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SEISMIC SOIL-TUNNELS INTERACTION VIA BEM PART I. MECHANICAL MODEL*

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Abstract. Two-dimensional elastodynamic problem for seismic response of unlined and lined tunnels located in a layered half-plane with free surface relief is solved. The computation tool uses the idea of the global matrix propagator method which allows derivation of a relation between the wave field quantities along different interfaces in the layered half-plane. The numerical realization of this idea is performed with the help of the sub-structured boundary element method (BEM) well suited when objects with arbitrary geometry are considered. A relation between displacements and tractions along the free surface and arbitrary interface of the soil stratum is derived. It works for arbitrary geometry of the interfaces between soil layers. Finally, in the companion paper, numerical results are presented which show both a validation study of the proposed computational methodology and extensive numerical simulations demonstrating the influence of some important factors as type and characteristics of the incident wave, dynamic tunnels interaction, soil-tunnel

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interaction, free surface relief, type of the tunnel construction and mechanical properties of the layered half-plane on the complex seismic field near and far-away from the underground structures.

Key words: Harmonic and transient P-SV waves, soil-tunnels interaction, seismograms, stress concentration field, boundary element method (BEM).

1. Introduction

The seismic vulnerability of underground structures which are critical elements in transportation and utility networks (e. g. rail-way and road tunnels, hydraulic tunnels and hydroelectric caverns, lifelines for transportation of water, oil, natural gas, etc.) attracted the attention of researchers in recent years. The associated risk may be relevant, since even a low level of damage may affect the serviceability of a wide network, although the widespread opinion that underground structures experience a lower rate of damage comparing to surface structures. Seismic response analysis of underground tunnel structures involves selection of seismic input motions based on site-specific seismic hazard assessment and seismic performance criteria, site response analysis, and soil-structure interaction analysis. A way of shedding some light on the understanding of the seismic soil-tunnel response phenomena consists in developing of high-performance methods for simulations of seismic wave propagation in complex geological sites with different type of heterogeneities.

This paper aims to propose a computational tool based on sub-structuring boundary element method (BEM) for solution of two-dimensional in-plane elastodynamic problem for frequency and time dependent seismic response of unlined and lined tunnels located in a layered half-plane with free surface relief.

Essentially, three main groups of approaches treat the problem for seismic wave propagation in complex geological media with different type of heterogeneities of natural or urban type. They are analytical, numerical and hybrid. The analytical tools are mainly based on the ray method presented in Babich [1] and its modifications such as the Maslov asymptotic theory in Chapman et al. [2], the Kirchhoff-Helmholtz methods [3], Gaussian beams [4], generalized ray method [5], Born approximation method [6], reflectivity method [7], the generalized Reflection/Transmission (R/T) coefficient method [8–9], wave number integration method [10] all restricted to media with simple geometry. The known computational techniques supporting more complex mechanical models with arbitrary geometry are numerical ones as the finite element method, boundary element method and finite difference method. In addition to the tools mentioned above, it is worth noting that a kind of other methods as distinct element method [11], fundamental-solution-less boundary element method [12] and the scaled boundary finite-element method [13–15] has been developed recently for exterior wave problems. The state of the art review in the field of wave scattering around different inclusions in complex half space shows that many problems had already been resolved by many scholars by using different advanced models/methods, but the most of them do not model the whole path of the seismic wave starting from the source, propagating through the regional inhomogeneous model and entering the civil structure resting on a local geological profile with specific geotechnical properties.

The modelling of soil-tunnel system subjected to seismic load includes seismic wave propagation in a complex layered geological media consisting of travel path of the signal plus wave motion in the near local site where the tunnel is located. It comprises two types of large-scaled models: (a) all-in-one (regional travel path-local site with heterogeneities) single computational tool demanding an extreme computer memory and time, especially in the cases when the distance between the travel path and the local site is measured in dozens of km and (b) hybrid approach [16–27] based on a two-step procedure that combines the travel path effects computed by one method and local site effects evaluated by other method, using the first method's wave field as input. The main disadvantage of the hybrid two-step techniques is that in subsequent steps past the first, any interaction between the backscattering waves from the local heterogeneity with the incoming wave fields emanating from the deeper layers of the geological profile is neglected. At the same time, there is almost no computational tools and models considering and treating simultaneously the whole system: regional model-local model-existing heterogeneities inside the local region.

That is the reason why we focus on development of all-in-one (regional travel path-local site with soil – tunnels system) single computational model based on BEM using the sub-structural approach and the idea of global matrix propagator method for derivation of a relation between displacements and tractions along the free surface and the arbitrary interface of the soil stratum. The special advantages of the BEM when solving seismic wave propagation in infinite geological media are discussed widely in the literature [28] and they are: (a) reduce the size of the problem dimensionality and the size of the resulting algebraic system in contrast to other numerical domain methods; (b) possibility to model lateral inhomogeneity in contrast to other analytical approaches; (c) solution at each internal point in the domain is expressed in terms of boundary values without recourse to domain discretization and this main facility is very important when wave propagation problems are being solved in multilayered solids, because only the boundaries between layers are discretized, not their volumes as it is the case when domain discretization methods as finite element or finite difference methods are used; (d) flexibility to model relief peculiarities in contrast to analytical methods and finite difference method having problems with implementing conditions on boundaries of complex geometric shapes; (e) the possibility to obtain directly, with no other intermediate source of error, the dynamic regime — displacements and tractions; (f) the semi-analytical character of the method as far as it is based on the fundamental solution or on the Green's function of the considered problem; (g) high level of accuracy is achieved since numerical quadrature techniques are directly applied to the boundary integral equations, which are an exact solution of the considered problem.

