace, defence, transportation, and civil engineering increasingly demand materials with superior mechanical properties, and bio-inspired laminated composites-mimicking layered natural architectures such as shells, bones, and wood have emerged as promising candidates offering strength, toughness, and light weight [1]. Helicoidal arrangements in particular enhance impact resistance and damage tolerance. Sandwich structures with honeycomb cores have been widely studied for their natural geometric efficiency [2–4], while Bouligand [5] first introduced the helicoidal structure concept, later investigated by Archipatbut and RaviChandar [6]

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and shown by Karlsson et al. [7] to offer significant mechanical improvements, including reduced through-thickness damage under impact [8]-leading to applications in tanks, aircraft carriers, and turbine blades.

Advanced theories have furthered laminate analysis: Khare et al. [9] proposed a shear-correction-free NEHSDT for multi-layered laminates, and Rodrigues et al. [10] coupled Radial Point Interpolation Method with higher-order theories for cross-ply plates. In vibration/stability research, Jafari and Mohammadimehr [11] studied piezoelectric-controlled sandwich beams, Akbaş [12] examined size-dependent buckling via modified couple stress theory, and Soltani [13] compared CARALL and GLARE fibre-metal laminates. Yüksel and Akbaş [14] assessed porosity effects on buckling, while Zenkour and El-Shahrany [15, 16] studied magnetostrictive and velocity-feedback vibration control. Bouazza and Zenkour [17] developed a refined nth-order theory for inhomogeneous plates, Santana et al. [18] proposed a unified spectral-element ESL theory, and Shishesaz et al. [19] modelled post-buckling of delaminated composite plates.

Helicoidal bio-inspired structures have advanced further: Karamanli et al. [20] and Bayat et al. [21] analysed bending, buckling, and vibration via Ritz and FSDT-FEM; Garg et al. [22–25] extended this to Miura-shells, panels, and perforated plates using iso-geometric/surrogate modelling; Garg et al. [26] studied impact damage via SPH-FEM; Karamanli et al. [27] developed quasi-3D FEM models; and Ding et al. [28] introduced self-stitching CFRP laminates. Manufacturing advances include 3D-printed helicoidal laminates [29], nacre-inspired CFRPs [30], self-healing [31] and fibre-reorientation [32] capabilities, HSDT-IGA dynamic analysis [33], interlaminar-stress studies of curved beams [34], and further reviews and designs [32, 35, 36].

Machine learning has aided optimization via active learning [37], SVM/surrogate models [24, 38], and studies on corrugated-core sandwich fracture [39], helicoidal plate responses via FSDT-2D-DQM [40], impact behaviour of ply-angle variants [41], biomimetic beetle-elytron sandwich cores [42], and thermoelastic HSDT-IGA models for CNT-reinforced TPMS structures [43].

To the authors' knowledge, free vibration of bio-inspired laminated beams with graded thickness remains unreported. This study therefore develops a novel HSDT to analyse BIHSLC beams, examining boundary conditions, lamination patterns, orthotropy ratios, and aspect ratios via a detailed parametric study with exponentially graded layer thickness.

2 GEOMETRIC MODEL AND MATERIAL PROPERTIES

A bio-inspired helicoidal symmetric laminated composite (BIHSLC) beam is depicted in Fig. 1. In this study, four different helicoidal designs, namely the helicoidal-recursive (HECR), helicoidal exponential (HECE), helicoidal semi-circular (HECSS) and Fibonacci helicoidal (FIBHEC) are analysed along with the variation of thickness

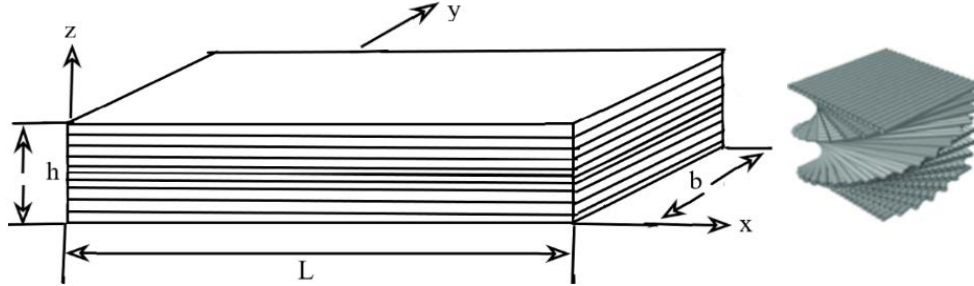


Figure 1. BIHSLC beam with its geometric description and lamination scheme.

of each layer using the exponential or power-law grading as given in Eq. (1). An isometric view of these configurations is given in Refs. [20, 33], which are summarized in Table 1.

