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SOLID MECHANICS

NUMERICAL SIMULATION OF DENSITY DISTRIBUTION DURING COMPACTION OF POROUS MATERIALS*

N. BONTCHEVA, R. IANKOV

1. Introduction

The porous metal undergoes a density change during the metal forming. Since the mechanical proporties of the final P/M products are greatly affected by the density or density distribution, it is very important to estimate them. There are efficient experimental or semi–experimental methods for obtaining the density distribution. Analytical methods of approach, based on the slab method [2, 3], the slip–line method [2, 3] and the upper–bound method [4, 3] for different metal forming problems are applied. Evaluation of density, stress and strain fields may be done effectively by means of numerical analysis avoiding experimental approaches. The elastic–plastic finite element approach is applied for this purpose by [5].

In the present study, the rigit–plastic finite element approach is used for analysis of forming processes of porous metals. This method is based on the upper bound theorem and the finite element solution in velocities.

2. Mathematical model

The plastic flow due to the metal forming of porous metals is charaterised by large plastic strains and volume changes. A yield criterion containing the first stress invariant should be applied. In the present model a yield criterion, proposed by Betten is used [6].

(1) $$F \equiv AII_s^2 + BI_1{}^2 + CII_s + DI_1 - \sigma_p^{*2} = 0,$$

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where

$$
\begin{aligned}
II_s &= \sqrt{\tfrac{1}{2}s_{ij}s_{ij}}, \\
s_{ij} &= \sigma_{ij} - \tfrac{1}{3}\sigma_{kk}\delta_{ij}, \\
I_1 &= \sigma_{ij}\delta_{ij} = \sigma_{kk},
\end{aligned} \tag{2}
$$

and σ_{ij} is the Cauchy stress tensor, A, B, C and D are experimentally obtained material functions depending on the relative density, σ_p^* is the generalized yield limit which depends on the yield limit of the base material σ_p^b and on the material function $\beta = \beta(\rho)$

$$\sigma_p^* = \beta\sigma_p^b. \tag{3}$$

The relative density is present by the ratio of the volume of the base metal V_b to the total volume of the porous metal V and $V = V_b + V_{por}$, V_{por} is the volume of pores in V

$$\rho = V_b/V. \tag{4}$$

The yield limit of the base material depends on the effective strain–rate $\dot{\bar{\varepsilon}}_b$ of the base material, on its effective strain $\bar{\varepsilon}_b$ and on temperature T

$$\sigma_p^b = \sigma_p^b(\bar{\varepsilon}_b, \dot{\bar{\varepsilon}}_b, T) \tag{5}$$

where

$$\bar{\varepsilon}_b = \int_0^t \dot{\bar{\varepsilon}}_b dt. \tag{6}$$

According to the associated flow rule, the plastic strain–rates and the volumetric strain–rate are:

$$
\begin{aligned}
\dot{\varepsilon}_{ij} &= \dot{\lambda}\left(\frac{\partial F}{\partial I_1}\delta_{ij} + \frac{\partial F}{\partial II_s{}^2}s_{ij}\right), \\
\dot{\varepsilon}_v &= \dot{\varepsilon}_{kk} = 3\dot{\lambda}\frac{\partial F}{\partial I_1},
\end{aligned} \tag{7}
$$

where

$$\dot{\varepsilon}_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}) \tag{8}$$

is the strain–rate tensor of the porous material and v_i is the velocity vector.

The rate of plastic energy dissipation is given by

$$\dot{W}_p = \sigma_{ij}\dot{\varepsilon}_{ij} = \sigma_p^*\dot{\varepsilon}_{ij}. \tag{9}$$

The density–rate kinetic equation is

$$\dot{\rho} = -\rho\dot{\varepsilon}_v. \tag{10}$$

The plastic flow is considered as quasi–static and the equilibrium equations are

$$\sigma_{ij,j} = 0. \tag{11}$$

Friction conditions at the die–material interface greatly influence metal flow, formation of surface and internal defects, stresses acting on the die as well as load and energy requirements. We assume Siebel friction law with friction factor depending on the relative density

$$\tau_t = \frac{1}{\sqrt{3}} m(\rho)\sigma_p^b, \tag{12}$$

where τ_t is the friction stress, directed opposite to the tangential velocity.

The surface roughness parameter R_{max} evolution is given by the following kinetic equation [7]

$$\frac{dR_{max}}{d\bar{\varepsilon}} = R\rho^{\gamma}, \tag{13}$$

where γ and $R = kG$ are material constants, G is the mean cristal grain diameter and k is a given constant, associated with the sintered preform.