The paper is structured in two parts. This first part starts with the problem statement in Section 2, followed by its BEM solution in Section 3 and ending with a discussion about advantages of the proposed computational tool in Section 4. Validation and convergence study plus intensive parametric study will be presented in the second part of this work.

2. Problem statement

Let us consider inhomogeneous in depth half-plane Ω_{hp} with flat free surface boundary S_{FS} and free surface relief V with boundary S_V of arbitrary shape, subjected to incident time-harmonic or transient P and SV wave. The half-plane is presented by N homogeneous elastic isotropic layers, defined with Lame constants and density λ_i, μ_i, ρ_i, $i = 1, 2, \ldots, N$ and with non-horizontal interfaces Λ_i rested on the seismic bed with material properties λ_0, μ_0, ρ_0, see Fig. 1. Cartesian coordinate system $Ox_1x_2x_3$ is inserted and the in-plane wave motion in the plane $x_3 = 0$ is studied. The plane wave propagates under angle of incidence θ with respect to axis Ox_1. Two infinite cylindrical circular segment tunnels Ω_{1t} and Ω_{2t} are embedded in the first layer of the half-plane at a distance $2d$ and the internal and external radii are denoted as R_{int} and R_{ext}. The material properties of the tunnel liner are denoted by λ_t, μ_t, ρ_t. The external boundaries of the tunnels form the interface with the surrounding rock which is denoted as $\Gamma_2 = \Gamma_{2l} \cup \Gamma_{2r}$, while the internal ones are denoted as $\Gamma_1 = \Gamma_{1l} \cup \Gamma_{1r}$. Here, the internal and external boundaries at the left tunnel are Γ_{1l} and Γ_{2l}, respectively while those at the right tunnel are Γ_{1r} andΓ_{2r}. For a state of plane strain and respectively in-plane wave motion, the only non-zero field quantities are displacement components u_1, u_2 and stresses σ_{11}, σ_{22}, σ_{12} all dependent on the coordinates (x_1, x_2) and on time t or frequency ω.

The governing equation of motion is:

$$\sigma_{ij,j} = \begin{cases} \dfrac{\partial^2 u_i}{\partial t^2} & \text{for transient wave} \\ -\rho\omega^2 u_i & \text{for time-harmonic wave} \end{cases} , \tag{1}$$

where:$\sigma_{ij} = C_{ijkl}u_{k,l}$, $C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$, δ_{ij} is Kronecker's delta symbol and comma subscripts denote partial differentiation with respect to the spatial coordinates, while the summation convention over repeated indices is implied.

The total wave field u_i, t_i outside the layers comes from a superposition of the free-field motion u_i^{ff}, t_i^{ff} and of the scattered by the tunnels, layers and free surface relief waves u_i^{sc}, t_i^{sc}, i.e.:

$$u_i\left(\mathbf{x}\right) = u_i^{ff}\left(\mathbf{x}\right) + u_i^{sc}\left(\mathbf{x}\right) \quad \text{and} \quad t_i\left(\mathbf{x}\right) = t_i^{ff}\left(\mathbf{x}\right) + t_i^{sc}\left(\mathbf{x}\right). \tag{2}$$

The boundary conditions for the total wave field are as follows: $(*)$ along the free-surface $S = S_{FS} \cup S_V$; the tractions $t_i = \sigma_{ij}n_j$ are zero, n_i are the components of the outward normal to the surface; $(*)$ compatibility and equilibrium conditions of displacements and tractions, respectively are satisfied along the layers interfaces Λ_i; $(*)$ Sommerfeld radiation condition is satisfied at infinity; $(*)$ tractions free hold at the internal boundary Γ_1 of the tunnels, while compatibility and equilibrium conditions are satisfied along the interface between the soil and tunnels' walls Γ_2; $(*)$ Along the last interface Λ_0, between the seismic bed and the first soil layer, the following boundary condition states: $t_j^{ff(0)} + t_j^{sc(0)} = t_j^{(0)} = -t_j^{(1)}$ for $(x_1, x_2) \in \Lambda_0$. Here superscripts in brackets denote the corresponding domain.

