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FLUID MECHANICS

FLOW PROPERTIES IN THE PROBLEM OF SPOTS CHAIN WITH INTERVALS

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ABSTRACT: Authors solves the problem of spots chain dynamics with initial intervals of mixed liquid in stratified environment, and viscous flow using the numerical splitting method for studying incompressible fluid flows (SMIF method). The SMIF method is characterized by minimal scheme dissipation and dispersion. The finite difference scheme has the second order of approximation in terms of spatial variables. The fields of the stream function are obtained, which are used to study the spots chain dynamics using the example of characteristic variant with given parameters of the values of the Reynolds and Froude numbers and initial intervals between spots.

KEY WORDS: Incompressible viscous fluid, stratification, salinity, stream function, Boussinesq approximation. Splitting Method for Incompressible Fluid flows (SMIF).

1 INTRODUCTION

In this paper, we consider the problem of numerical simulation of mixed fluid spots dynamics in stratified medium, and viscous flow. Atmospheric and oceanic processes in inhomogeneous media are studied. In particular, the process of overturning of internal waves, as a result of which, under the action of a force, the accumulation of mixed liquid increases. This paper found studies on spots chain dynamics of mixed liquid. The density of ocean water depends on temperature, pressure and salinity and changes vertically. The instantaneous distribution of hydro physical parameters (density, temperature, salinity) over depth is of a stepwise nature: there are areas where hydrodynamic characteristics are constant, and areas where it replaces by areas with large gradients. This is due to the fact that in a turbulent flow with a strongly stable stratification, turbulence is spread in spots. The heterogeneous and highly anisotropic turbulence character under conditions of strong stable stratification was

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predicted by A. N. Kolmogorov in the late 1940s. The existence of flat turbulence spots in the ocean was pointed out in [1]. Spots of mixed liquid are a consequence of the interaction of internal waves, which, under certain conditions, can overturn. Turbulence is concentrated in flat layers - spots extending horizontally over distances significantly exceeding their vertical dimensions [2–5]. These flat spots turn out to be sharply limited and long-lasting. Therefore, the emergence and development of mixed fluid spots in a stratified environment is of significant interest in connection with the ocean structure study. The spots evolve, gradually flattening and intruding into the environment with tongues - intrusions. The liquid mixing in the slick creates in it an excess pressure in comparison with the environment, which generates the intrusion driving force. Under the influence of this force, blurring (collapse) of the spot occurs.

The problem of studies on spots chain dynamics of mixed liquid with intervals is of interest both for theorists and for experimenters.

2 Statement of the Problem

For mathematical modeling of the dynamics of a chain of spots with discontinuities, we consider the plane unsteady problem of the flow arising from the collapse (collapse along the vertical direction) of a homogeneous fluid region A surrounded by a fluid that is stably and continuously stratified in density (for certainty, let the according to a linear law), Fig. 1.

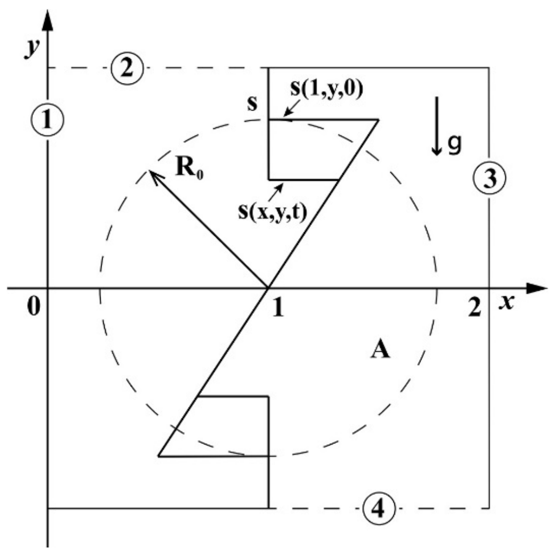

Fig. 1: Initial $s(1, y, 0)$ and stabilized $s(x, y, t)$ salinity perturbation fields.

