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A FAST METHOD FOR PREDICTION OF RELIABILITY AND FATIGUE-LIFE

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Notations and symbols:

N – estimate of the expected number of cycles to fracture

σ_c – stress-fatigue limit

N_o – basic number of cycles in a fatigue strength curve

m – exponent of a fatigue strength curve of the type: $N.\sigma^m = N\sigma_c^m$

$p(\sigma)$ – probability density of stress amplitude levels

σ_i – i-th stress amplitude level

X_p – quantiles of the approximating distribution $F(X)$ for the normalized quantity $X = \sigma/\sigma_{\max}$

$V = \left[\int X^b.P(\sigma).d\sigma\right]^{1/b}$ – form factor of the spectrum

W_x – variance coefficient of the stress levels

K_m – conversion factor

$b \leq m$ – exponent according to the applied linear hypothesis

$n_\sigma = X_c/V$ – safety factor of fatigue fracture

$A = X_c$ – reduction factor accounting for overloading

$R = 1 - P > 0.5$ – probability of avoiding fracture

N_R – probability estimate of the cycles to fracture

$\gamma = \dfrac{1.28}{m\,[\ln(1+W_{\sigma c}^2)]^{1/2}}$ – parameter of the fatigue-life distribution

$Q = N/(\ln 2)^{1/\gamma}$ – a fatigue-life characteristics after Weibull's model

$W_{\sigma c}$ – variance coefficient of the stress-fatigue limit

U_p – quantile of the normal distribution $P(0,1)$

1. Introduction

It is well known [1] that in designing of components and structures the expected fatigue-life is predicted using a linear hypothesis about the accumulation of damages. The general form of this dependence is:

$$N = \frac{N_o \sigma_c^m}{\int_{\sigma_{\min}}^{\sigma_{\max}} \sigma^m . p(\sigma) d\sigma} \tag{1}$$

The analyses performed in [2] and [3] indicate that expression (1) can be simplified and transformed into a relation of the following type:

$$N/N_o = A^m . V^{-b} = A^{m-b} . n_\sigma^b \tag{2}$$

This relation is a basis for the estimation and prediction of the fatigue-life and the corresponding reliability index of the boundary strength condition of "fatigue fracture", the problem being reduced to establishing the form factor of the load spectrum.

The purpose of the present work is to develop a methodological framework for a fast computational procedure for estimation of fatigue-life and its probability distribution through approximation of the variable load with adequate statistical models.

2. A method for presumptive estimation of fatigue-life

As it is well-known, the initial phase of processing of the load spectrum includes the following major steps: (i) modelling of the load and its reduction into a loading unit of amplitude levels with corresponding recurrence frequencies; (ii) approximation of the unit with an appropriate statistical model and (iii) application of a selected linear hypothesis for accounting of the damaging action. Each one of these steps contributes to the precision of the final engineering estimate of fatigue-life, but of decisive importance is the generalized description of the damaging action of the spectrum, expressed by means of its form factor. For its fast determination, a combined graphic and analytical approach, based on the following estimating relation is proposed in [3]:

$$V = \left[0.1 \sum_{P=0.1}^{0.9} X_P^b \right]^{1/b} \tag{3}$$

The accuracy of this estimation can be improved by means of a computational procedure, where the quantile values of the employed distributions are represented as a function of their corresponding parametres, i.e. of the empirical statistical characteristics of the load spectrum respectively. Thus, relation (3) is transformed into an analytical dependence of the type:

$$V = K_m . \overline{X} \tag{4}$$

where the expressions for determining the conversion factor K_m are systematized in Table 1.

Table 1.

Model	K_M^b
Normal	$0.1 \sum_{P=0.1}^{0.9} (1 + U_P W_X)^b$
Lognormal	$\frac{0.1}{(1+W_x^2)^{b/2}} \sum_{P=0.1}^{0.9} e^{b.U_p[ln(1+W_x^2)]^{1/2}}$
Weibull	$\frac{0.1 e^{0.577b/\beta}}{(1+W_x^2)^{b/2}} \sum_{P=0.1}^{0.9} \left(\ln \frac{1}{1-P} \right)^{b/\beta}$
Rayleigh	$0.10.8^b \sum_{P=0.1}^{0.9} \left(\ln \frac{1}{1-P} \right)^{b/2}$
Exponential	$0.1 \sum_{P=0.1}^{0.9} \left(\ln \frac{1}{1-P} \right)^{b}$

From the basic relation (2) it follows that with similar load spectra, characterized by the same load form factor, the decisive factor for life expectancy is the index of overloading. Overloading is defined as the ratio between the stress-fatigue limit and the maximum stress in the load spectrum. It can be predicted with a preset level of confidence and depends both on the type of the statistical model and on the volume of empirical information:

(5) $$F(X > X_{\max}) = 1 - P = n_{bl}^{-1},$$

where n_{bl} – volume of the loading unit.