Free field motion is produced by P and SV waves, propagating through the elastic homogeneous half-plane with material properties λ_0, μ_0, ρ_0 in the absence of any layers and relief. It gives the following displacement field at a point $\mathbf{x} = (x_1, x_2)$ for an incident wave angle θ:

A) P-wave case:

- Incident wave with amplitude A_P^{in}:

$$\begin{pmatrix} u_1^{in}(\mathbf{x},t) \\ u_2^{in}(\mathbf{x},t) \end{pmatrix} = A_P^{in}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}\begin{cases} f\left[t-\dfrac{x_1\cos\theta+x_2\sin\theta}{V_P}\right]^{\text{TRANSIENT CASE}} \\ \exp\left[ik_P\left(V_Pt-x_1\cos\theta-x_2\sin\theta\right)\right]^{\text{TIME−HARMONIC CASE}} \end{cases}; \tag{3}$$

- Free field transient motion [29]:

$$\begin{aligned} &\begin{pmatrix} u_1^{ff}(\mathbf{x},t) \\ u_2^{ff}(\mathbf{x},t) \end{pmatrix} \\ &= A_P^{in}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} f\left[t-\frac{x_1\cos\theta+x_2\sin\theta}{V_P}\right]+^{\text{INCIDENT P−WAVE}} \\ &+A_P^{ref}\begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix} f\left[t-\frac{x_1\cos\theta-x_2\sin\theta}{V_P}\right]+^{\text{REFLECTED P−WAVE}}; \\ &+A_{SV}^{ref}\begin{pmatrix} -\psi_2 \\ -\psi_1 \end{pmatrix} f\left[t-\frac{x_1\psi_1-x_2\psi_2}{V_{SV}}\right],^{\text{REFLECTED SV−WAVE}} \end{aligned} \tag{3a}$$

where:

$$\begin{aligned} &p=\sqrt{\left(\frac{V_P}{V_{SV}\cos\theta}\right)^2-1}, && s=\sqrt{\frac{1}{\cos^2\theta}-1}, \\ &\psi_1=\frac{V_{SV}}{V_P}\cos\theta, && \psi_2=\sqrt{1-\psi_1^2}, \\ &A_P^{ref}=\frac{4sp-\left(1-p^2\right)^2}{4sp+\left(1-p^2\right)^2}A_P^{in}, && A_{SV}^{ref}=\frac{4s\left(1-p^2\right)V_P}{\left(4sp+\left(1-p^2\right)^2\right)V_{SV}}A_P^{in}. \end{aligned} \tag{3b}$$

- Free field time-harmonic motion [30]:

$$\text{(3c)}\quad \begin{pmatrix} u_1^{ff}(\mathbf{x},t) \\ u_2^{ff}(\mathbf{x},t) \end{pmatrix}$$

$$\begin{aligned} &= A_P^{in}\left\{ \begin{array}{c} k/k_P \\ -i\nu/k_P \end{array} \right\} \exp(-\nu x_2 - ikx_1) + {}^{\text{INCIDENT P-WAVE}} \\ &+ A_P^{in} R_{PP}\left\{ \begin{array}{c} k/k_P \\ i\nu/k_P \end{array} \right\} \exp(\nu x_2 - ikx_1) + {}^{\text{REFLECTED P-WAVE}} \quad ; \\ &+ A_P^{in} R_{PS}\left\{ \begin{array}{c} i\nu'/k_P \\ -k/k_P \end{array} \right\} \exp(\nu' x_2 - ikx_1), {}^{\text{REFLECTED SV-WAVE}} \end{aligned}$$

where:

$$\text{(3d)}\qquad \begin{aligned} &k = k_P \cos\theta,\ \nu = ik_P \sin\theta, \\ &\nu' = \sqrt{k^2 - k_S^2},\ \Delta(k) = (2k^2 - k_S^2)^2 - 4k^2\nu\nu', \\ &R_{PP} = -[(2k^2 - k_S^2)^2 + 4k^2\nu\nu']/\Delta(k), \\ &R_{PS} = -4ik\nu(2k^2 - k_S^2)/\Delta(k). \end{aligned}$$

B) SV-wave case:

- Incident wave with amplitude A_{SV}^{in}:

$$\text{(4)}\quad \begin{pmatrix} u_1^{in}(\mathbf{x},t) \\ u_2^{in}(\mathbf{x},t) \end{pmatrix}$$

$$= A_{SV}^{in} \begin{pmatrix} \sin\theta \\ -\cos\theta \end{pmatrix} \begin{cases} f\left[t - \dfrac{x_1\cos\theta + x_2\sin\theta}{V_{SV}}\right] \text{TRANSIENT CASE} \\ \exp[ik_S(V_{SV}t - x_1\cos\theta - x_2\sin\theta)]^{\text{TIME-HARMONIC CASE}} \end{cases} ;$$

