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AN APPROACH TO THE ESTIMATION OF FATIGUE LIFE UNDER BLOCK AND RANDOM LOADING*

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1. Introduction

Mechanical systems are usually subjected to cyclically or randomly varying loads in which amplitude and/or mean levels fluctuate irregularly with time. Although there are many theories of fatigue damage accumulation, the results of useful life estimation are not always satisfactory. This is due to the fact that none of the hypotheses is sufficiently general to consider the effects of all the interacting factors which influence useful life. Therefore the prediction of fatigue life continues to be a problem, especially when mean stresses are present and investigators are still trying to improve the methods for life estimation under such conditions.

The accumulation of fatigue damage in metals is a result of cyclic plastic deformation. Under cyclic loading, stress-strain hysteresis loops result. The hysteresis loop defines the best released in cyclic deformation and is considered to be the best definition of a damaging event.

There are many relations [1] describing the mean stress effect in high cycle fatigue, but none of them is energy based. In this study a simple procedure to predict the fatigue life under a varying loading condition was developed by using the hysteresis energy as damage parameter and a relationship between hysteresis energy, mean stress and stress amplitude.

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2. Damage parameter and mean stress effect

A number of authors[2] have advanced several combinations of variables, known as damage parameters, with the goal of describing the fatigue damage process. This could lead to different damage parameter/life curves. These curves are used for the evaluation of cycles with arbitrary combinations of stress and strain ranges and mean stress.

$$P(\Delta\sigma, \Delta\varepsilon, \sigma_m) = P(N_f) \tag{1}$$

In the present work the inelastic strain energy per cycle W was used as a damage parameter. W is the area of the hysteresis loop. The measured hysteresis loop at half-life was taken to be representative and was used to calculate the hysteresis energy by digital integration along the hysteresis branches. For this purpose a programme was written in Fortran 77 on an IBM PC.

It was assumed that the same fatigue life would be observed if W from the completely reversed test was equal to W_m from the mean stress test. Therefore a relationship can be written for a given fatigue life N_f as:

$$W = W_m = \text{constant} \tag{2}$$

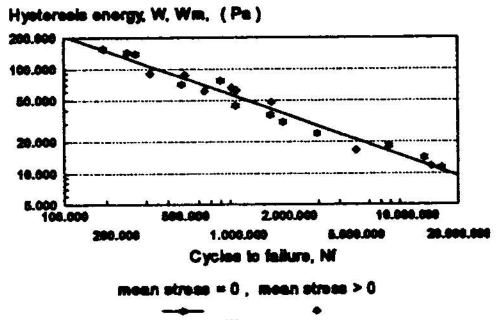


Fig.1. Damage parameter - life curve
for load - conrtolled cyclic tests.

High cycle fatigue push-pull tests were carried out on specimens from St.45 low carbon steel. Information about test method, material and specimens is given in the next section. The hysteresis energy W_m vs number of cycles to failure N_f is plotted in Fig. 1. The log-log linear regression analysis was done using total and only $\sigma_m = 0$ test data points. The equations are given as:

$$W = a.N_f^d = 1.717.10^8.N_f^{-0.584} \tag{3}$$

(4) $$W_m = p.N_f^q = 1.789.10^8.N_f^{-0.591}$$

It was found that equations (3) and (4) look similar. Therefore we can conclude that the hysteresis energy-life curve is not affected by the mean stress and N_f for test with $\sigma_m \neq 0$ can be evaluated from the hysteresis energy-life curve for fully reversed test, if W_m is known.

One of the basic causes of the effect of mean stress on fatigue life is that the mean stress affects the stable stress-strain behaviour of materials.[1]

Fatigue tests with constant stress amplitude and different mean stress were carried out [3] and was suggested by experimental results a power low type relationship between W_m, σ_a, and σ_m as:

(5) $$W_m = k_1.\sigma_a^{n_1}.A^{\sigma_m}$$

where k_1, n_1, A are material constants, determined from the linear regression analysis – Table 2. The hysteresis energy calculated by proposed model, eqn.(5), was in good agreement with the measured values.[3] Therefore, the hysteresis energy as damage parameter vs number of cycles to failure may be expressed as:

(6) $$k_1.\sigma_a^{n_1}.A^{\sigma_m} = a.N_f^d$$

Comparison was made for constant amplitude loading with other proposed mean stress models as the Morrow model, SWT parameter and the Lorenzo-Laird model [3]. The energy-based damage parameter and mean stress model best represented the data. It is the objective of this paper to apply this approach to block and random loading.