Table 1. The configuration of the helicoidal layup

Lamination scheme	Description	Lamination sequence for 32 layers
Helicoidal recursive	$\theta_i = \theta_1/\theta_2/\dots/\theta_N = \theta_{i-1} + \beta(i-1)$ if $\beta = 1$, HECR1; $\beta = 2$, HECR2; $\beta = 3$, HECR3; β : angle increment per layer, also called the helicoidal pitch increment, i : layer number (from 1 to N).	HECR1: [0/1/3/6/.../120] _s HECR2: [0/2/6/.../240] _s HECR3: [0/3/9/.../360] _s
Helicoidal exponential	$\theta_1/\theta_2/\dots/\theta_N = \gamma^i$; γ is the growth factor; i is the layer index start from 1. If $\gamma = 2$, HECE1; $\gamma = 2.5$, HECE2; $\gamma = 3$, HECE3	HECE1: [2/4/8/.../256] _{2s} HECE2: 2.5/6.3/.../1525.9] _{2s} HECE3: [3/9/27/.../6561] _{2s}
Helicoidal semi-circular	$\theta_i = \sqrt{\theta_{\max}^2 - \left[\frac{2\theta_{\max}}{N-1}(i-1) - \theta_{\max} \right]^2}$ if $\theta_{\max} = 45^\circ$, HECSS1, $\theta_{\max} = 180^\circ$, HECSS2	HECSS1: [0/15.9/.../45] _s HECSS2: [0/63.6/.../180] _s
Fibonacci helicoidal	$\theta_i = \theta_{i-1} + \theta_{i-2}$; $\theta_1 = 0^\circ$, $\theta_2 = 10^\circ$ $\text{fib_seq}(i) = \text{mod}(\text{fib_seq}(i-1) + \text{fib_seq}(i-2), 360)$ Generates a Fibonacci-based helicoidal sequence of fiber orientations for the first half of a symmetric laminated composite beam. They wrap around 360° .	FIBBHEC: [0/10/10/.../340] _s

$$(1) \quad h_i = h \frac{e^{\alpha(i-1)}}{\sum_{j=1}^N e^{\alpha(j-1)}} .$$

Here α is the grading exponent, which controls the distribution profile of the layer thicknesses within the beam structure; h_i is the i^{th} layer thickness; h is the total thickness of beam; N represents the Number of Layers, which is set to 32 unless otherwise stated in this study.

While helicoidal angle variation redistributes directional stiffness without altering mass distribution, thickness grading offers a distinct mechanism: it redistributes material volume through the thickness, affecting both bending stiffness and mass moment of inertia. By shifting stiffness toward or away from the neutral axis, it controls bending–shear interaction directly—a negative exponent ($\alpha < 0$, thicker mid-plane plies) raises bending stiffness and frequencies, while a positive exponent ($\alpha > 0$, thicker outer plies) lowers the stiffness-to-mass ratio and frequencies, an effect helicoidal variation alone cannot achieve. This decoupling of stiffness alignment from mass concentration enables finer vibration tailoring. Natural Bouligand structures also show non-uniform lamella thickness alongside fiber rotation, so thickness grading improves biomimetic fidelity, justifying its inclusion as a novel, essential design parameter.

3 FORMULATION OF HSDT

The displacement field of the higher order shear deformation theory is expressed as

$$(2) \quad \begin{cases} U_x = u + \psi(z) w_{b,x} + (z + 2\psi(z)) w_{s,x} , \\ U_z = w_b + w_s , \end{cases}$$

where u is the axial displacement, w_b is the bending component of the transverse displacement, w_s is the shear component of the transverse displacement. In the present study $\psi(z) = \left(\frac{2z^5}{5h^4} - \frac{3z^3}{2h^2} \right)$ is considered and $(,)$ represents the derivative to the variable.

The displacement field adopted here follows the higher-order shear deformation theory (HSDT) framework, distinct from common shear- and normal-deformable beam theories, and aligns closely with the quasi-3D theory of Ref. [20] (used for validation), the refined shear deformation theory of Ref. [46], and the higher-order/quasi-3D theories of Ref. [47]. Building on these, the present field is formulated as a novel HSDT that captures transverse shear effects accurately without shear correction factors, with its accuracy confirmed against quasi-3D results in the literature.

