Journal of Theoretical
and Applied Mechanics
Sofia 1993, Year XXIV No 2

ELASTIC-HYDRODYNAMIC LUBRICATION OF DYNAMICALLY LOADED FINITE JOURNAL BEARINGS

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1. Introduction

Journal bearing surfaces are sometimes lined with materials that are much softer than the usual bearing metals. The elastic distortion of journal bearing housings can have a significant influence on the lubricant film thickness and, consequently, upon the pressure distribution.

There are a number of publications where the influence of elastic deformations of the liner on the journal bearing performance is considered (p.e. [1],[2]). However in these studies it is only bearings subjected to constant unidirectional load that are dealt with.

The objective of the present study is to analyze the influence of the deformation of plastic liner around the shaft on the performance of dynamically loaded journal bearings.

A 2D-problem of a finitely long journal bearing is considered by a simplified elasticity theory in accordance with the Winkler hypothesis. The problem is studied under isothermal and isoviscous conditions. The solution is obtained for prescribed loci of the shaft centre. The contribution of the inertial forces to the bearing dynamics is assumed negligible.

2. Notations (see Fig.1)

c – radial clearance;

d – thickness of the shaft liner;

E – Young modulus;

e – eccentricity;

h_0 – film thickness (rigid case; $h_0 = c + e\cos\vartheta$);

h_1 – film thickness (flexible case; $h_1 = h_0 + \delta$);

L – axial length of the bearing;

p_0 – pressure (rigid case);

p_1 – pressure (flexible case);

r – outer radius of the shaft liner;

t – time;

u, v, w – velocity components in x, y, z directions;

W – total load;

γ – angular deviation;

δ – radial distortion ($\delta = \dfrac{2(1-\mu^2)d}{\pi E}p$);

η – viscosity of the lubricant;

μ – Poisson ratio;

ω – angular velocity of the shaft.

Dimensionless parameters

$$H = \frac{h}{c}; \quad \Pi = \frac{c^2}{6\eta\omega r^2}p; \quad \tau = \frac{\omega t}{2}; \quad z_1 = \frac{2z}{L}; \quad \bar{\delta} = \frac{\delta}{c};$$

$\alpha = \left(\dfrac{2r}{L}\right)^2$ – diameter-to-length ratio;

$\varepsilon = \dfrac{e}{c}$ – eccentricity ratio;

$\vartheta = \dfrac{x}{r}$ – angular coordinate measured from the line of centres;

$K_h = \dfrac{6\eta\omega r^2}{c^3}\dfrac{2(1-\mu^2)}{\pi E}d$ – elasticity factor;

$S_0 = \dfrac{Wc^2}{\eta L\omega r^3}$ – Sommerfeld number.

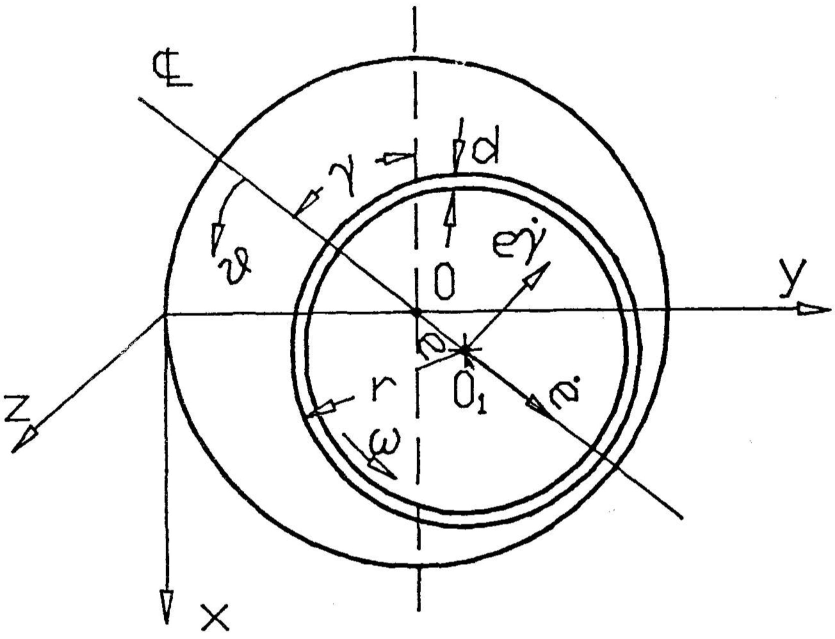


Fig. 1.