- Free field transient motion [29]:

$$\text{(4a)}\quad \begin{pmatrix} u_1^{ff}(\mathbf{x},t) \\ u_2^{ff}(\mathbf{x},t) \end{pmatrix}$$

$$= A_{SV}^{in}\begin{pmatrix} \sin\theta \\ -\cos\theta \end{pmatrix} f\left[t - \frac{x_1\cos\theta + x_2\sin\theta}{V_{SV}}\right] + ^{\text{INCIDENT SV-WAVE}}$$

$$+A_{SV}^{ref}\begin{pmatrix} -\sin\theta \\ -\cos\theta \end{pmatrix} f\left[t - \frac{x_1\cos\theta - x_2\sin\theta}{V_{SV}}\right] + ^{\text{REFLECTED SV-WAVE}} \quad ;$$

$$+A_{P}^{ref}\begin{pmatrix} \psi_1 \\ -\psi_2 \end{pmatrix} f\left[t - \frac{x_1\psi_1 - x_2\psi_2}{V_{P}}\right], ^{\text{REFLECTED P-WAVE}}$$

where:

$$p = \sqrt{\frac{1}{\cos^2\theta} - 1}, \qquad s = \sqrt{\left(\frac{V_{SV}}{V_P\cos\theta}\right)^2 - 1},$$

$$\text{(4b)}\quad \psi_1 = \frac{V_P}{V_{SV}}\cos\theta, \qquad \psi_2 = \sqrt{1-\psi_1^2} \quad ;$$

$$A_P^{ref} = -\frac{4p\left(1-p^2\right)V_{SV}}{\left(4sp + (1-p^2)^2\right)V_P}.A_{SV}^{in}, \quad A_{SV}^{ref} = \frac{4sp - \left(1-p^2\right)^2}{4sp + (1-p^2)^2}A_{SV}^{in}$$

- Free field time-harmonic motion [30]:

$$\text{(4c)}\quad \begin{pmatrix} u_1^{ff}(\mathbf{x},t) \\ u_2^{ff}(\mathbf{x},t) \end{pmatrix}$$

$$= A_{SV}^{in}\left\{ \begin{matrix} -i\nu'/k_S \\ -k/k_S \end{matrix} \right\}\exp(-\nu' x_2 - ikx_1) + ^{\text{INCIDENT SV-WAVE}}$$

$$+A_{SV}^{in}R_{SS}\left\{ \begin{matrix} i\nu'/k_S \\ -k/k_S \end{matrix} \right\}\exp(\nu' x_2 - ikx_1) + ^{\text{REFLECTED SV-WAVE}} \quad ;$$

$$+A_{SV}^{in}R_{SP}\left\{ \begin{matrix} k/k_S \\ i\nu/k_S \end{matrix} \right\}\exp(\nu x_2 - ikx_1), ^{\text{REFLECTED P-WAVE}}$$

where:

$$
\text{(4d)}\quad \begin{array}{ll}
k = k_S \cos\theta, & \nu = \sqrt{k^2 - k_P^2}, \\
\nu' = i k_S \sin\theta, & \Delta(k) = \left(2k^2 - k_S^2\right)^2 - 4k^2\nu\nu' \\
R_{SP} = 4ik\nu'(2k^2 - k_S^2)/\Delta(k), & R_{SS} = -[(2k^2 - k_S^2)^2 + 4k^2\nu\nu']/\Delta(k)
\end{array} .
$$

The quantities $V_P = \sqrt{\dfrac{\lambda_0 + 2\mu_0}{\rho_0}}$, $V_{SV} = \sqrt{\dfrac{\mu_0}{\rho_0}}$, $k_P = \dfrac{\omega}{V_P}$, $k_S = \dfrac{\omega}{V_{SV}}$ in equations (3)–(4d) are velocities and wave numbers of longitudinal and shear waves, respectively. The corresponding tractions of the free field motion are computed by:

$$
t_i^{ff} = \left(\lambda_0 u_{k,k}^{ff} \delta_{ij} + \mu_0 \left(u_{i,j}^{ff} + u_{j,i}^{ff}\right)\right) n_j. \tag{5}
$$

In sum, our aim is to compute the resulting wave signals along the free surface of the half-plane and to evaluate the stress concentration field near the tunnel walls.

3. Problem solution

The relation between the wave displacements and the tractions along any layers' interfaces is derived by extension of the idea of the global matrix propagator method by the use of the BEM based on the sub-structuring approach. The main disadvantage of the global matrix propagator is that it can be applied only for stratified geological region with parallel line interfaces of the layers. This strong limitation is overcome here by the usage of the BEM based on the substructure approach.

The BEM description of the total wave field in the frequency domain is as follows, see [28]:

$$
\begin{aligned}
\text{(6a)}\quad c_{ij} u_j^{(k)}(\mathbf{x},\omega) = & \int\limits_{S_{\Omega_k}} U_{ij}^{*(k)}(\mathbf{x},\mathbf{y},\omega)\, t_j^{(k)}(\mathbf{y},\omega)\, dS_{\Omega_k}(\mathbf{y}) - \\
& - \int\limits_{S_{\Omega_k}} P_{ij}^{*(k)}(\mathbf{x},\mathbf{y},\omega)\, u_j^{(k)}(\mathbf{y},\omega)\, dS_{\Omega_k}(\mathbf{y}),
\end{aligned}
$$

where: $\mathbf{x} \in S_{\Omega_k}$, $k = 1, 2, 3, \ldots, N$.