Then the strain-displacement relations are defined by

$$(3) \quad \begin{cases} \varepsilon_{xx} = U_{x,x} = u_{,x} + \psi(z) w_{b,xx} + (z + 2\psi(z)) w_{s,xx}, \\ \varepsilon_{xz} = U_{x,z} + U_{z,x} = (1 + \psi(z)_{,z}) (w_{b,x} + 2w_{s,x}). \end{cases}$$

The stress-strain relations are in the form of:

$$(4) \quad \sigma_{xx} = \bar{Q}_{11} \varepsilon_{xx},$$

$$(5) \quad \sigma_{xz} = \bar{Q}_{55} \varepsilon_{xz},$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11} \cos^4(\theta) + 2(Q_{12} + 2Q_{66}) \cos^2(\theta) \sin^2(\theta) + Q_{22} \sin^4(\theta), \\ \bar{Q}_{55} &= Q_{55} \cos^2(\theta) + Q_{44} \sin^2(\theta), \\ Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}}; \quad Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}}; \quad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}; \\ Q_{44} &= G_{23}; \quad Q_{55} = G_{13}; \quad Q_{66} = G_{12}; \quad \nu_{21} = \frac{E_2}{E_1} \nu_{12}; \end{aligned}$$

θ is the fibre angle of the i^{th} layer.

The strain energy (U_s) of the BIHSLC beam is given by:

$$(6) \quad U_s = \frac{1}{2} \int_v (\sigma_{xx} \varepsilon_{xx} + \sigma_{xz} \varepsilon_{xz}) dv.$$

By substituting Eqs. (3–5) into Eq. (6), the strain energy equation can be expressed as:

$$(7) \quad U_s = \frac{1}{2} \int_v \bar{Q}_{11} \left\{ u_{,x}^2 + \psi(z)^2 w_{b,xx}^2 + (z + 2\psi(z))^2 w_{s,xx}^2 + 2\psi(z) u_{,x} w_{b,xx} \right. \\ \left. + 2\psi(z) (z + 2\psi(z)) w_{b,xx} w_{s,xx} + 2(z + 2\psi(z)) u_{,x} w_{s,xx} \right. \\ \left. + \bar{Q}_{55} (1 + \psi(z)_{,z})^2 [w_{b,x}^2 + 4w_{s,x}^2 + 4w_{b,x} w_{s,x}] \right\} dv.$$

The kinetic energy is calculated as follows:

$$(8) \quad U_s = \frac{1}{2} \int_v \rho \{ U_{x,t}^2 + U_{z,t}^2 \} dv \\ = \frac{1}{2} \int_v \rho \left\{ u_{,t}^2 + \psi(z)^2 w_{b,xt}^2 + (z + 2\psi(z))^2 w_{s,xt}^2 \right. \\ \left. + 2\psi(z) u_{,t} w_{b,xt} + 2\psi(z) (z + 2\psi(z)) w_{b,xt} w_{s,xt} \right. \\ \left. + 2(z + 2\psi(z)) u_{,t} w_{s,xt} + w_{b,t}^2 + w_{s,t}^2 + 2w_{b,t} w_{s,t} \right\} dv.$$

4 RITZ FORMULATION

Numerical solutions for the free vibration analysis of the bio-inspired helicoidal symmetric laminated composite beam, based on the developed variational formulation, are obtained using the Ritz method. This method has been acknowledged widely by researchers in the literature for its accuracy and computational efficiency in structural vibration problems [20, 44, 45]. The displacement functions $u(x)$, $w_b(x)$, and $w_s(x)$ are defined as follows:

$$(9) \quad u(x) = \sum_{j=1}^k A_j \phi_j(x), \quad \phi_j(x) = x^{p_u}(x-L)^{q_u} x^{j-1},$$

$$(10) \quad w_b(x) = \sum_{j=1}^k B_j \chi_j(x), \quad \chi_j(x) = x^{p_{w_b}}(x-L)^{q_{w_b}} x^{j-1},$$

$$(11) \quad w_s(x) = \sum_{j=1}^k C_j \Omega_j(x), \quad \Omega_j(x) = x^{p_{w_s}}(x-L)^{q_{w_s}} x^{j-1}.$$

The unknown coefficients are A_j , B_j and C_j , while the trial functions are given by $\phi_j(x)$, $\chi_j(x)$, and $\Omega_j(x)$. Based on the boundary conditions (BCs) listed in Table 2, the boundary exponents p_u , p_{w_b} , p_{w_s} , q_u , q_{w_b} , and q_{w_s} are provided in Table 3.