3. Theoretical foundations

Under the assumption that $\dfrac{\partial^2 q}{\partial x^2} \ll \dfrac{\partial^2 q}{\partial z^2} \ll \dfrac{\partial^2 q}{\partial y^2}$, (where $q = u, v, w$) and that $\frac{\partial p}{\partial x}$ and $\frac{\partial p}{\partial z}$ do not vary across the film, the Navier-Stokes equations lead to:

$$
(1) \qquad \begin{aligned} u(x,y,z,t) &= \frac{1}{2\eta}\frac{\partial p}{\partial x}(y-h)y + \frac{1}{h}u_h y; \\ w(x,y,z,t) &= \frac{1}{2\eta}\frac{\partial p}{\partial z}(y-h)y. \end{aligned}
$$

Since the fluid film is thin enough, the radial velocity is expressed in the form

$$
(2) \qquad v(x,y,z,t) = C_1(x,z,t)y + C_2(x,z,t).
$$

The boundary conditions are taken (Fig.1)

$$
(3) \qquad \begin{aligned} &u(x,0,z,t) = u_0 = 0; && u(x,h,z,t) = u_h = \omega r + \dot{e}\sin\vartheta - e\dot{\gamma}\cos\vartheta; \\ &v(x,0,z,t) = v_0 = 0; && v(x,h,z,t) = v_h = \omega r \tfrac{\partial h}{\partial x} + \dot{e}\cos\vartheta + e\dot{\gamma}\sin\vartheta; \\ &w(x,0,z,t) = w_0 = 0; && w(x,h,z,t) = w_h = 0. \end{aligned}
$$

The above lead for the radial velocity to

$$v(x,y,t) = \frac{1}{h}(\omega r \frac{\partial h}{\partial x} + \dot{e}\cos\vartheta + e\dot{\gamma}\sin\vartheta)y. \tag{4}$$

After substituting the expressions for u, v, w and the above boundary values in the continuity condition across the film

$$\int_0^h \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) dy = 0, \tag{5}$$

the final partial differential equation looks like

$$\begin{aligned} \frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right) &= 6\eta\omega r\frac{\partial h}{\partial x} - \\ &-6\eta(\dot{e}\sin\vartheta - e\dot{\gamma}\cos\vartheta) + 12\eta(\dot{e}\cos\vartheta + e\dot{\gamma}\sin\vartheta). \end{aligned} \tag{6}$$

This equation differs from the classical Reynolds equation in the last two terms on the right-hand side only. They take into account the pressure changes due to the temporal variation of the film thickness.

The physical meaning of (6) is that the generation of hydrodynamic pressure within the film depends on three terms, the so-called wedge, stretch and squeeze contributions to the load support. The magnitude of the stretch term, however, is small enough compared to the wedge or squeeze terms. The squeeze contribution is due to the fact that the surfaces get closer in normal direction with or without the occurrence of sliding between them. This event could be caused by impact, slow speed squeezing action, or more commonly by mutual surface vibrations in normal direction.