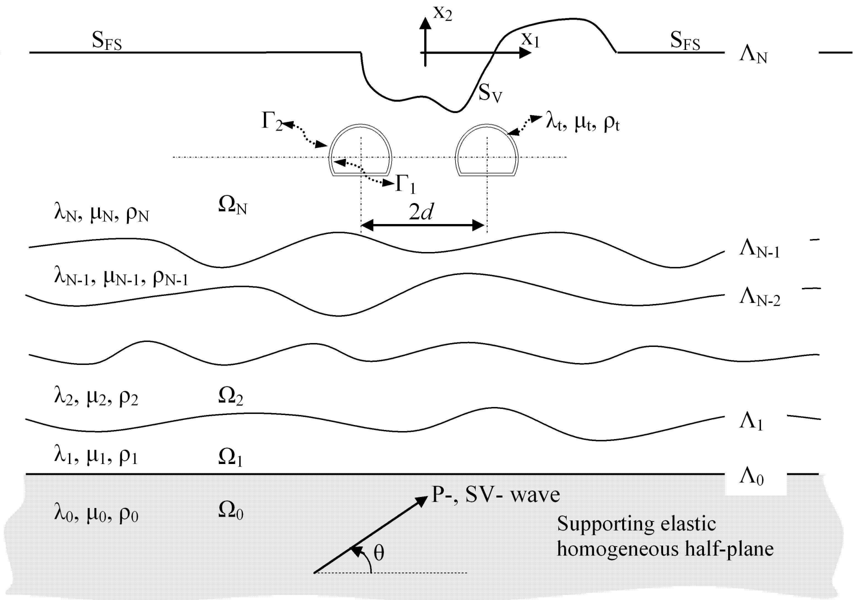

Fig. 1. The geometry

Here: c_{ij} is the jump term depending on the local geometry at the collocation point $\mathbf{x}(x_1, x_2)$, $\mathbf{x}$ and $\mathbf{y}$ are the position vectors of the source and the field points, $U_{ij}^{*(k)}$ is the displacement fundamental solution of the governing equation (1), $P_{ij}^{*(k)} = C_{iqsl} U_{sj,l}^{*(k)} n_q$ is its corresponding traction in the k-th layer, $S_{\Omega_k} = \Lambda_{k-1} \cup \Lambda_k$. Note, that the upper boundary of the layer Ω_N is $\Lambda_N = S_{FS} \cup S_V$ and $S_{\Omega_N} = \Lambda_{N-1} \cup \Lambda_N \cup \Gamma_1 \cup \Gamma_2$, see Fig. 1. Remind, that the following boundary condition is satisfied along the interface between the first layer and the supporting elastic homogeneous half-plane (i.e. seismic bed) Ω_0: $t_j^{ff(0)} + t_j^{sc(0)} = t_j^{(0)} = -t_j^{(1)}$ for $(x_1, x_2) \in \Lambda_0$, where $t_j^{(0)}$ is the total traction in the region Ω_0, while $t_j^{(1)}$ is the total traction at contour Λ_0 for layer Ω_1. The total traction $t_j^{(0)}$, the free field traction $t_j^{ff(0)}$ and the free field displacement $u_j^{ff(0)}$ are all developed in the region Ω_0 and are connected in the following BIE along the interface Λ_0:

$$\text{(6b)}\quad c_{ij}\left(u_j^{(0)}(\mathbf{x},\omega)-u_j^{ff(0)}(\mathbf{x},\omega)\right)= \\ =\int\limits_{\Lambda_0}U_{ij}^{*(0)}(\mathbf{x},\mathbf{y},\omega)\,(t_j^{(0)}(\mathbf{y},\omega)-t_j^{ff(0)}(\mathbf{y},\omega))d\Lambda_0- \\ -\int\limits_{\Lambda_0}P_{ij}^{*(0)}(\mathbf{x},\mathbf{y},\omega)\,(u_j^{(0)}(\mathbf{y},\omega)-u_j^{ff(0)}(\mathbf{y},\omega))d\Lambda_0,$$

where: $\mathbf{x}\in\Lambda_0$.

The solution of the problem for transient waves is solved by the usage of direct and inverse Fast Fourier Transform (FFT), so the BIE has the same form as Eq. (6) but written for the Fourier transforms of the field quantities.

In this work, BEM based on sub-structural approach is applied where one modification is made by consecutive exclusion of the soil layers, which leads to the reduced system of linear algebraic equations. The idea of this consecutive elimination of the layers will be explained in what follows. Let's first consider the elastic bed as a single domain Ω_0. The matrix system of equations generated for this region in terms of scattered field quantities, in line with equations (6), is:

$$\text{(7)}\qquad \mathbf{G}\mathbf{t}^{sc}-\mathbf{H}\mathbf{u}^{sc}=\mathbf{0}.$$

Matrices $\mathbf{G}$ and $\mathbf{H}$ are the final, fully populated influence matrices, of size $2N\times 2N$, where N is the total number of nodes of the domain under consideration, while vectors $\mathbf{u}^{sc}$ and $\mathbf{t}^{sc}$ contain the scattered boundary displacements and tractions. Here, the BEM implementation is performed in terms of scattered field, which should satisfy the radiation condition at infinity. It is necessary to replace the scattered field tractions and displacements with $\mathbf{t}^{sc}=\mathbf{t}-\mathbf{t}^{ff}$ and $\mathbf{u}^{sc}=\mathbf{u}-\mathbf{u}^{ff}$, in line with equations (2), in order to derive the free term due to incident wave impact:

$$\text{(7a)}\qquad \mathbf{G}\mathbf{t}-\mathbf{H}\mathbf{u}=\mathbf{G}\mathbf{t}^{ff}-\mathbf{H}\mathbf{u}^{ff}.$$