Table 2. Kinematic BCs used in this study

BC	$x = 0$	$x = L$
Simple support (SS)	$u = w_b = w_s = 0$	$w_b = w_s = 0$
Clamped-clamped (CC)	$u = w_b = w_s = w_{b,x} = w_{s,x} = 0$	$u = w_b = w_s = w_{b,x} = w_{s,x} = 0$
Clamped-free (CF)	$u = w_b = w_s = w_{b,x} = w_{s,x} = 0$	—
Clamped-simply (CS)	$u = w_b = w_s = w_{b,x} = w_{s,x} = 0$	$w_b = w_s = 0$

Table 3. Boundary exponents used to present various BCs

BC	Left end			Right end		
	p_u	p_{w_b}	p_{w_s}	q_u	q_{w_b}	q_{w_s}
SS	1	1	1	0	1	1
CC	1	2	2	1	2	2
CF	1	2	2	0	0	0
CS	1	2	2	0	1	1

Lagrange's equations are employed to derive the equations of motion:

$$(12) \quad \Pi = U - K,$$

$$(13) \quad \frac{\partial \Pi}{q_j} - \frac{\partial}{\partial t} \left(\frac{\partial \Pi}{\partial \dot{q}_j} \right) = 0,$$

where q_j represents the values of A, B, C, D . The equations of motion can be obtained as:

$$(14) \quad \left(\begin{bmatrix} [S_{11}] & [S_{12}] & [S_{13}] \\ [S_{12}]^T & [S_{22}] & [S_{23}] \\ [S_{13}]^T & [S_{32}] & [S_{33}] \end{bmatrix} - \omega^2 \begin{bmatrix} [M_{11}] & [M_{12}] & [M_{13}] \\ [M_{12}]^T & [M_{22}] & [M_{23}] \\ [M_{13}]^T & [M_{32}] & [M_{33}] \end{bmatrix} \right) \begin{Bmatrix} \{A\} \\ \{B\} \\ \{C\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{0\} \end{Bmatrix},$$

where $[S_{kl}]$ and $[M_{kl}]$ are the stiffness and the mass matrices. The elements of $[S]$ and $[M]$ are given as

$$(15) \quad \begin{aligned} S_{11}(ij) &= \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \bar{Q}_{11} dz \right) \phi_{i,x} \phi_{j,x} dx, \\ S_{12}(ij) &= \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \bar{Q}_{11} \psi(z) dz \right) \phi_{i,x} \chi_{j,xx} dx, \\ S_{13}(ij) &= \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \bar{Q}_{11} (z + 2\psi(z)) dz \right) \phi_{i,x} \Omega_{j,xx} dx, \\ S_{22}(i,j) &= \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \bar{Q}_{11} \psi(z)^2 dz \right) \chi_{i,xx} \chi_{j,xx} dx \\ &+ \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \bar{Q}_{55} (1 + \psi(z)_{,z})^2 dz \right) \chi_{i,x} \chi_{j,x} dx, \\ S_{23}(i,j) &= \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \bar{Q}_{11} \psi(z) (z + 2\psi(z)) dz \right) \chi_{i,xx} \Omega_{j,xx} dx \\ &+ 2 \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \bar{Q}_{55} (1 + \psi(z)_{,z})^2 dz \right) \chi_{i,x} \Omega_{j,x} dx, \\ S_{33}(i,j) &= \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \bar{Q}_{11} (z + 2\psi(z))^2 dz \right) \Omega_{i,xx} \Omega_{j,xx} dx \\ &+ 4 \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \bar{Q}_{55} (1 + \psi(z)_{,z})^2 dz \right) \Omega_{i,x} \Omega_{j,x} dx. \end{aligned}$$

$$\begin{aligned}
M_{11}(ij) &= \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \rho dz \right) \phi_i \phi_j dx, \\
M_{12}(ij) &= \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \rho \psi(z) dz \right) \phi_i \chi_{j,x} dx, \\
M_{13}(ij) &= \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \rho (z + 2\psi(z)) dz \right) \phi_i \Omega_{j,x} dx, \\
(16) \quad M_{22}(i, j) &= \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \rho \psi(z)^2 dz \right) \chi_{i,x} \chi_{j,x} dx \\
&\quad + \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \rho dz \right) \chi_i \chi_j dx, \\
M_{23}(i, j) &= \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \rho \psi(z) (z + 2\psi(z)) dz \right) \chi_{i,x} \Omega_{j,x} dx \\
&\quad + \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \rho dz \right) \chi_i \Omega_j dx, \\
M_{33}(i, j) &= \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \rho (z + 2\psi(z))^2 dz \right) \Omega_{i,x} \Omega_{j,x} dx \\
&\quad + \int_0^L \left(\sum_{k=1}^{NL} \int_{z_k}^{z_{k+1}} \rho dz \right) \Omega_i \Omega_j dx.
\end{aligned}$$

5 NUMERICAL RESULTS

To validate the proposed higher-order shear deformation theory, several comparative studies are conducted on both conventional and bio-inspired laminated composite beams. A comprehensive investigation and discussion are carried out to examine the effects of length-to-thickness ratios (L/h), lay-up sequences, and various boundary conditions, including simply supported (SS), clamped–clamped (CC), clamped–free (CF), and clamped–simply supported (CS) configurations. The mechanical properties of the constituent materials used in the laminated composites are presented in Table 4.