After introducing non-dimensional variables, the governing differential equation takes the final form

$$\frac{\partial}{\partial\vartheta}\left(H^3\frac{\partial\Pi}{\partial\vartheta}\right) + \alpha\frac{\partial}{\partial z_1}\left(H^3\frac{\partial\Pi}{\partial z_1}\right) = \frac{\partial H}{\partial\vartheta} + (\dot{\varepsilon}\cos\vartheta + \varepsilon\dot{\gamma}\sin\vartheta). \tag{7}$$

4. Numerical solution

In order to solve partial differential equation (7), the finite difference method is used. The derivatives in (7) are expressed by finite differences in two variables. The boundary conditions are taken

$$\begin{aligned} &\Pi = 0 & & at \quad \vartheta = 0; \\ &\Pi = 0, & \frac{\partial\Pi}{\partial\vartheta} = 0 \quad & at \quad \vartheta = \vartheta_1, \end{aligned} \tag{8}$$

where ϑ_1 is the local cavitation angle. The numerical procedure is iterative and the successive over-relaxation method is used to improve the convergence rate.

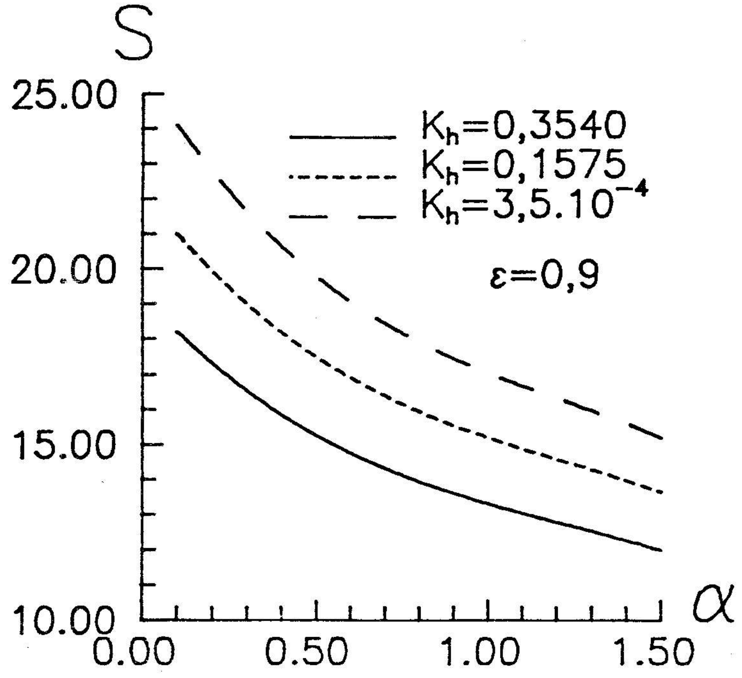


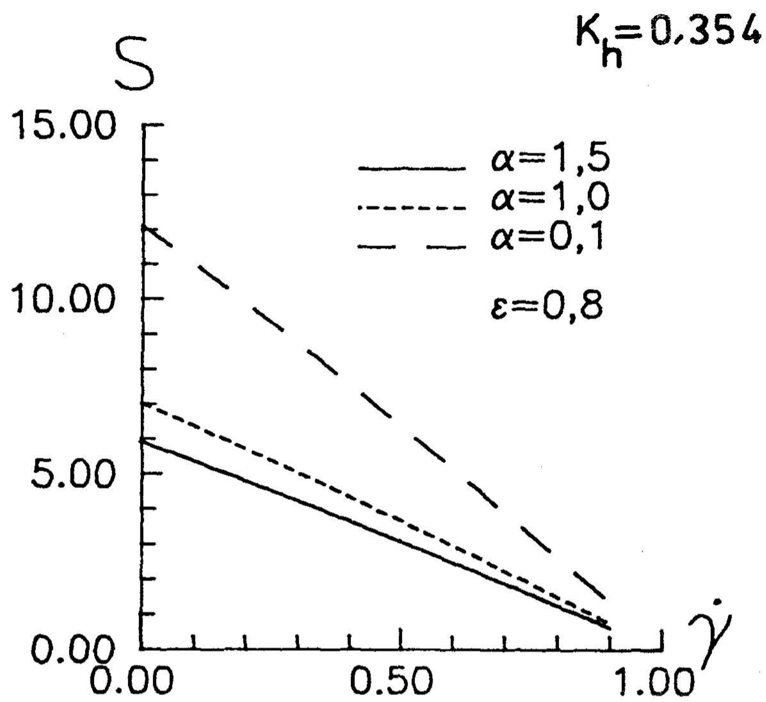
Fig. 2.