Assuming different values of the material constants in eq.(1) we obtain the models of *Doraivelu, Shima and Oyane* and etc. For example if $C = 0, D = 0$ and A, B are appropriate material functions, the model proposed by *Doraivelu* [8] is obtained.

$$\begin{aligned} \sigma_p^* &= \sqrt{2\rho^2 - 1}\sigma_p^b \\ A(\rho) &= 2 + \rho^2, \\ B(\rho) &= 1 - \tfrac{1}{3}A = \tfrac{1}{3}(1 - \rho^2). \end{aligned} \tag{14}$$

3. Method of solution

The Finite Element Method will be applied for solving the boundary value problem, based on the obove mentioned model. The case $C = D = 0$ will be considered. The method is based on the requirement for stationarity of the following functional [9]

$$\pi = \int_V \sigma_p \dot{\bar{\varepsilon}} dV + \int_{S_f} \tau_t v_t dS - \int_{S_p} p_i v_i dS \Rightarrow \text{stat}, \tag{15}$$

Taking into account (9) and the rigid–plastic FEM approach, the boundary value problem leads to solving the following linear system of equations:

$$[K]\{q\} = \{F\} + \{F_\tau\}, \tag{16}$$

where $\{q\}$ is the global unknown nodal–point velocity vector and

$$[K] = \int_V \frac{\sigma_p^*}{\dot{\bar{\varepsilon}}} [P]\, dV \tag{17}$$

is the global stiffness matrix.

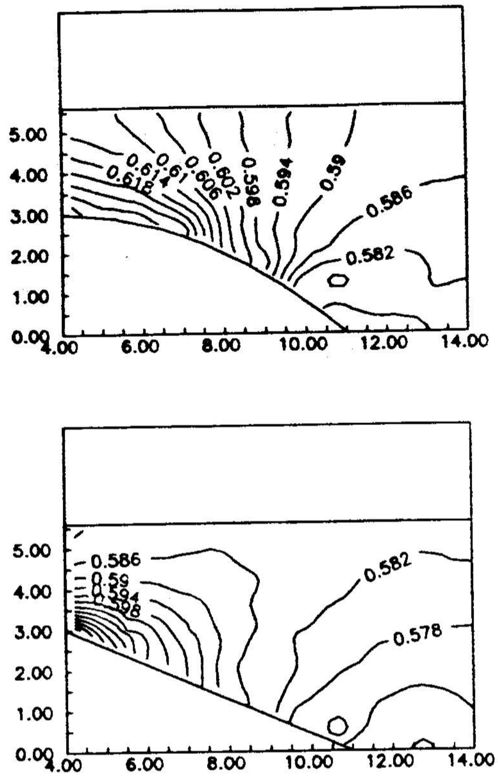

Fig. 1

The load vector due to the prescribed tractions $\{F\}$ and the friction load vector $\{F_\tau\}$ are presented by

$$\{F\} = \int_{S_p} [N]^T \{P\}\, dS, \quad \{F_\tau\} = -\int_{S_f} [N]^T \{\tau\}\, dS \tag{18}$$

where the matrix $[P]$ is

$$[P] = [P_1] + [P_2], \quad [P_1] = [B]^T [D_1] [B], \quad [P_2] = [B]^T [D_2] [B] \tag{19}$$

and

$$[D_1] = \frac{2}{3A}(3[I_1] - [I_2]), \quad [D_2] = \frac{1}{9B}[I_2], \tag{20}$$

$[I_1]$ and $[I_2]$ are unique matrices and $[N]$ is the matrix of the shape functions. $[B]$ is the matrix, giving the relation between the strain rate vector and the nodal velocities.