3. Test method

3.1. Material and specimens

The material used in this study was a (0.45% C) normalized low carbon steel (880^0 C ×1.2 hr, cooled in air) and its mechanical properties are shown in Table 1. Test specimens, the dimension of which is shown in Fig.2, were turned from a bar of 20mm diameter and were polished carefully with a fine emery paper.

Table 1. Mechanical properties of material.

Yield strength, σ_y =320 MPa	Modulus of elasticity, E=200340 MPa
Ultimate strength, σ_u =640 MPa	Elongation, δ=25 %

3.2. Testing machine and fatigue tests

Fatigue tests were conducted using a "SCHENCK HYDROPULS" push-pull servo-hydraulic testing machine with load capacity of 40 kN. An IBM personal computer

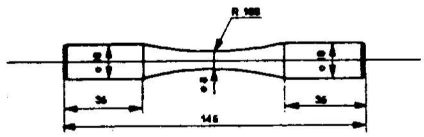


Fig. 2. Test specimen

(PC) was used for the generation of loading and for the record of data, continuously during fatigue testing. A strain gauge was mounted on the specimen to measure the cyclic strain. Thus, the continuously controlled data were load amplitude and mean load, and the continuously recorded data were measured values of stress and strain. Test frequency was 15 Hz.

Two different type of fatigue test programmes were carried out: 1) Gaussian type block loading without a mean load, with constant and varying mean load – Fig.3.1., 2) Gaussian type block loading of randomly arranged cycles obtained by applying the rainflow cycle counting method to the random loading process – Fig.3.2. In this study, a Gaussian type frequency distribution ($\nu = 0.75$), in which 10^6 cycles (block loading, Fig.4a) and 10^2 cycles (randomly arranged cycles, Fig.4b) were defined as one block, was used.

4. Results and discussions

4.1. Fatigue properties of the material used

The basic fatigue properties of the material used in this study were tabulated in Table 2. All relationships were obtained by analyzing the test results of fully reversed stress controlled fatigue tests.

4.2. Fatigue life prediction under varying loading

The fatigue life prediction was commonly carried out using the linear cumulative damage rule known as the Palmgren-Miner rule. In most cases, however, the evaluation of the influence of the change in the amplitude was only discussed and that of the mean load was usually ignored. In this study, the life prediction by using the hysteresis energy