Results are portrayed in dimensionless form for fundamental frequency in Tables 5–7 and assessed with Karamanli et al. [20], Nguyen et al. [46], Thuc P. Vo et

Table 4. Material properties of composite beams

Material	E_1 GPa	E_2 GPa	G_{12} GPa	G_{13} GPa	G_{23} GPa	ν_{12}	ν_{13}	ν_{23}	ρ
1	Open	Open	E_2	$0.5E_2$	$0.5E_2$	$0.2E_2$	0.25	0.25	0.25
2	144.8	9.65	9.65	4.14	4.14	3.45	0.3	0.3	0.3
3	40	1	1	0.6	0.6	0.5	0.25	0.25	0.25

al. [47]. For convenience, the following dimensionless parameters are used:

$$(17) \quad \text{DFF}(\bar{\omega}) = \begin{cases} \frac{\omega L^2}{h} \sqrt{\frac{\rho}{E_1}} & \text{for Materials 1 and 2} \\ \frac{\omega L^2}{h} \sqrt{\frac{\rho}{E_2}} & \text{for Material 3} \end{cases}$$

From Tables 5 and 6, it can be observed that the comparative results of the proposed numerical model validate its accuracy. The present results show excellent agreement with those reported in the literature for analysing the free vibration characteristics of laminated composite beams using refined shear deformation theory (RSDT) [46], quasi-3D theory [20, 47], and higher-order beam theory (HBT) [47], across all boundary conditions (BCs).

The proposed formulation is validated against Ref. [20] under identical modeling conditions (beam geometry, material properties, lamination schemes, and SS/CC/CF/CS boundary conditions), using the dimensionless fundamental frequency (DFF) as a robust metric that emphasizes global stiffness and inertia consistency over minor kinematic differences. As shown in Table 7, results agree closely with the quasi-

Table 5. Verification of DFF for $(0^\circ/90^\circ)$ laminated composite beams under various BCs (Material 2)

L/h	Theory	SS	CC	CF
5	RSDT [46]	1.17220	1.98600	0.4492
	$\varepsilon_z \neq 0$ [47]	—	1.99350	—
	Quasi-3D [20]	1.16760	1.99460	0.4536
	Present $\varepsilon_z = 0$	1.16350	1.99879	0.45078
15	RSDT [46]	1.33140	2.84430	0.4792
	$\varepsilon_z \neq 0$ [47]	—	2.87090	—
	Quasi-3D [20]	1.33930	2.86930	0.4834
	Present $\varepsilon_z = 0$	1.33311	2.85488	0.48073

Table 6. Verification of DFF for $(0^\circ/90^\circ/0^\circ)$ and $(0^\circ/90^\circ)$ laminated composite beams under various BCs and L/h (Material 3)

L/h	Theory	SS		CF		CC	
		$0^\circ/90^\circ/0^\circ$	$0^\circ/90^\circ$	$0^\circ/90^\circ/0^\circ$	$0^\circ/90^\circ$	$0^\circ/90^\circ/0^\circ$	$0^\circ/90^\circ$
5	HBT [47]	9.206	6.058	4.23	2.381	11.601	10.022
	Quasi-3D [47]	9.208	6.075	4.221	2.391	11.479	9.99
	Present	9.2160	6.0744	4.2383	2.385	11.6899	10.0985
20	HBT [47]	16.327	7.204	6.02	2.289	29.643	15.65
	Quasi-3D [47]	16.328	7.228	6.03	2.589	29.639	15.708
	Present	16.3370	7.2103	6.0673	2.590	29.677	15.668

3D solutions of Ref. [20] across L/h ratios, boundary conditions, and stacking sequences, with differences confined to 4–6%; the slightly higher frequencies reflect HSDT's more accurate capture of transverse shear effects, confirming that validation relies on physically meaningful agreement rather than identical displacement fields. Robustness is further confirmed by analysing the DFF of BIHSLC beams for lamination schemes HECR3, HECSS2, and FIBHEC under all four boundary conditions and $L/h = 10$ and 20 ($E_1/E_2 = 3$). Table 7 shows HECSS2 consistently yields the lowest fundamental frequency and FIBHEC the highest across all cases, reinforcing the model's reliability and supporting its use as a benchmark for future validation studies.

