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ON THE NONSTATIONARY RANDOM VIBRATIONS OF THREE-DIMENSIONAL SHEARWALL BUILDINGS

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Introduction

Shearwall buildings are widely employed in regions with a seismic activity potential. The Bulgarian seismic code requires a static analysis of the structure loaded by story earthquake forces. The seismic response of shearwall buildings can be also obtained using a direct integration of the equations of motion. In both cases the direction of the seismic excitation is to be taken into consideration. The present paper utilizes the random vibration theory and the work of Vanmarcke [1], Irschik [2], Ziegler [3] and Spanos [4], where single-degree-of-freedom or plane multy-degree-of-freedom systems are examined. Such analysis of space structures leads also to the problem of accounting for the direction of the seismic input. Liu and Wilson [5] point out that the ground motion records in any two horizontal directions are generally correlated to some extend and use cross spectra, determined from given earthquake records.This requires an empoyment of a great number of suitable accelerogramms. For the analysis of a three-dimensional shearwall structure we are going to employ the Kanai-Tajimi spectral density function $S_g(\omega)$ which represents a white noise filtered through a simple oscilator with a natural frequency ω_g and viscous damping ξ_g related to the soil conditions

$$S_g(\omega) = \frac{[1 + 4\xi_g^2(\frac{\omega}{\omega_g})^2]S_0}{[1 - (\frac{\omega}{\omega_g})^2]^2 + 4\xi_g^2(\frac{\omega}{\omega_g})^2} \tag{1}$$

This earthquake discription enables an easy computation of the integrals for the response variances but gives no possibility for the input cross spectra to be realistically determined. To minimize the number of earthquake directions for the analysis an initial determination of most dangerous directions is performed using the structural eigen modes.

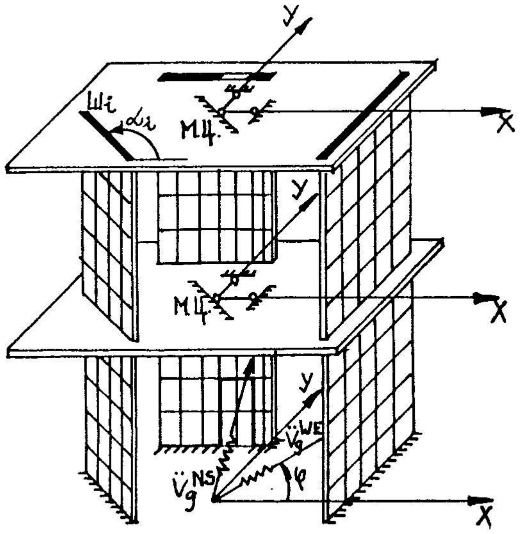

Fig. 1

Structural model

A separate shearwall is discretized by a mesh of rectangular plane elements, connected by nodes with two degrees of freedom. The Finite Element Method is employed and superelements are used in order to reduce the shearwalls degrees of freedom. The shearwalls are connected by rigid horizontal floor slabs. Each slab is assumed to have three degrees of freedom – two displacements in the slab plane and one rotation about a vertical axis, passing through the slab center of gravity (Fig.1). This leads to a diagonal form of the structure mass matrix. The dynamic behaviour of the structure is described by the system of equations

$$\mathbf{M}\ddot{\mathbf{v}} + \mathbf{C}\dot{\mathbf{v}} + \mathbf{K}\mathbf{v} = -\mathbf{M}\mathbf{T}_g\ddot{v}_g \tag{2}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the structure mass, damping and stiffness matrices, $\mathbf{v}$, $\dot{\mathbf{v}}$ and $\ddot{\mathbf{v}}$ are vectors of relative displacements, velocities and accelerations, $\ddot{v}_g$ is the ground acceleration, and

$$\mathbf{T}_g = [\cos\varphi \sin\varphi \; 0 \cos\varphi \sin\varphi \; 0 \; \ldots]^T \tag{3}$$

is the ground acceleration influence vector. The normal equation of motion for the j-the mode of (2) is

$$\ddot{Y}_j(t) + 2\omega_j\xi_j\dot{Y}_j(t) + \omega_j^2 Y_j(t) = F_j(t) \tag{4}$$

in which ω_j denotes the eigen frequency, ξ_j – the damping percent and F_j – the generalized load

$$F_j(t) = -\frac{\mathbf{\Phi}_j^T \mathbf{M}\mathbf{T}_g \ddot{v}_g(t)}{m_j} = \frac{L_j \ddot{v}_g(t)}{m_j}, \tag{5}$$

computed using the eigen vectors $\mathbf{\Phi_j}$ and the normalized mass m_j. The generalized displacement is determined from

$$v_i(t) = \sum_{j=1}^{n} \Phi_{ij} Y_j(t), \tag{6}$$

where n is the number of eigen modes examined. The inertia force associated to the mass M_i is

$$Z_i(t) = \sum_{j=1}^{n} M_i \Phi_{ij} \omega_j^2 Y_j(t). \tag{7}$$

Nonstationary random response variances and barrier problem

Following [2] and [4] the nonstationary process is assumed to be a product of the stationary spectral density (1) and the time modulating function $\psi(t)$ suggested in the form $\psi(t) = \sin(\pi t/T)$ if T is the duration of the seismic excitation

$$S_g(\omega, t) = S_g(\omega)\psi(t). \tag{8}$$

The spectral power density function for the j-the mode of vibration is then

$$S_j(\omega, t) = S_g(\omega)\psi(t)|H_j(\omega, t)|^2, \tag{9}$$

where the time-dependant transfer function can be assumed

$$|H_j(\omega, t)|^2 = [(\omega_j^2 - \omega^2)^2 + 4\xi_{jt}\omega_j^2\omega^2]^{-1}. \tag{10}$$