5. Numerical results

The numerical results are shown in Figs 2 – 4. They have been obtained for a wide range of values for $\alpha, \dot{\gamma}, \dot{\varepsilon}$ and K_h. Fig. 2 shows the load parameter S_0 (Sommerfeld number) vs. the diameter-to-length ratio α for various liner stiffness K_h. The effect of the tangential velocity of the journal centre on the same parameter S is shown in Fig. 3 for different values of α. The variation of S with the radial velocity of the journal centre for various α is given in Fig. 4.

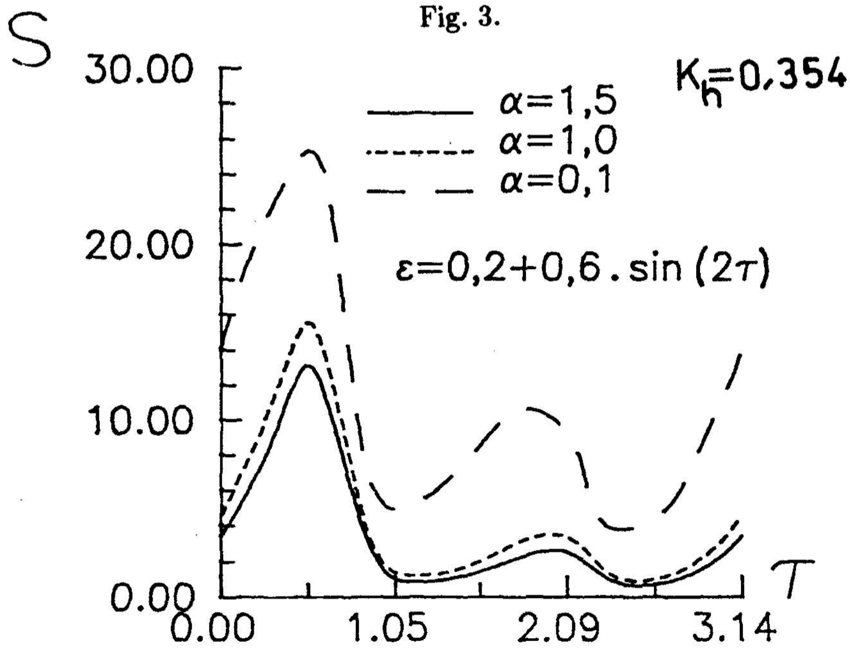
6. Conclusions

Journal bearing research has afforded numerous theoretical results in the field of the EHD lubrication. Most of the authors conclude that due to the deformation of the bearing liner the side leakage of the oil is negligible and the pressure variation along the axis is insignificant. However the solution presented in this paper shows the noticeable influence of the L/D ratio on the load capacity of the bearing. Some authors (p.e. [3]) assume that the squeeze film action is of little importance. The results in the present paper demonstrate that this assumption is an over-simplification.



K_h=0,354
S
15.00
10.00
5.00
0.00
α=1,5
α=1,0
α=0,1
ε=0,8
0.00
0.50
1.00
$\dot{\gamma}$

Fig. 3.



S
30.00
20.00
10.00
0.00
α=1,5
α=1,0
α=0,1
K_h=0,354
ε=0,2+0,6 . sin (2τ)
0.00
1.05
2.09
3.14
τ

Fig. 4.

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Presented at the 7th Congress on Theoretical and Applied Mechanics, Sofia, August 30 – September 1, 1993
section: Applied Mechanics