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GENERAL MECHANICS

THE INFLUENCE OF THE PIEZOELECTRIC EFFECT ON THE ANALYSIS OF ELASTIC STRESSES FOR A COMPOSITE DISK UNDER THE PARABOLIC DISTRIBUTION OF TEMPERATURE

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ABSTRACT: In this paper, we studied the influence of the piezoelectric effect under the variation of the parabolic temperature, for this purpose, a thermo-elastic composite disk reinforced by steel fibers is used for the analysis of the radial and tangential elastic thermal stresses. The solutions of the stresses were obtained analytically. In addition, a comparison between these expressions for different temperature values is calculated.

KEY WORDS: piezoelectric, radial, temperature, tangential, elastic.

1 INTRODUCTION

Thermoplastic composites offer many advantages in industrial applications due to such as high-improved fracture toughness, specific stiffness, and increased impact resistance. They can be remelted, reformed and reformed. Thermal stress analysis in composite discs is a considerable subject due to the wide range of applications in practice [1]. The fiber reinforced composite materials are widely used in many structural applications such as marine, automotive and aviation industries, on account of their high strength, low weight, good fatigue life and corrosion resistance [2]. Piezoelectric materials have electro-mechanical coupling characteristics, and have been widely used in sensors and actuators [3, 4].

Materials capable of tolerating material properties with spatial dimensions offer reliable engineering solutions for industrial applications involving severe thermal and mechanical loading as in the case of tribology, combustion processes, aerospace structures and high temperature technologies [5].

An analytical method to investigate thermo-elastic stress analysis on an orthotropic composite disc with aluminium metal-matrix reinforced towards x- and y-direction has been used in Ref. [6]. The elastic stress analysis of the hollow composite disc under the linearly increasing distribution of temperature has been carried out analytically with the help of a computer program [7].

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The experimental and theoretical study of dielectric, piezoelectric and mechanical properties of these materials is performed in our laboratories, using various methods, including interferometric method, operating in a wide temperature range [8].

The variation of the elastic thermal stresses with the presence of the piezoelectric effect was analysed under a parabolic temperature loading for different values of (r) for a thermoplastic composite disc.

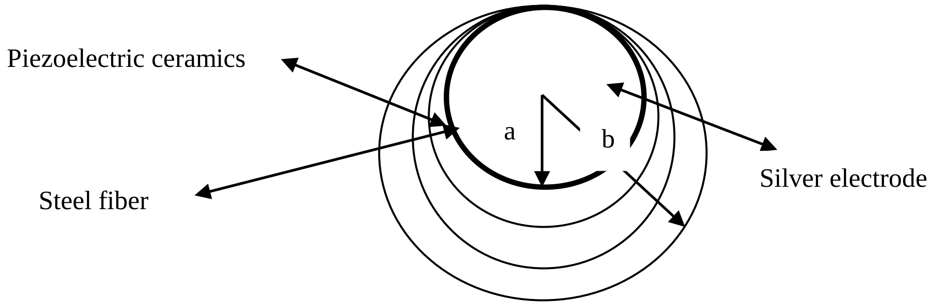

Fig. 1. Structure of the piezoelectric composite disc.

The stress-strain relationship for a piezoelectric composite disk under the variation of a parabolic temperature is expressed as follows:

$$\varepsilon_r = \frac{1}{E_r}\sigma_r - \frac{\upsilon_{r\theta}}{E_\theta}\sigma_\theta + \alpha_r T\,, \tag{1}$$

$$\varepsilon_\theta = \frac{\upsilon_{r\theta}}{E_\theta}\sigma_r - \frac{1}{E_\theta}\sigma_\theta + \alpha_\theta T\,, \tag{2}$$

where

σ_r – the radial stress;

σ_θ – the tangential stress;

ε_r – the radial strain;

ε_θ – the tangential strain;

α_r – the thermal expansion coefficients in the radial direction;

α_θ – the thermal expansion coefficients in the tangential direction;

E_r – modulus of elasticity in the radial direction;

E_θ – modulus of elasticity in the tangential direction.

The equation of equilibrium and the relation between deformations is written as follows:

$$r\frac{\partial \sigma_r}{\partial r} + \sigma_r - \sigma_\theta = 0\,, \tag{3}$$

$$\varepsilon_r = \varepsilon_\theta + r\frac{\partial \varepsilon_r}{\partial r} + d_{31}E_3\,. \tag{4}$$