5.1 PARAMETRIC STUDY OF DFF

After validation, a parametric study evaluates the dimensionless fundamental frequencies (DFF) of BIHSLC beams across lamination architecture, orthotropy ratio (E_1/E_2), thickness grading exponent (α), aspect ratio (L/h), and boundary conditions. Four helicoidal stacking sequences are considered-recursive (HECR), exponential (HECE), semi-circular (HECSS), and Fibonacci-inspired (FIBHEC)-under SS, CC, CF, and CS conditions. Results are presented in Tables A1–A4 and Figs. 2–4 (below) and A1–A12 (Appendix at: <https://github.com/bathinisiddareddy-arch/exponentially-graded-thickness>).

From Tables A1–A4, DFF consistently decreases with increasing α across all cases. Positive α (thinner edges, thicker centre) concentrates mass at the outer surfaces, reducing bending stiffness and the stiffness-to-mass ratio, lowering frequencies. Negative α (thicker edges, thinner centre) concentrates stiffness near the neutral axis where dynamic stresses are highest, raising bending stiffness and frequencies. $\alpha = 0$ denotes uniform thickness.

Orthotropy (E_1/E_2) also strongly affects DFF: increasing it from 3 to 50 mono-

Table 7. Verification of DFF for bio-inspired helicoidal symmetric lamination schemes, BCs, and L/h (Material 1)

BCs	Theory	$L/h = 10$			$L/h = 20$		
		HECR3	HECSS2	FIBHEC	HECR3	HECSS2	FIBHEC
SS	Quasi-3D [20]	2.3539	2.0698	2.388	2.4135	2.1029	2.4375
	Present	2.4748	2.1711	2.5317	2.5389	2.2086	2.5878
	Error	4.89%	4.67%	5.68%	4.94%	4.79%	5.81%
CC	Quasi-3D [20]	4.8274	4.4164	4.9894	5.3198	4.6928	5.4045
	Present	5.0607	4.6000	5.2547	5.5871	4.9141	5.7218
	Error	4.61%	3.99%	5.05%	4.78%	4.50%	5.55%
CF	Quasi-3D [20]	0.8541	0.7464	0.8638	0.864	0.7516	0.8719
	Present	0.8977	0.7829	0.9160	0.9088	0.7893	0.9256
	Error	4.86%	4.66%	5.70%	4.92%	4.77%	5.80%
CS	Quasi-3D [20]	3.5451	3.1647	3.6224	3.734	3.2678	3.7791
	Present	3.7216	3.3095	3.8292	3.9251	3.4278	4.0075
	Error	4.74%	4.38%	5.40%	4.87%	4.67%	5.70%

tonically decreases DFF, as stiffer longitudinal fibres dominate over less effective transverse fibres—an effect most pronounced at higher α . Low orthotropy ($E_1/E_2 = 3$) yields relatively uniform vibrational behaviour, while higher ratios increase sensitivity to fibre orientation and grading.

Helicoidal architecture effects depend on α and boundary conditions. Recursive schemes (HECR1–HECR3) give the highest DFF at non-positive α (especially under SS, CC, CS), owing to small pitch angles (e.g., 1° in HECR1) enabling quasi-isotropic, uniform stiffness—though performance drops at positive α . Exponential schemes (HECE1–HECE3) are adaptive: HECE1 excels at lower α , HECE3 at higher α . Semi-circular schemes (HECSS1, HECSS2) are more complex—HECSS2 (180° coverage) gives the lowest DFF at negative α /low anisotropy but performs exceptionally under CF at higher α , likely due to stiffness concentration near the clamped end. FIBHEC delivers consistent performance across all parameters due to its quasi-random stiffness distribution, suiting uncertain or multidirectional loading.

Boundary conditions follow the classical trend $CC > CS > SS > CF$, reflecting increasing constraint rigidity. Recursive stacking dominates under CC, CS, and SS, and remains competitive under CF, owing to its evenly distributed stiffness resisting both global and local deformation.

Overall, recursive stacking suits highly constrained systems; exponential and semi-circular configurations suit asymmetric or flexible supports; and Fibonacci-inspired

designs offer balanced performance for uncertain or variable loading—useful guidance for tailoring dynamic response in bio-inspired composite design.

The complete parametric dataset (32-layer BIHSLC beams, all nine schemes, three orthotropy ratios, seven grading exponents, all four boundary conditions) is provided as Tables A1–A4 in the Appendix.