The model of Weibull is commonly used for description of the probability distribution of fatigue-life. Considering this, a relation of the following type is derived in [2]:

(6) $$N_R = K_R \Theta,$$

where the probability factor K_R is tabulated in Table 2 for typical values of normalized reliability and of the parameter of the distribution model:

Table 2.

γ	R				
	0.9	0.95	0.99	0.995	0.999
1	0.105	0.050	0.010	0.005	0.001
2	0.324	0.226	0.100	0.071	0.032
3	0.472	0.372	0.216	0.171	0.100
4	0.570	0.476	0.317	0.266	0.178

In the presence of several critical sections with respect to the discussed boundary strength condition, the probability of avoiding fracture is a function of the corresponding partial values treated as a system of consecutively linked elements:

(7) $$R = \prod_i R_i$$

In the particular case when the critical points have equal probability distributions of failures, the guaranteed life will be determined by the expression:

(8) $$N_R = K_R \Theta_1 / Z^{1/\gamma}$$

Z – number of critical points.

In the case of different expected partial characteristics, the overall estimate of guaranteed life will be obtained through the relation:

$$N_R = K_R\Theta_1 / \left[1 + \sum_{i=1}^{z}\left(\frac{\Theta_1}{\Theta_i}\right)^{\gamma}\right]^{1/\gamma} \tag{9}$$

3. Practical application

For verification of the proposed computational procedure are used data from an investigation of the service life of a truck trailer frame with respect to the boundary strength condition of "fatigue fracture" [4]. The monitored sample has been tested after a (n, U, n) plan and failures have been observed in the form of fatigue-induced cracks in the presence of stress concentrators. The life limiting failures are in the area of the so-called "swan neck" (section 1) and in the area of the rear cross-pieces (section 2). The results of the experimental investigation of the failures in these two considered sections are presented in Fig. 1 in Weibull's probability grid $(\lg N; \lg\ln\frac{1}{1-p})$, where $P = \frac{i}{n+1}$. By means of strain measurement during exploitation, a representative loading unit with the following characteristics has been formed: $n_{bl} = 10^6$cycles $\cong$ $4.2.10^3$ km.$\sigma_{\max} = 140$ MPa; $\overline{X} = 0.206$; $W_x = 0.537$.

According to the literature, Wohler's curve of the investigated structure has the following parameters: $\sigma_c = 75MPa$; $N_o = 2.10^6$ cycles; $m = 3.5$; $W_{\sigma_c} = 0.12$.

According to the proposed methodology the estimate of expected service life with respect to failures in section 1, and using Corten-Dolan's hypothesis, ($b = 0.8$ m) will be derived from the expression:

$$N_1 = 2.10^6(75/140)^{3.5}(K_m.0.206)^{-2.8} = 15.9.10^6 \text{ cycles} = 67.10^3 \text{ km},$$

where, after approximating the spectrum according to Weibull's model, $K_m = 1.06$.

The parameters of the model of fatigue-life distribution in this case are:

$$\gamma = 3.058;\quad \Theta_1 = 17.9.10^6 \text{ cycles} \cong 75.10^6\text{km}.$$

In case of a ratio between the partial fatigue-lives in the considered critical sections ~ 1.5, the values of guaranteed service life of the whole structure in real exploitation will be obtained by means of relation (9): $N_R = K_R\frac{\Theta_1}{(1+1.5^{\gamma})^{1/\gamma}} =$ $K_R 10.74.10^6$, where for $R = 0.9$ $N_{0.9} = 5.0710^6 \cong 21.3.10^3$ km

$R = 0.99$ $N_{0.99} = 2.32.10^6 \cong 9.7.10^3$ km

The distribution of predicted service life is also presented in Fig. 1.