For the two stresses a certain function is used to determine the differential equation:

$$\sigma_r = \frac{F}{r}\,,\,, \tag{5}$$

$$\sigma_\theta = \frac{\partial F}{\partial r}\,. \tag{6}$$

Using equations (1), (2), (5), and (6) and substituting in (4) we obtain

$$\begin{aligned} \frac{1}{E_r}\frac{F}{r} + \frac{\upsilon_{r\theta}}{E_\theta}\frac{\partial F}{\partial r} + \alpha_r T = \frac{\upsilon_{r\theta}}{E_\theta}\frac{F}{r} + \frac{1}{E_\theta}\frac{\partial F}{\partial r} \\ + \alpha_\theta T + \frac{\upsilon_{r\theta}}{E_\theta}\frac{\partial F}{\partial r} + \frac{r}{E_\theta}\frac{\partial^2 F}{\partial r^2} + r\alpha_\theta\frac{\partial T}{\partial r} + d_{31}E_3\,. \end{aligned} \tag{7}$$

Another way,

$$\frac{r^2}{E_\theta}\frac{\partial^2 F}{\partial r^2} + \frac{r}{E_\theta}\frac{\partial F}{\partial r} + \left(\frac{\upsilon_{r\theta}}{E_\theta} - \frac{1}{E_r}\right)F = -r^2\alpha_\theta\frac{\partial T}{\partial r} + (r\alpha_r - r\alpha_\theta)T - d_{31}E_3\,. \tag{8}$$

In this study the formula of parabolic variation of temperature is given by

$$T = \frac{T_i}{(b^2 - a^2)}\left(b^2 - r^2\right)\,. \tag{9}$$

The differential equation of function F is written as follows:

$$\begin{aligned} \frac{r^2}{E_\theta}\frac{\partial^2 F}{\partial r^2} + \frac{r}{E_\theta}\frac{\partial F}{\partial r} + \left(\frac{\upsilon_{r\theta}}{E_\theta} - \frac{1}{E_r}\right)F \\ = \left(-2r^3\frac{\alpha_\theta}{(b^2 - a^2)} - \frac{(r\alpha_r - r\alpha_\theta)\left(b^2 - r^2\right)}{(b^2 - a^2)}\right)T + d_{31}E_3\,. \end{aligned} \tag{10}$$

Another way to reduce this equation

$$r^2\frac{\partial^2 F}{\partial r^2} + r\frac{\partial F}{\partial r} + \left(\upsilon_{r\theta} - \frac{E_\theta}{E_r}\right)F + L_1 T + E_\theta d_{31}E_3\,, \tag{11}$$

where

$$L_1 = (-3\frac{E_\theta \alpha_\theta}{(b^2 - a^2)} + \frac{E_\theta \alpha_r}{(b^2 - a^2)})r^3 + (\frac{E_\theta \alpha_r b^2}{(b^2 - a^2)} + \frac{E_\theta \alpha_\theta b^2}{(b^2 - a^2)})r\,, \tag{12}$$

$$r^2 \frac{\partial^2 F}{\partial r^2} + r\frac{\partial F}{\partial r} + \left(\upsilon_{r\theta} - \frac{E_\theta}{E_r}\right) F + L_2\,, \tag{13}$$

where

$$L_2 = L_1 T + E_\theta d_{31} E_3\,. \tag{14}$$

The solution of the differential equation gives

$$F = -\frac{L_2 E_r}{\upsilon_{r\theta} E_r - E_\theta} + C_1 r^{\frac{\sqrt{-E_r(\upsilon_{r\theta} E_r - E_\theta)}}{E_r}} + C_2 r^{\frac{-\sqrt{-E_r(\upsilon_{r\theta} E_r - E_\theta)}}{E_r}}\,, \tag{15}$$

where C_1, C_2 are the constants of integrations,

$$P = -\frac{L_2 E_r}{\upsilon_{r\theta} E_r - E_\theta}\,, \tag{16}$$

$$G = \frac{\sqrt{-E_r\left(\upsilon_{r\theta} E_r - E_\theta\right)}}{E_r}\,. \tag{17}$$

With a simpler expression F is written as follows:

$$F = P + C_1 r^G + C_2 r^{-G}\,. \tag{18}$$

We substitute the expression of F in the two formulas of the radial and tangential stresses:

$$\sigma_r = \frac{F}{r} = \frac{P}{r} + C_1 r^{G-1} + C_2 r^{-G-1}\,, \tag{19}$$

$$\sigma_\theta = \frac{\partial F}{\partial r} = P_1 + C_1 G r^{G-1} - C_2 G r^{-G-1}\,, \tag{20}$$

where

$$P_1 = \frac{\partial P}{\partial r} = 3(-3\frac{E_\theta \alpha_\theta}{(b^2 - a^2)} + \frac{E_\theta \alpha_r}{(b^2 - a^2)})r^3 + (\frac{E_\theta \alpha_r b^2}{(b^2 - a^2)} + \frac{E_\theta \alpha_\theta b^2}{(b^2 - a^2)})\,, \tag{21}$$