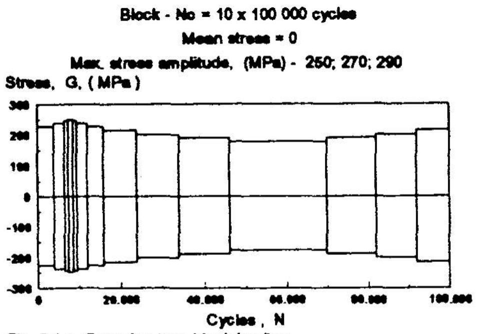


Fig. 3.1.a. Gaussian type block loading

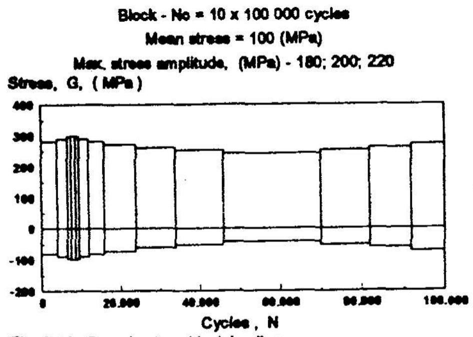


Fig. 3.1.b. Gaussian type block loading

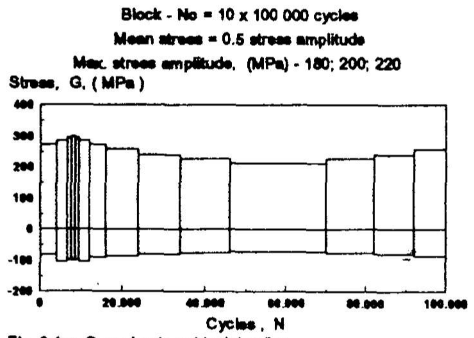


Fig. 3.1.c. Gaussian type block loading

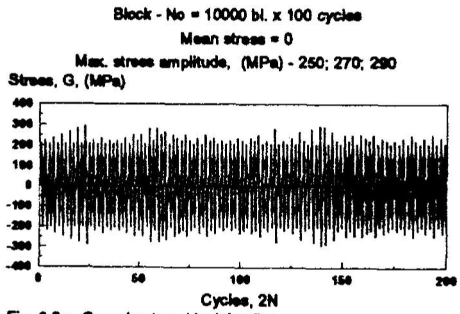


Fig. 3.2.a. Gaussian type block loading
of randomly arranged cycles

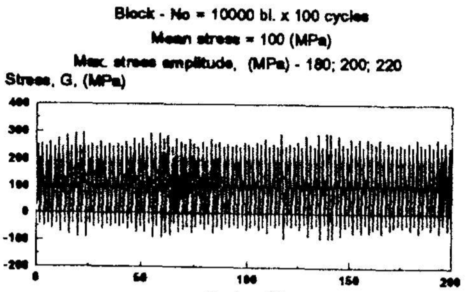


Fig. 3.2.b. Gaussian type block loading
of randomly arranged cycles

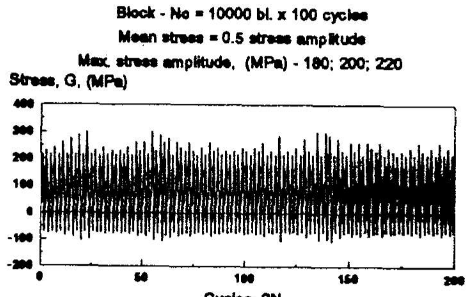


Fig. 3.2.c. Gaussian type block loading
of randomly arranged cycles

Table 2. Basic fatigue properties of material.

n	k(MPa)	σ_f(MPa)	ε_f	b	c
0.198	1221	540.78	0.025	-0.069	-0.357

n_1	k_1 (Pa)	d	a(Pa)	A	ψ	σ_{-1} (MPa), N_b=10^7
8.178	$6.429.10^{-15}$	0.584	$1.717.10^8$	1.004	0.75	175.02

Cyclic stress-strain curve $\varepsilon_a = \frac{\sigma_a}{E} + (\frac{\sigma_a}{K})^{1/n}$
Coffin-Manson relation $\varepsilon_a = \frac{\sigma_f}{E}.N_f^b + \varepsilon_f.N_f^c$
Damage parameter-life curve $W = W_m = k_1.\sigma_a^{n_1}.A^{\sigma_m} = a.N_f^{-d}$

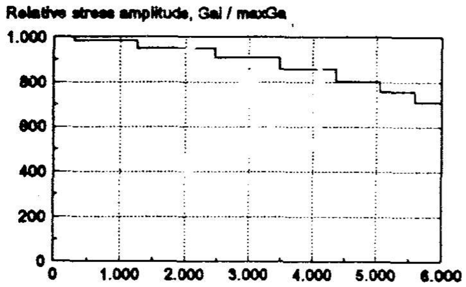


Fig. 4a. Cumulative frequency distribution of load amplitude

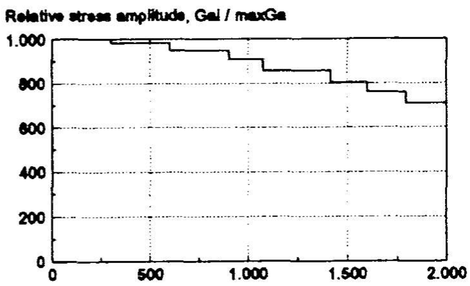
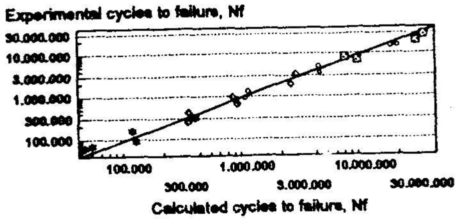