The flow develops in a uniform gravity field with the acceleration of gravity $\boldsymbol{g}$. Unperturbed linear density distribution:

$$\rho(x,\, y) = \rho_0(1 - \frac{y}{\Lambda} + s(x,y)) \tag{1}$$

is characterized by the stratification scale

$$\Lambda = \Big|\frac{1}{\rho_0}\Big(\frac{\partial\rho}{\partial y}\Big)\Big|^{-1}, \qquad a = \frac{1}{\rho_0}\Big(\frac{\partial\rho}{\partial y}\Big)_0 < 0\,,$$

buoyancy frequency $N = \sqrt{g/\Lambda}$, buoyancy period $T_b = 2\pi/N$; $R_0 < 1$ – spot radius at the initial moment of time, s – salinity perturbation (stratification component), includes salt ratio of compression.

Choosing as a characteristic linear dimension: spot radius R_0 at time $t = 0$, characteristic density ρ_0 in the spot at the initial moment of time, characteristic time N^{-1}, we pass to dimensionless variables:

$$\begin{array}{llll} \tilde f = (\tilde x,\, \tilde y,\, \tilde t,\, \tilde u,\, \tilde v,\, \tilde p,\, \tilde\rho), & x = \tilde x R_0\,, & y = \tilde y R_0\,, & t = \tilde t/N\,, \\ u = \tilde u R_0 N\,, & v = \tilde v R_0 N\,, & p = \tilde p \rho_0 R_0^2 N^2\,, & \rho = \tilde\rho\rho_0\,. \end{array}$$

We will also assume that p is the pressure minus the hydrostatic one. Then the Navier-Stokes equations, initial and boundary conditions take the following form in dimensionless variables in the Boussinesq approximation (the tilde is omitted):

$$\begin{aligned} &\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\nabla p + \frac{1}{Re}\Delta\boldsymbol{v} + \frac{1}{Fr} s\,\frac{\boldsymbol{g}}{g}\,,\\ &\frac{\partial s}{\partial t} + (\boldsymbol{v}\cdot\nabla)s = \frac{1}{Sc\cdot Re}\Delta s + \frac{\nu}{C}\,,\\ &\nabla\cdot\boldsymbol{v} = 0\,,\\ &u = \nu = 0\,, \qquad (x,y) \in \mathbf{R}^2\,,\\ &\rho = 1\,, \qquad (x,y) \in A\,,\\ &\rho = 1 - \frac{y}{C} + s\,, \qquad (x,y) \in \mathbf{R}^2\backslash A\,,\\ &s = \begin{cases} \dfrac{y}{C}\,, & (x,y) \in A \\ 0\,, & (x,y) \in \mathbf{R}^2\backslash A \end{cases}\,. \end{aligned} \tag{2}$$

Here $Re = \rho_0 R_0^2 N/\mu$ is the Reynolds number, $Fr = R_0 N^2/g$ – the Froude number, $Sc = \mu/\rho_0 k_s$ – the Schmidt number, k_s – the diffusion coefficient of salts, μ – dynamic viscosity coefficient, $C = \Lambda/R_0$ – scale ratio.

The solution to the problem will search in the rectangular area $\{x, y : 0 \leq x \leq 2, -Y \leq y \leq Y\}$. On the left boundary (line 1 in Fig. 1) and the right boundary (line 3 in Fig. 1) of this region, the conditions for the periodicity of the flow are set. The upper (line 2 in Fig. 1) and lower (line 4 in Fig. 1) boundaries should be chosen at a sufficiently large distance from the disturbance source (from the spot) so that the setting of any boundary conditions at these boundaries, necessary for solving the problem, does not have a significant effect on fluid flows.