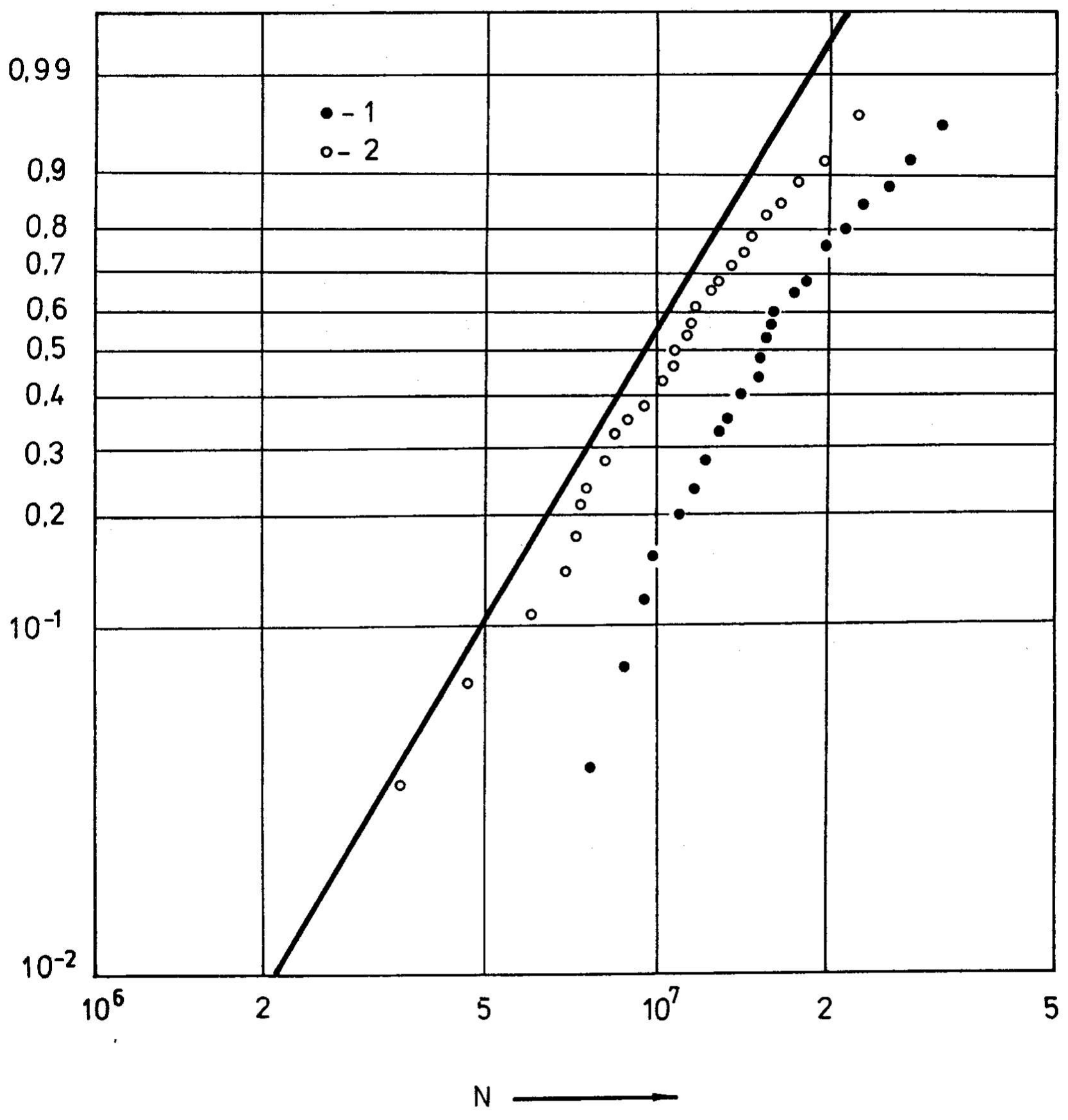

Fig. 1.

4. Conclusions

– A fast method for prediction of reliability and fatigue-life of mechanical components and structures during the stage of design, using fundamental empirical characteristics of the load spectrum, is proposed and programmed.

– The predicted values agree with the results of exploitation tests with an accuracy, sufficient for practical purposes.

– The method can be used for proximate analysis of engineering designs.

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