In what follows, superscripts denote sub-regions (i for domain Ω_i, $i=0:N$) while subscripts denote interfaces (i for Λ_i, $i=0:N$), where all source and the receiver points are located. We after solving eq. (7a) with respect to the tractions get:

$$\mathbf{t}_0^{(0)} = \left(\mathbf{G}_{0,0}^{(0)}\right)^{-1} \mathbf{H}_{0,0}^{(0)}\mathbf{u}_0^{(0)} + \mathbf{t}_0^{(0)ff} - \left(\mathbf{G}_{0,0}^{(0)}\right)^{-1} \mathbf{H}_{0,0}^{(0)}\mathbf{u}_0^{(0)ff}. \tag{7b}$$

The equilibrium and compatibility conditions on the interface between seismic bed and the first layer, belonging to the seismic bed domain Ω_0 on one hand, and to the first layer Ω_1 on the other hand, can be written as:

$$\mathbf{t}_0^{(1)} = -\mathbf{t}_0^{(0)} \quad \text{and} \quad \mathbf{u}_0^{(1)} = \mathbf{u}_0^{(0)}. \tag{8}$$

Let us replace the product $\left(\mathbf{G}_{0,0}^{(0)}\right)^{-1} \mathbf{H}_{0,0}^{(0)}$ from eq. (7b) with $-\mathbf{B}_{0,0}^{(0)}$ and the free term $\mathbf{t}_0^{(0)ff} - \left(\mathbf{G}_{0,0}^{(0)}\right)^{-1} \mathbf{H}_{0,0}^{(0)}\mathbf{u}_0^{(0)ff}$ with $\mathbf{F}$ ($\mathbf{F} = \mathbf{t}_0^{(0)ff} + \mathbf{B}_{0,0}^{(0)}\mathbf{u}_0^{(0)ff}$). We after substituting eq. (7b) in (8) for tractions and displacements of the interface Λ_0 have:

$$\mathbf{t}_0^{(1)} = \mathbf{B}_{0,0}^{(0)}\mathbf{u}_0^{(1)} - \mathbf{F}. \tag{8a}$$

Next, the first layer is considered as a single domain Ω_1 delineated by interfaces Λ_0 and Λ_1 (Fig. 1). The standard matrix system of equations generated for this region holds:

$$\mathbf{G}^{(1)}\mathbf{t}^{(1)} - \mathbf{H}^{(1)}\mathbf{u}^{(1)} = \mathbf{0}, \tag{9}$$

or for the tractions of the first layer contours we get:

$$\mathbf{t}^{(1)} = \left(\mathbf{G}^{(1)}\right)^{-1} \mathbf{H}^{(1)}\mathbf{u}^{(1)} = \mathbf{C}^{(1)}\mathbf{u}^{(1)}, \tag{10}$$

where matrix product $\left(\mathbf{G}^{(1)}\right)^{-1} \mathbf{H}^{(1)}$ is substituted by matrix $\mathbf{C}^{(1)}$. Equations (10) written in an expanded form in accordance with sub- and super- script notations mentioned above, take the following form:

$$\left\{ \begin{array}{c} \mathbf{t}_0^{(1)} \\ \mathbf{t}_1^{(1)} \end{array} \right\} = \left[\begin{array}{cc} \mathbf{C}_{0,0}^{(1)} & \mathbf{C}_{0,1}^{(1)} \\ \mathbf{C}_{1,0}^{(1)} & \mathbf{C}_{1,1}^{(1)} \end{array} \right] \left\{ \begin{array}{c} \mathbf{u}_0^{(1)} \\ \mathbf{u}_1^{(1)} \end{array} \right\}. \tag{11}$$

We get a relation between displacements of both contours by substituting equations (8a) in the first row of (11), Λ_0 and Λ_1, delineating the domain under consideration:

$$\mathbf{u}_0^{(1)} = (\mathbf{B}_{0,0}^{(0)} - \mathbf{C}_{0,0}^{(1)})^{-1}\mathbf{C}_{0,1}^{(1)}\mathbf{u}_1^{(1)} + (\mathbf{B}_{0,0}^{(0)} - \mathbf{C}_{0,0}^{(1)})^{-1}\mathbf{F}. \tag{12}$$

After replacing the equation (12) into the second expression of (11) the tractions of the contour, Λ_1 are written as:

$$\mathbf{t}_1^{(1)} = \left(\mathbf{C}_{1,0}^{(1)}(\mathbf{B}_{0,0}^{(0)} - \mathbf{C}_{0,0}^{(1)})^{-1}\mathbf{C}_{0,1}^{(1)} + \mathbf{C}_{1,1}^{(1)}\right)\mathbf{u}_1^{(1)} + \mathbf{C}_{1,0}^{(1)}(\mathbf{B}_{0,0}^{(0)} - \mathbf{C}_{0,0}^{(1)})^{-1}\mathbf{F}. \tag{13}$$