3 Solution Method

To solve the problem, we used the Splitting method for studying incompressible fluid flows (SMIF method) [6]. The finite-difference scheme has such properties as the second order approximation in spatial variables, minimal scheme dissipation and dispersion, performance in a wide range of Reynolds and Froude numbers and, most importantly, in the study of wave processes, the property of monotony. The splitting scheme is as following:

$$\frac{\tilde{\boldsymbol{v}} - \boldsymbol{v}^n}{\tau} = -(\boldsymbol{v}^n \cdot \nabla)\boldsymbol{v}^n + \frac{1}{Re}\Delta \boldsymbol{v}^n + \frac{1}{Fr} s^n \frac{\boldsymbol{g}}{g}\,, \tag{3}$$

$$\tau \Delta p = \nabla \cdot \tilde{\boldsymbol{v}}\,, \tag{4}$$

$$\frac{\boldsymbol{v}^{n+1} - \tilde{\boldsymbol{v}}}{\tau} = -\nabla p\,, \tag{5}$$

$$\frac{s^{n+1} - s^n}{\tau} = -(\boldsymbol{v}^{n+1} \cdot \nabla)s^n + \frac{1}{Sc \cdot Re}\Delta s^n + \frac{\nu^{n+1}}{C}\,. \tag{6}$$

At the beginning the salinity perturbation is calculated (6). At Stage I (3), it is assumed that the transfer of momentum occurs only due to convection, diffusion, and buoyancy forces. At Stage II (4), given the intermediate velocity field $\tilde{\boldsymbol{v}}$ found earlier and taking into account the solenoidality of the velocity vector $\boldsymbol{v}^{n+1}$, the pressure field is found by solving the Poisson equation (over-relaxation method is used here). At Stage III (5), it is assumed that the transfer of momentum occurs only due to the pressure gradient. In Stage IV (6) on found velocity field $\boldsymbol{v}^{n+1}$ the salinity perturbation is calculated.

Finite-difference scheme. The flow region under examination is covered by grid of cells (the grid is uniform with respect to x and y)

$$\Omega = \begin{cases} x_{i+1/2} = ih_x\,, & h_x > 0\,, \quad i = 0, 1, \ldots, L\,; \quad Lh_x = X\,, \\ y_{j+1/2} = jh_y\,, & h_y > 0\,, \quad j = 0, 1, \ldots, M\,; \quad Mh_y = 2Y, \end{cases}$$

where h_x and h_y are the grid sizes, and L and M are the numbers of grid cells in the directions x and y, respectively (the point with the coordinates $(i,\, j)$ coincides with the cell center). Here, as in the original splitting method, the staggered grid is used. This allows us to visually interpret each cell as an element of the medium volume; such an element is characterized by the pressure $p_{i,j}$, (and possibly by the temperature, energy, *etc*.), and the divergence $D_{i,j}$ (D determines the presence of a source or sink in the volume, depending on its sign) computed at the cell center. The knowledge of the normal velocity component on a cell side makes it possible to directly calculate the momentum flux through this side. The finite-difference scheme for the problem of spot collapse in stratified fluid for 2D case was published in details in [7].

4 Calculation Results

Let us consider a characteristic variant of a spots chain with intervals in a stratified viscous fluid, which has the following parameters: initial spot radius $R_0 = 0.8$, Reynolds number $Re = 3162$, Froude number $Fr = 1$, are presented. The spots are arranged horizontally in a row at some distance from each other. Computation area is $0 < x < 2$, $-8 < y < 8$. On the left boundary $x = 0$ and on the right boundary $x = 2$, the periodicity conditions are set. At the upper boundary, the conditions of a solid cover are set. At the lower boundary, the fluid is at rest. The rectangular computational grid has a computational step $h = 0.02$. The size of the computational grid is 20×160. The coefficients and parameters are: $\mu/\rho_0 = 0.01$ cm^2/s, $k_s = 1.41 \times 10^{-5}$ cm^2/s, $T_b = 2\pi s$, $\Lambda = 10$ cm, $Sc = 709.2$. The computational time step is $\tau = 0.002$.