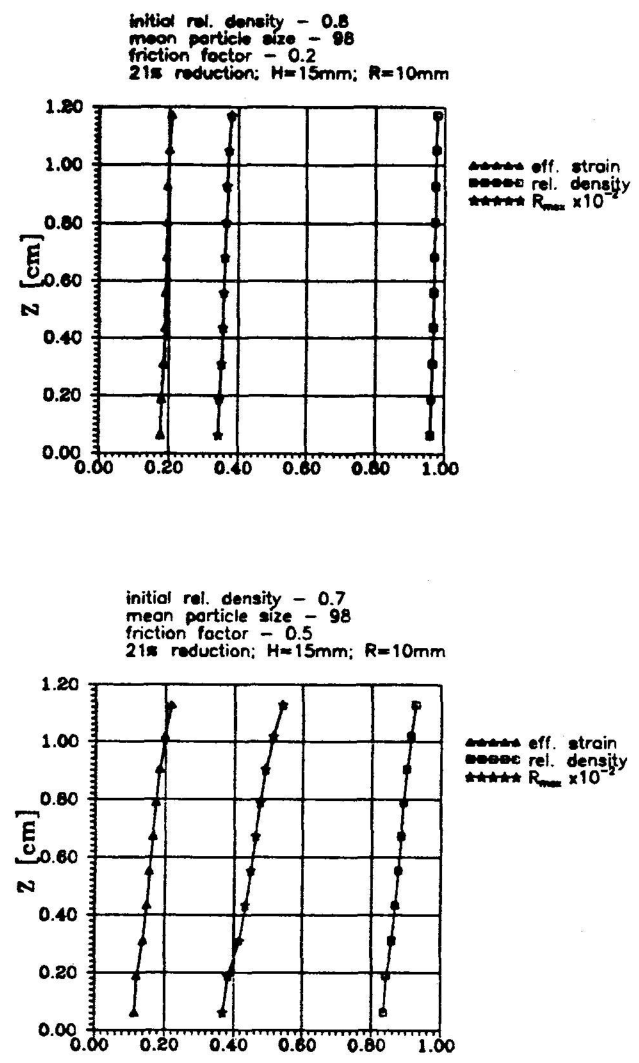

Fig. 2

On the base of the above mentioned equations, the programme system **PORO** was created. The programme gives the possibility to incorporate different density functions $A(\rho), B(\rho), \sigma_p(\bar{\varepsilon}, \dot{\bar{\varepsilon}}, T)$.

4. Numerical results

As a numeriacal example, the compaction process of ring powder billet of different geometry is considered. The aim of this numerical example is the investigation of the influence of geometry on relative density distribution into the body due to compaction. The material functions of the model are taken in the form proposed in [8] (see (14)). The material yield limit function corresponds to alluminium [11]: $\sigma_p^b = \sigma_{p_0}^b(1 + a\bar{\varepsilon}_b - b\bar{\varepsilon}_b^2)$, where $a = 0.4286, b = 0$. The problem is solved as axisymmetric with 4-node elements by means of the finite elemnent programme **PORO**. The density distribution into the body after a 30% reduction is plotted in Fig.1.

As a second example, the evolution of the surface roughness parameter R_{max} along the wall during the compation of a cylindrical powder billet is considered. The R_{max} distribution along the wall of the billet at different values of the initial relative density is plotted in Fig.2.

REFERENCES

[1] OH HUNG-KUK and JEONG-KEUN LEE. A study of the extrusion of sintered porous metal. *Journal of Mechanical Working Technology*, **11** (1985), 53-69.

[2] OH HUNG-KUK and HEE SON. A basic study of the hardness measurement of sintered porous methals. *Journal of Mechanical Working Technology*, **1** (1986), 175-187.

[3] OYANE M. and S. SHIMA. Consideration of basic equations and their applications, in the forming of metal powders and porous metals. *Journal of Mechanical Working Technology*, **1** (1978), 325-341.

[4] OH H. K. and J. H. MUN. An analysis of the ring upsetting of sintered material. *Journal of Mechanical Working Technology*, **9** (1984), 279-290.

[5] ZIENKIEWICZ O. C., G. C. HUANG and Y. C. LIU. Adaptive fem computation of forming processes – application to porous and non-porous matherials. *Int. J. Num. Methods Ehg.*, **30** (1990), 1527-1553.

[6] BETTEN J., H.-G. FROSCH and M. BORRMANN. Pressure-dependent yield behaviour of metals and polymers. *MAtherials Sicence and Engineering*, **56** (1982), 233-246.

[7] IANKOV R. Numeriacal simulation of sufface roughness evolution during the compaction of powder based materials. *ZAMM*, **74** (1993), in print.

[8] DORAIVELU S. M., H. L. GEDEL, J. S. GUNASEKERA, J. C. MALAS, J. T. MORGAN and J. F. THOMAS. A new yield function for compressible p/m materials. *Int. J. Mech. Sci.*, **26**, *9/10* (1984), 527-535.

[9] KOBAYASHI S., S. I. OH and T. ALTAN. Metal Forming and the Finite Element Method. Oxford University Press, New York Oxford, 1989.

[10] SHIMA S. and M. OYANE. Plasticity theory for porous metals. *Int. J. Mach. Sci.*, **18** (1976), 285-291.

[11] DUNG N. L. Beitrag zur berechnung instation "arer star-plastischer form" anderungen mit ainer finite-elemen-methode. Technical Report 6, Fortschritt-Berichte VDI, VDI Verlag, 1981.

[12] TAKAKURA N., YAMAGUVHI K. and FUKUDA M. Surface roughness development in upsetting of sintered powder metals. *Advanced Technolojy of Plasticity*, **2** (1990), 903-908.

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