Figures 2–4 show DFF versus L/h and α for SS, CC, and CS ($\beta = 1^\circ$). DFF

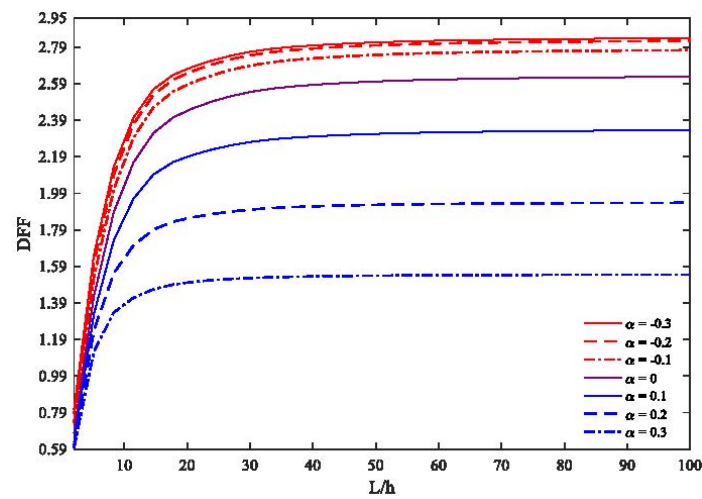


Figure 2. DFF of BIHSLC beams under simply supported BCs ($\beta = 1^\circ$), $E_1/E_2 = 25$.

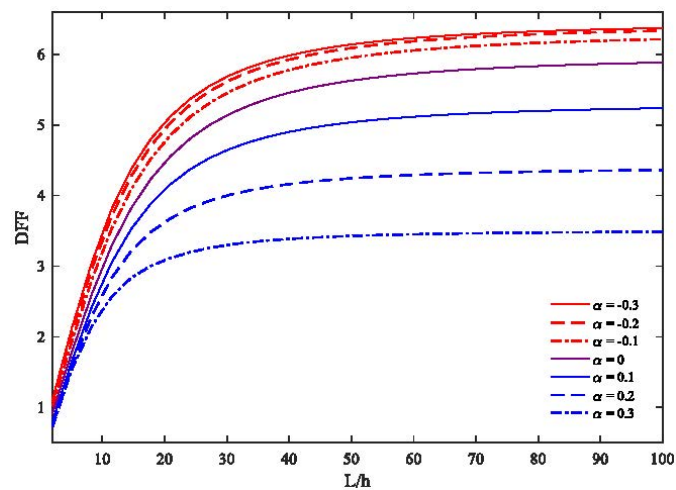


Figure 3. DFF of BIHSLC beams under clamped-clamped BCs ($\beta = 1^\circ$), $E_1/E_2 = 25$.

decreases with increasing α in all cases, more sharply in slender beams and at higher α , due to stiffness shifting away from the neutral axis. DFF rises rapidly with L/h up to 20 (shear-deformation dominated), then more slowly beyond $L/h = 20$ as behaviour transitions toward thin-beam response.

CC beams (Fig. 3) show markedly higher DFF than SS (Fig. 2) and CS (Fig. 4) due to stronger end constraints, with greater sensitivity to α and L/h . CS beams fall between SS and CC, with asymmetric constraints producing nonlinear α – L/h interactions; at negative α and low L/h , CS values approach CC, showing gradation can offset geometric/boundary asymmetry. Overall, boundary conditions rank $CC > CS > SS$ for vibrational stiffness, confirming that thickness gradation and geometric parameters must be jointly optimized for BIHSLC dynamic performance.

The mode shapes of bio-inspired helicoidal laminated composite beams, illustrated in Figs. A13–A16 (Appendix) for four boundary conditions, reveal distinct vibrational characteristics influenced by the helicoidal architecture and boundary constraints. In the simply supported case (Fig. A13), the mode shapes exhibit symmetric, sinusoidal patterns typical of classical beam theory, though slightly modified by the orthotropic helicoidal layering. The clamped-clamped beam (Fig. A14) shows higher stiffness and pronounced curvature near the ends, with multiple nodes and inflection points due to the restriction of both displacement and rotation. For the clamped-free configuration (Fig. A15), the mode shapes display increasing amplitude toward the free end, highlighting the cantilever behaviour with noticeable bending-torsion cou-

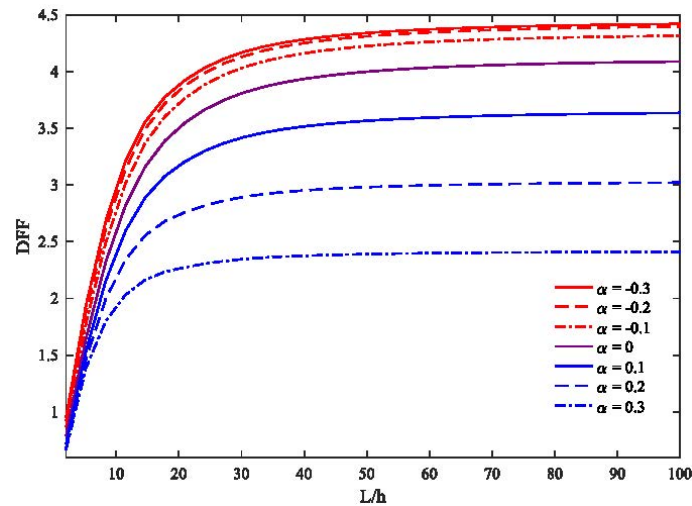


Figure 4. DFF of BIHSLC beams under clamped-simply supported BCs ($\beta = 1^\circ$), $E_1/E_2 = 25$.