$$\sigma_r = 0 \quad \text{at } r = a \text{ and } r = b\,. \tag{22}$$

The expressions of C_1 and C_2 are given by

$$C_1 = -\frac{P\left(b^{-G} - a^{-G}\right)}{(a^G b^{-G} - b^G a^{-G})}\,, \tag{23}$$

$$C_2 = -\frac{P\left(a^G - b^G\right)}{(a^G b^{-G} - b^G a^{-G})}\,. \tag{24}$$

2 Results and Discussion

$E_\theta = 11300$ Mpa, $E_r = 260$ Mpa, $\upsilon_{r\theta} = 0.43$, $\alpha_r = 130 \times 10^{-6}$ 1/°C, $\alpha_\theta = 12.8 \times 10^{-6}$ 1/°C, $a = 30$ mm, $b = 70$ mm, $d_{31} = -5.35 \times 10^{-7}$ mm/v, $E_3 = 7150$ v/mm.

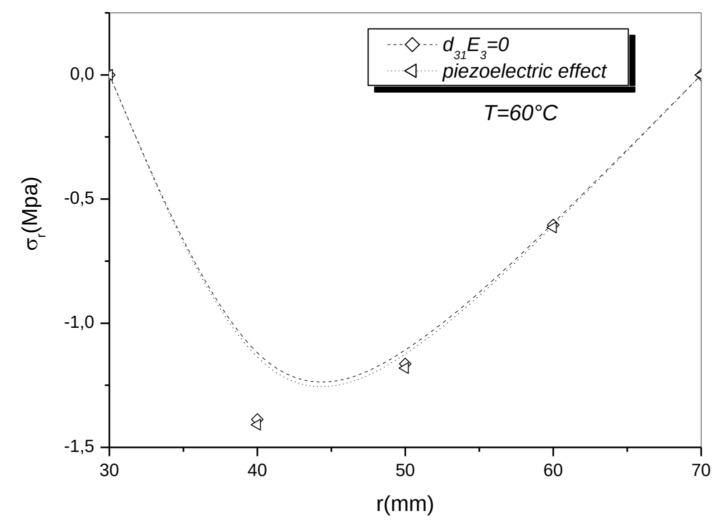

Fig. 2. Variation of the radial stress at 60°C.

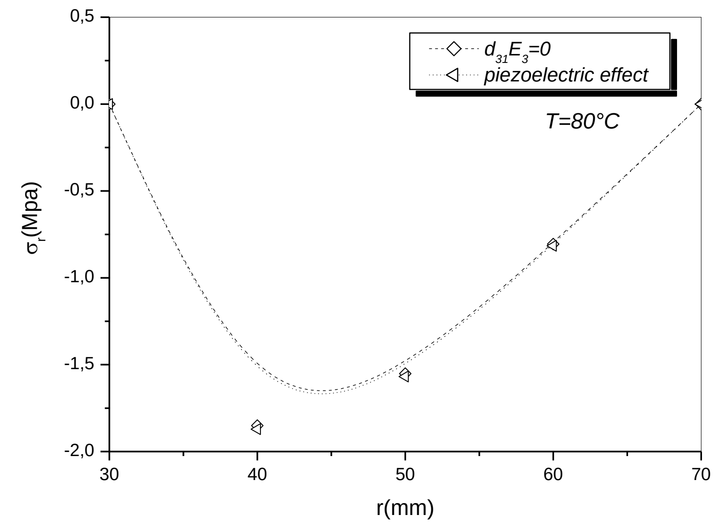

Fig. 3. Variation of the radial stress at 80°C.

The variation of the thermo-elastic radial stresses for the three different temperature values 60°C, 80°C and 100°C with the presence of the piezoelectric effect are

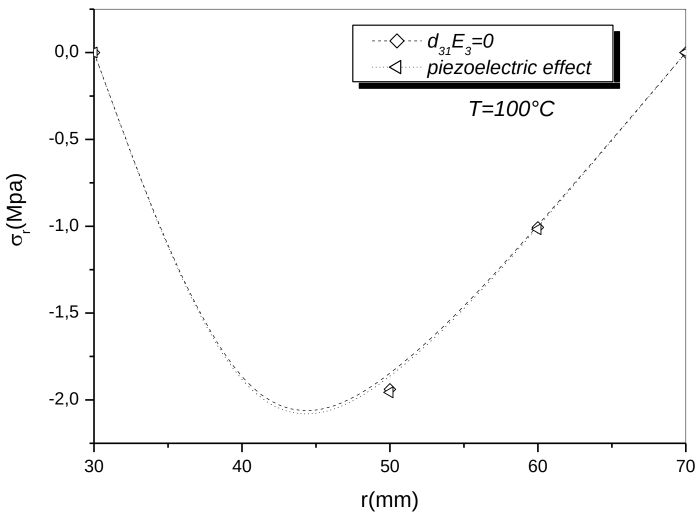

Fig. 4. Variation of the radial stress at 100°C.

shown in Figs. 2, 3 and 4, respectively. These stresses augmented with small ratios include this effect; these figures show that all the values of these stresses give the form of compression between the two ends, inner and outer diameter.