The spots chain dynamics results with intervals will be illustrated by the stream function fields. In Figs. 2 and 3, the isoclines of the current function are shown in color. The minimum value of the current function corresponds to magenta, and the maximum value of the current function corresponds to red. The rest of the stream function values are sorted by color according to the colors of the rainbow.

In the first second of motion in the center of the computational domain, 4 perturbation areas of the stream function of oval shape appear. 2 areas of low values and 2 areas of high values with extremums in the center of these areas, as shown in Fig. 2a ($t = 1$). In the 2 second of motion, the shape and orientation of the oval perturbations of the stream function change slightly. The area of extrema expands, while the perturbation areas retain their size and the absolute values of extrema increase slightly, see Fig. 2b ($t = 2$). At $t = 3$, 8 perturbation areas are formed around the center of the computational domain. Regions alternate in low and high values of the stream function The regions shape takes the form of a lemniscate with a common center (Fig. 2c). Already at the fourth second, we again see only 4 areas of perturbation of the stream function. The shape of the perturbations becomes more rounded. Compared to $t = 2$, at $t = 4$, the extrema are in the reflected order, which means the

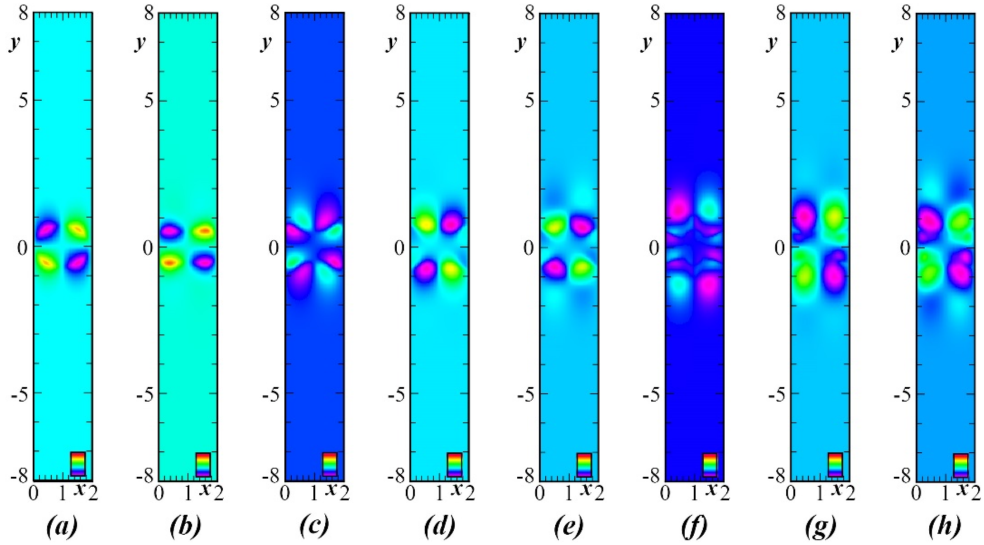

Fig. 2: Stream function isoclines for $R_0 = 0.8$, $Re = 3162$, $Fr = 1$, $Sc = 709.2$, $\mu/\rho_0 = 0.01$ cm^2/s, $k_s = 1.41 \times 10^{-5}$ cm^2/s, computation area is 2×8: (**a**) $t = 1$; (**b**) $t = 2$; (**c**) $t = 3$; (**d**) $t = 4$, (**e**) $t = 5$, (**f**) $t = 6$; (**g**) $t = 7$, (**h**) $t = 8$.

oscillatory nature of the movement (Fig. 2d).

At $t = 5$, the perturbations shape of the stream function changes insignificantly, and the orientation of these areas changes. Around the main perturbations, a second level of perturbations begins to form from below and from above (Fig. 2e). At $t = 6$, the main perturbation areas of the stream function expands significantly. Areas of low and high extremums alternate. Compared to the previous moment of time, here the oval perturbations are located in a mirror image. In the center of the computational domain there are crescent-shaped perturbations, then oval perturbations (Fig. 2f). At $t = 7$, the perturbation areas become larger, oval areas absorb small crescent-shaped areas (Fig. 2g). At $t = 8$, a noticeable increase in the perturbation areas of the stream function is again observed. Above the ovals of low values of the current function, ovals of high values are formed, and above the ovals of high values of the current function, ovals of low values are formed. Ovals of perturbations rotate around their axis over time (Fig. 2h).