The equilibrium and compatibility conditions at interface Λ_1 between first and second layers are:

$$\mathbf{t}_1^{(2)} = -\mathbf{t}_1^{(1)} \quad \text{and} \quad \mathbf{u}_1^{(2)} = \mathbf{u}_1^{(1)}, \tag{14}$$

and after substituting equations (12) and (13) into (14), we get:

$$\mathbf{t}_1^{(2)} = \mathbf{B}_{1,1}^{(1)}\mathbf{u}_1^{(2)} - \mathbf{D}_{1,1}^{(1)}\mathbf{F}, \tag{14a}$$

where: $\mathbf{B}_{1,1}^{(1)} = -\left(\mathbf{C}_{1,0}^{(1)}(\mathbf{B}_{0,0}^{(0)} - \mathbf{C}_{0,0}^{(1)})^{-1}\mathbf{C}_{0,1}^{(1)} + \mathbf{C}_{1,1}^{(1)}\right)$ and $\mathbf{D}_{1,1}^{(1)} = \mathbf{C}_{1,0}^{(1)}(\mathbf{B}_{0,0}^{(0)} - \mathbf{C}_{0,0}^{(1)})^{-1}$. These matrices give the relation between the tractions and displacements of the interface Λ_1. They are a product of the influence matrices $\mathbf{G}^{(1)}$ and $\mathbf{H}^{(1)}$ generated for the domain Ω_1 and matrix $\mathbf{B}_{0,0}^{(0)}$ generated for the previously considered domain.

In a similar way, the tractions at any interface Λ_i could be expressed by the corresponding displacements. The matrices $\mathbf{B}_{i,i}^{(i)}$ and $\mathbf{D}_{i,i}^{(i)}$ that relate these tractions and displacements contain the influence of all matrices generated for the previously considered layers.

Apparently, for the discrete field quantities of any internal contour Λ_i (for i=0:N-1) the following relations are valid:

$$\begin{aligned} &\mathbf{u}_i^{(i+1)} = \mathbf{u}_i^{(i)} = (\mathbf{B}_{i,i}^{(i)} - \mathbf{C}_{i,i}^{(i+1)})^{-1}\mathbf{C}_{i,i+1}^{(i+1)}\mathbf{u}_{i+1}^{(i+1)} + (\mathbf{B}_{i,i}^{(i)} - \mathbf{C}_{i,i}^{(i+1)})^{-1}\mathbf{D}_{i,i}^{(i)}\mathbf{F}; \\ &\mathbf{t}_i^{(i+1)} = \mathbf{B}_{i,i}^{(i)}\mathbf{u}_i^{(i)} - \mathbf{D}_{i,i}^{(i)}\mathbf{F} = \mathbf{B}_{i,i}^{(i)}\mathbf{u}_i^{(i+1)} - \mathbf{D}_{i,i}^{(i)}\mathbf{F}, \end{aligned} \tag{15}$$

where:

$$\begin{array}{l}
\left.\begin{aligned}
\mathbf{B}^{(i)}_{i,i} &= -(\mathbf{C}^{(i)}_{i,i-1}(\mathbf{B}^{(i-1)}_{i-1,i-1}-\mathbf{C}^{(i)}_{i-1,i-1})^{-1}\mathbf{C}^{(i)}_{i-1,i}+\mathbf{C}^{(i)}_{i,i})\\
\mathbf{D}^{(i)}_{i,i} &= \mathbf{C}^{(i)}_{i,i-1}(\mathbf{B}^{(i-1)}_{i-1,i-1}-\mathbf{C}^{(i)}_{i-1,i-1})^{-1}\mathbf{D}^{(i-1)}_{i-1,i-1}
\end{aligned}\right\}\text{for } i=1:N{-}1\\
\left.\begin{aligned}
\mathbf{B}^{(i)}_{i,i} &= -(\mathbf{G}^{(0)}_{0,0})^{-1}\mathbf{H}^{(0)}_{0,0}\\
\mathbf{D}^{(i)}_{i,i} &= [I]\ \text{(Identity matrix)}
\end{aligned}\right\}\text{for } i=0
\end{array}. \tag{16}$$

Finally, the most upper layer should be considered as a single domain. Rearranging the standard generated system of equations with respect to the tractions, the following equations are derived (in the absence of tunnels):

$$\begin{aligned}
\left\{\begin{array}{c}\mathbf{t}^{(N)}_{N-1}\\ \mathbf{t}^{(N)}_{N}\end{array}\right\} &= \left\{\begin{array}{c}\mathbf{B}^{(N-1)}_{N-1,N-1}\mathbf{u}^{(N)}_{N-1}-\mathbf{D}^{(N-1)}_{N-1,N-1}\mathbf{F}\\ \mathbf{t}^{(N)}_{N}\end{array}\right\} = \\
&= \left[\begin{array}{cc}\mathbf{C}^{(N)}_{N-1,N-1} & \mathbf{C}^{(N)}_{N-1,N}\\ \mathbf{C}^{(N)}_{N,N-1} & \mathbf{C}^{(N)}_{N,N}\end{array}\right]\left\{\begin{array}{c}\mathbf{u}^{(N)}_{N-1}\\ \mathbf{u}^{(N)}_{N}\end{array}\right\},
\end{aligned} \tag{17}$$

where:

$$\mathbf{C}^{(N)} = \left[\begin{array}{cc}\mathbf{C}^{(N)}_{N-1,N-1} & \mathbf{C}^{(N)}_{N-1,N}\\ \mathbf{C}^{(N)}_{N,N-1} & \mathbf{C}^{(N)}_{N,N}\end{array}\right] = \left(\mathbf{G}^{(N)}\right)^{-1}\mathbf{H}^{(N)}. \tag{18}$$