pling induced by the helicoidal structure. Similarly, the clamped-simply supported beam (Fig. A16) presents asymmetric deformation patterns, with higher curvature concentrated near the clamped end, illustrating the influence of support conditions on modal behaviour. Across all cases, the exponential variation and helicoidal design introduce subtle asymmetries and energy localization, emphasizing their potential for tailored dynamic responses in advanced structural applications.

The variation of the DFF with the number of layers in BIHSLC beams is presented in Figs. A1–A12 (Appendix) for four different boundary conditions: SS, CC, CF, and CS, considering three thickness grading exponents $\alpha = -0.3, 0$, and 0.3 , along with various helicoidal lamination schemes HECR1, HECR2, HECR3, HECE1, HECE2, HECSS, and FIBHEC. The results clearly show that the DFF is influenced by changes in the number of layers. Across all boundary conditions, the DFF consistently decreases as the number of layers increases, which can be attributed to the fact that increasing the number of layers results in thinner individual plies while maintaining the overall beam thickness. This reduction in ply thickness reduces the global bending stiffness and, consequently, the natural frequencies of the beam. An exception to this trend is observed in the SS with $\alpha = -0.3, 0$ and 0.3 and at CC, CF, CS, condition $\alpha = -0.3$ for the HECSS scheme (see Appendix Figs. A1–A4, A7, A10), where the DFF exhibits a slight increase with layer count, indicating a potential redistribution of stiffness favouring the HECSS geometry under full-end constraints. Among the lamination schemes, HECR1 and HECR2 typically produce higher DFFs, especially under SS and CC boundary conditions (Appendix Figs. A1–A6), suggesting that these recursive stacking sequences provide better stiffness alignment. In contrast, FIBHEC and HECSS consistently yield lower frequency responses across all boundary conditions, likely due to their smoother fiber orientation transitions and reduced directional stiffness. The influence of the grading index α is relatively subtle, though in most cases, $\alpha = -0.3$ (indicating reduced thickness toward the outer fibers) results in slightly higher DFFs compared to $\alpha = 0$ and $\alpha = 0.3$, particularly under SS and CS conditions (Appendix Figs. A1–A3 and A10–A12). This may be attributed to a more favourable distribution of stiffness closer to the beam's mid-plane. The full set of mode-shape plots for the SS, CC, CF, and CS boundary conditions is provided as Figs. A13–A16 in the Appendix. The complete set of DFF-versus-number-of-layers plots for all four boundary conditions, three grading exponents, and nine helicoidal lamination schemes is provided as Figs. A1–A12 in the Appendix. Boundary conditions significantly influence the DFF trends: clamped (CC) beams exhibit the highest frequencies due to full-end restraint (Appendix Figs. A4–A6), followed by clamped–simply supported (CS) (Appendix Figs. A10–A12), simply supported (SS) (Appendix Figs. A1–A3), and finally clamped-free (CF) (Appendix Figs. A7–A9), which show the lowest DFFs as a result of minimal constraint. These observations

highlight the complex interplay between layer count, lamination architecture, thickness grading, and boundary support in governing the vibrational behaviour of BIHSLC beams. Notably, the study confirms that increasing the number of layers does not inherently enhance the dynamic stiffness of helicoidal laminates and that careful selection of the stacking scheme and grading index is essential. Overall, the findings suggest that the mechanical behaviour and frequency characteristics of BIHSLC beams can be effectively tailored by optimizing the number of layers and stacking strategy for specific design requirements.

6 CONCLUSIONS

This study presents a free vibration analysis of bio-inspired helicoidal symmetric laminated composite (BIHSLC) beams using the higher-order shear deformation theory (HSDT) and the Ritz method. The effects of various parameters, including boundary conditions, lamination schemes, orthotropy ratio, aspect ratio, and thickness grading exponents, are systematically investigated. Notably, this work is the first to report the free vibration behaviour of BIHSLC beams with thickness gradation. The results reveal that the dimensionless fundamental frequencies (DFF) are significantly influenced by the thickness grading exponents and the rotation angle of the helicoidal structure. Furthermore, it is demonstrated that the dynamic response of BIHSLC beams can be effectively controlled through strategic variation in stacking sequence, thickness gradation, and boundary conditions. These findings offer valuable insights for the advanced design and optimization of next-generation composite structures.

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