Fig. 4b. Cumulative frequency distribution of load amplitude

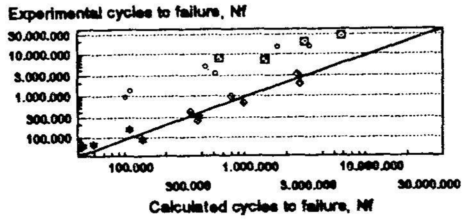
as damage parameter and energy based mean stress model to reflect the influence of mean load was examined. The procedure of analyses, counting and classifying of cycles with identical σ_{ai} and σ_{mj}, calculation and accumulation of fatigue damage, D_{ij}, by using the fatigue damage parameter, eqn.(6), and decision of failure at the time when the value of D_{ij} reached to the critical value, D_c, is shown below. In case under block loading, the prediction was done by assuming the value of $D_c = 0.7$ and in case under random loading, $D_c = 0.5$. The mean stress effects on fatigue life were evaluated with the Morrow model, eqn.(7), SWT parameter, eqn.(8) and ψ-method, eqn.(9). A comparison between experimental data and the predictions of the proposed method and other models are shown in Fig.5.



block loading, Gm = 0 ; block loading, Gm > 0

random loading, Gm > 0 ; random loading, Gm = 0

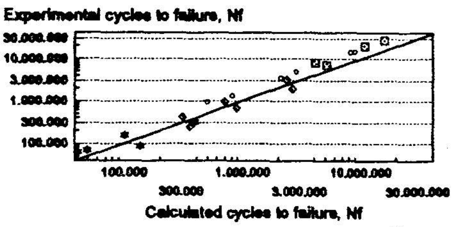
Fig.5.1. Comparison between experimental and calculated Nf by present method.



block loading, Gm = 0 ; block loading, Gm > 0

random loading, Gm > 0 ; random loading, Gm = 0

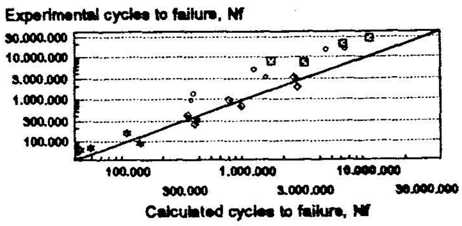
Fig.5.2. Comparison between experimental and calculated Nf by Morrow s model



block loading, Gm = 0 ; block loading, Gm > 0

random loading, Gm > 0 ; random loading, Gm = 0

Fig.5.3. Comparison between experimental and calculated Nf by SWT-parameter.



block loading, Gm = 0 ; block loading, Gm > 0

random loading, Gm > 0 ; random loading, Gm = 0

Fig.5.4. Comparison between experimental and calculated Nf by ψ - method.

(7) $$\varepsilon_a = \frac{\sigma_f - \sigma_m}{E}.N_f^b + \varepsilon_f.N_f^c$$

(8) $$P_{SWT} = \sqrt{\sigma_{max}.\varepsilon_a.E} = \sqrt{\sigma_f^2.N_f^b + \sigma_f.\varepsilon_f.E.N_f^{b+c}}$$

(9) $$\sigma_{am} = .\sigma_a + \psi.\sigma_m; \qquad N_f = (\frac{\sigma_{am}}{\sigma_f})^{1/b}$$

σ_{am} – equivalent stress amplitude; ψ – Table 2.

Procedure for fatigue life prediction under varying loads

input data		mean stress model		damage parameter
σ_{ai} , σ_{mj} ,n_{ij}	$\rightarrow$	$W_{mij} = k_1.\sigma_{aj}^{n_1}.A^{\sigma_{mj}}$	$\rightarrow$	$W_{mij} = a.N_{fij}^{d}$

$\boxed{D_{ij} = \frac{n_{ij}}{N_{ij}}} \rightarrow \boxed{\sum_1^j(\sum_1^i D_{ij}) \geq D_c} \xrightarrow{yes} \boxed{\text{Failure}}$

no

Palmger-Miner rule

Conclusion

1. The capability and accuracy of an energy based approach, describing the mean stress effect on the hysteresis energy, to predict fatigue life were evaluated using the results of Gaussian type block programmed fatigue tests. Predictions of the proposed method are in good agreement with the experimental data for various types of test.

2. The predicted life showed a good agreement with the test result, by assuming $D_c < 1$. It is connected with the effect of load sequence.

3. The proposed energy based damage parameter and mean stress model are better suited for a description of the mean stress effect on the high cycle fatigue life than the Morrow model, SWT parameter and ψ-method.

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