The first row of equation (17) can be obtained from the first relation of equations (15), when $i = N-1$. Starting from $i = 0$ for the seismic bed domain and contour Λ_0 and replacing $\mathbf{u}^{(i+1)}_{i+1}$ using first expression in equations (15) the following formula that relates displacements of the interfaces Λ_0 and Λ_N is reached:

$$\begin{aligned}
\mathbf{u}^{(0)}_{0} = &\left(\prod_{i=0}^{N-1}\left(\mathbf{L}^{(i)}\mathbf{C}^{(i+1)}_{i,i+1}\right)\right)\mathbf{u}^{(N)}_{N} + \\
&+ \mathbf{L}^{(0)}\left(\mathbf{F}+\sum_{i=1}^{N-1}\left(\left(\prod_{j=1}^{i}\left(\mathbf{C}^{(j)}_{j-1,j}\mathbf{L}^{(j)}\right)\right)\mathbf{D}^{(i)}_{i,i}\right)\mathbf{F}\right),
\end{aligned} \tag{19}$$

where: $\mathbf{B}^{(i)}_{i,i}$ and $\mathbf{D}^{(i)}_{i,i}$ are defined by equations (16), $\mathbf{L}^{(k)} = (\mathbf{B}^{(k)}_{k,k} - \mathbf{C}^{(k+1)}_{k,k})^{-1}$ for $k = 0, i, j$. In equation (19), N could be the number of the most upper interface of the model in Figure 1 or any number of interfaces between 0 and N. The relation between displacements of the free surface and arbitrary interface of the soil stratum, with running number m, reads:

$$\text{(19a)}\quad \mathbf{u}_m^{(m)} = \left(\prod_{i=m}^{N-1} \left(\mathbf{L}^{(i)} \mathbf{C}_{i,i+1}^{(i+1)} \right) \right) \mathbf{u}_N^{(N)} + \\ + \mathbf{L}^{(m)} \left(\mathbf{D}_{m,m}^{(m)} \mathbf{F} + \sum_{i=m+1}^{N-1} \left(\left(\prod_{j=m+1}^{i} \left(\mathbf{C}_{j-1,j}^{(j)} \mathbf{L}^{(j)} \right) \right) \mathbf{D}_{i,i}^{(i)} \right) \mathbf{F} \right).$$

The tractions at interface Λ_0, in line with equation (8a), expressed in terms of surface displacements could be written:

$$\text{(20)}\quad \mathbf{t}_0^{(1)} = \mathbf{B}_{0,0}^{(0)} \left(\left(\prod_{i=0}^{N-1} \left(\mathbf{L}^{(i)} \mathbf{C}_{i,i+1}^{(i+1)} \right) \right) \mathbf{u}_N^{(N)} + \\ + \mathbf{L}^{(0)} \left(\mathbf{F} + \sum_{i=1}^{N-1} \left(\left(\prod_{j=1}^{i} \left(\mathbf{C}_{j-1,j}^{(j)} \mathbf{L}^{(j)} \right) \right) \mathbf{D}_{i,i}^{(i)} \right) \mathbf{F} \right) \right) - \mathbf{F}.$$

The procedure of consecutive elimination of the layers is very suitable for modelling of soil regions with large number of stratums, because its application results to considerable reduction of the model degrees of freedom, respectively the computational time. This consecutive exclusion leads to the inversion of several smaller systems of linear algebraic equations instead of solving a single one but containing all nodal quantities of the whole layered model. Independently of the number of layers, the final system of linear algebraic equations contains only the degrees of freedom, respectively unknown field quantities of the last considered layer – most upper one or seismic bed domain. The computer memory and the computational time required for solving this system is the same as that for a single layer with two interfaces only, while the classical algorithm needs to calculate the system generated for the whole model with all layers. The reduction is proportional to the number of infinite layers, larger number of layers smaller degrees of freedom compared to classical BEM modelling.

Application of the technique proposed here allows establishing a relation between displacements of the free surface and each single interface of the soil stratum including the lowest contour of the seismic bed given by equations

(19) and (19a). This feature is very useful for synthesis of seismic signals in a layered half-plane with arbitrary geometry of interfaces between layers.

So, here, an extension of the idea of global matrix propagator method is presented by the use of BEM based on the sub-structuring approach. As a result, simple relations (Eq. 19–Eq. 20)) between displacements and tractions along any interface with those at the seismic bed boundary, assuming an arbitrary geometry of layer contours, is derived.

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