Further, a significant propagation of perturbations is observed. At $t = 9$, 12 perturbation areas of the stream function are formed in the form of lemniscates. Large areas alternate in extrema, there are also pairs of adjacent areas with close extrema (Fig. 3a). At $t = 10$ and further, the spots dynamics repeats, the perturbations grow larger (Fig. 3b). At $t = 20$, already 6 rows of areas of perturbations of the stream

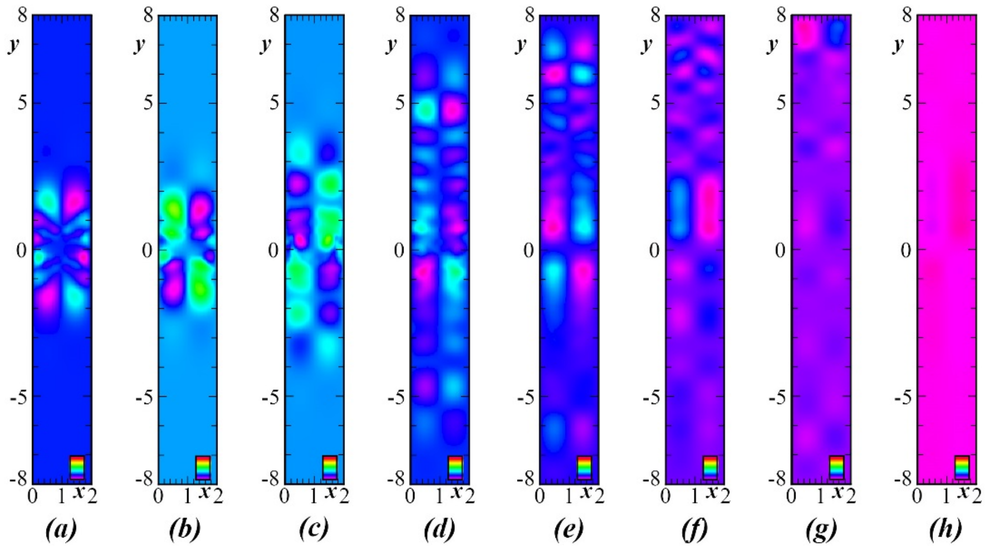

Fig. 3: Stream function isoclines for $R_0 = 0.8$, $Re = 3162$, $Fr = 1$, $Sc = 709.2$, $\mu/\rho_0 = 0.01$ cm^2/s, $k_s = 1.41 \times 10^{-5}$ cm^2/s, computation area is 2×8: (**a**) $t = 9$; (**b**) $t = 10$; (**c**) $t = 20$; (**d**) $t = 40$; (**e**) $t = 80$; (**f**) $t = 120$; (**g**) $t = 200$; (**h**) $t = 600$.

function are formed. Extremums are either alternate or arranged in pairs. When compared with the previous point in time, the location of the extremums also changes to the reflected one (Fig. 3c). At $t = 40$, the perturbations of the stream function already occupy the entire computational domain and decay. Central perturbations combine into long regions. Extreme perturbations have an oval shape, which alternate in the values of extremums (Fig. 3d).

At $t = 80$, perturbations of the stream function remain in the entire computational domain. When compared with the previous point in time, the location of the extremums is mirrored (Fig. 3e). At $t = 120$, the perturbations of the stream function decay (Fig. 3f). At $t = 200$, perturbations of the stream function are barely visible (Fig. 3g). At $t = 600$, the perturbations of the stream function completely disappear (Fig. 3h).