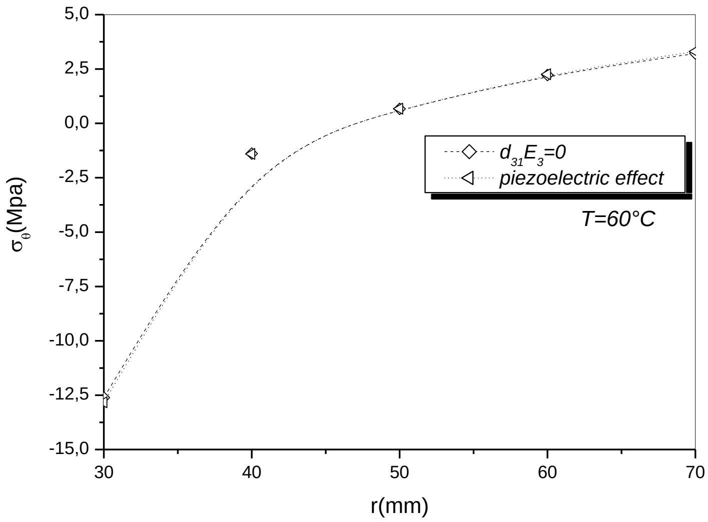

Fig. 5. Variation of the tangential stress at 60°C.

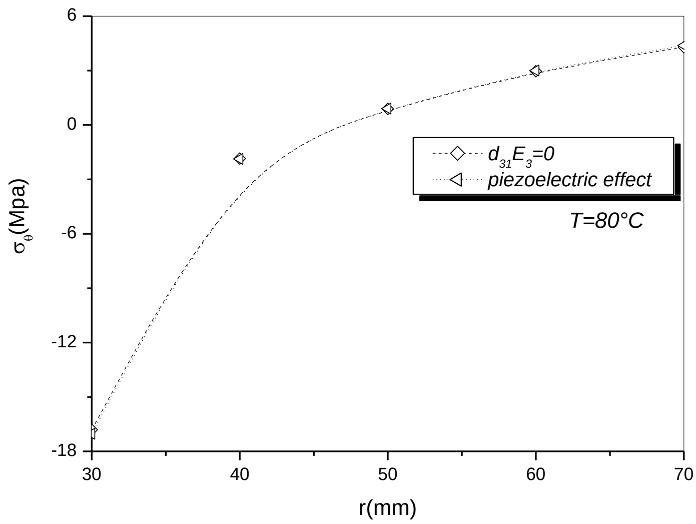

Fig. 6. Variation of the tangential stress at 80°C.

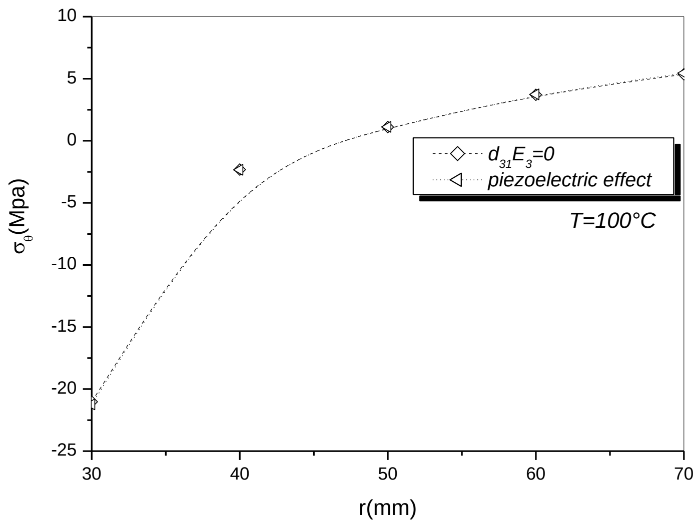

Fig. 7. Variation of the tangential stress at 100°C.

Figures 5, 6 and 7 show the tangential stress variation for $T = 60$°C, 80°C and 100°C, the influence of the piezoelectric effect is clear in this part, in this part the two phenomena s' is produced, compression and traction, especially in areas where $a = 30$ mm and $b = 70$ mm, the figures show that the tangential stresses at the inner diameter are higher than at the outer diameter.

3 CONCLUSION

In this study, the variation of the radial and tangential thermo-elastic stresses for a composite disk reinforced by steel fibers is of such great importance that the different values obtained along the diameter, the piezoelectric effect positively affects two types of stresses, a convergence of all the values at zero at the level $r = a$ and $r = b$, the compression and tensile phenomena is illustrated in all the graphs, the radial stresses reaches its maximum in the middle between a and b which causes the risk that this disc will be spread.

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