Consider the salinity perturbation fields for a chain with intervals with the same parameters: initial spot radius $R_0 = 0.8$, Reynolds number $Re = 3162$, Froude number $Fr = 1$. Up to $t = 11$, oscillatory movements of salinity perturbations are observed (Fig. 4a-e). By $t = 40$ and beyond, perturbations propagate throughout the area (Fig. 4f-g). Finally, at $t = 600$, a horizontal streak of salinity perturbations is formed (Fig. 4h).

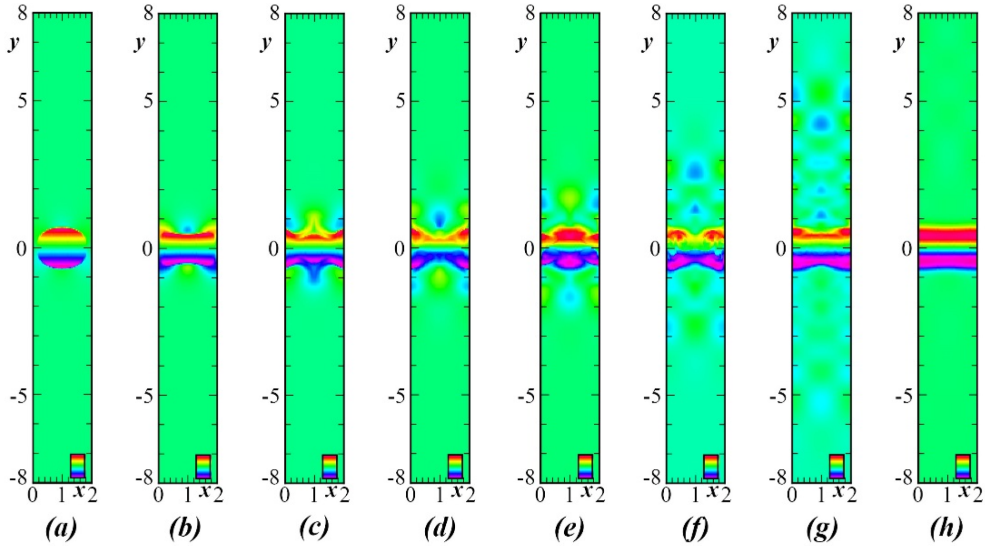

Fig. 4: Salinity perturbation isoclines for $R_0 = 0.8$, $Re = 3162$, $Fr = 1$, $Sc = 709.2$, $\mu/\rho_0 = 0.01$ cm^2/s, $k_s = 1.41 \times 10^{-5}$ cm^2/s, computation area is 2×8: (**a**) $t = 1$; (**b**) $t = 2$; (**c**) $t = 5$; (**d**) $t = 9$; (**e**) $t = 11$; (**f**) $t = 20$; (**g**) $t = 40$; (**h**) $t = 600$.

5 Conclusion

A characteristic variant of the spots chain dynamics with intervals is considered on the example of the stream function fields. Perturbations begin to form in the center of the computational domain and propagate to the end of the computational domain in 40 s. Local areas of perturbation rotate around their own axis. Areas with low extrema are located, alternating at the same time with areas of high extrema. In this case, over time, areas of high extrema are replaced by areas of low extrema, and areas of low extrema are replaced by areas of high extrema. During the first 40 s, periodically repeating dynamics of the stream function isoclines is observed, then, during 560 s, the field of the stream function decays and the flow is established. The period of current establishment is more than 10 times longer than the period of active current dynamics.

The stream function fields and the salinity perturbation fields are compared on the same characteristic variant of the spots chain dynamics with intervals. Coincidence of the period of oscillatory movements of salinity perturbation and the period of damping of oscillations with the formation of a horizontal streak is shown. One of the latest variants of the Splitting Method for Incompressible Fluid (SMIF) [6] developed by the authors is used to solve the problem. The finite difference scheme of the method has the following properties: the second order of approximation on spatial variables, minimal scheme viscosity and dispersion, monotonicity.

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