The time dependant damping percent for the j-the mode is

$$\xi_{jt} = \frac{\xi_j}{1 - e^{-2\xi_j\omega_j t}}. \tag{11}$$

Using the results of Spanos [3] the j-the mode response variance will be

$$\sigma_j^2(t) = \int_0^\infty S_j(\omega,t)\,d\omega = \frac{\pi}{2\omega_j^2} e^{-2\xi_{jt}\omega_j t} S_g(\omega_j)\left(\frac{L_j}{m_j}\right)^2 \int_0^t e^{2\xi_{jt}\omega_j \tau}\psi(\tau)\,d\tau \tag{12}$$

and for a sinus modulating function $\psi(t)$

$$\sigma_j^2(t) = \frac{\pi S_g(\omega_j)}{2\omega_j^2(a^2+b^2)}\left(\frac{L_j}{m_j}\right)^2 [a\sin(bt) + b(e^{-at} - \cos(bt))], \tag{13}$$

where

$$a = 2\xi_{jt}\omega_j \quad and \quad b = \frac{\pi}{T}. \tag{14}$$

The response variance for the generalyzed displacement v_i is computed according Vanmarcke [1]. If the indexes j and k denote the modes number ($j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, n$)

$$\sigma_{v_i}^2(t) = \sum_{j=1}^n \alpha_{ij}(t)\Phi_{ij}^2\sigma_j^2(t), \tag{15}$$

in which

$$\alpha_{ij}(t) = 1 + \sum_{k\neq j}^n \frac{\Phi_{ik}}{\Phi_{ij}} A_{jk}(t) \tag{16}$$

and the factor A_{jk} is

$$A_{jk}(t) = \frac{6r\xi_{jt}(\xi_{kt} + \xi_{jt}r)[\rho - 4r(\xi_{jt} - \xi_{kt}r)(\xi_{kt} - \xi_{jt}r]}{8r^2[(\xi_{jt}^2 + \xi_{kt}^2)\rho - 2(\xi_{kt}^2 - \xi_{jt}^2 r^2)(\xi_{jt}^2 - \xi_{kt}^2 r^2)] + \rho^2} \tag{17}$$

and depends on the frequency ratio $r = \omega_k/\omega_j$ and $\rho = (1 - r^2)$.

The two-sided upcrossing probability for a given barrier $\bar{a}$ of the i-the generalized displacement is obtained as for a Poisson process from

$$H_i(t, \bar{a}) = 1 - e^{-t_0\beta_i(t)}, \tag{18}$$

where

$$\beta_i(t) = \frac{\Omega_i}{\pi}\frac{1 - e^{-\delta\sqrt{0.5\pi\bar{a}^2/\lambda_{0i}(t)}}}{e^{0.5\bar{a}^2/\lambda_{0i}(t)} - 1}. \tag{19}$$

The central frequency Ω_j for the j-the mode is

$$\Omega_j(t) = \sqrt{\frac{\lambda_{2j}(t)}{\lambda_{0j}(t)}} = \frac{\sigma_{\dot{v}_j}}{\sigma_{v_j}} \tag{20}$$

and the central frequency Ω_i according to [1] and [2] is computed using the weighting factors p_{ij}

$$\Omega_i(t) = \sqrt{\sum_{j=1}^{n} p_{ij}(t)\Omega_j^2(t)}. \tag{21}$$

The dispersion parameter δ employed in (19) is

$$\delta_i(t) = \sqrt{1 - \frac{\lambda_{1i}^2(t)}{\lambda_{0i}(t)\lambda_{2i}(T)}} \tag{22}$$

The spectral moments related to the i-th generalized displacement above are determined from the expression

$$\lambda_{li}(t) = \sum_{j=1}^{n} \alpha_{ij}(t)\Phi_{ij}^2\lambda_{lj}(t), \tag{23}$$

where the j-the component of the l-the spectral moment

$$\lambda_{lj}(t) = \int_0^\infty \omega^l S_j(\omega, t)\, d\omega \tag{24}$$

is computed through numerical integration, using the simplifications proposed in [1]. The time parameter t_0 in (18) is

$$t_0 = te^{-2[\frac{\lambda_{0i}(t)}{\lambda_{0i}(t/2)} - 1]}. \tag{25}$$

The upcrossing probability for inertia forces or other quantities is determined in a similar way. The direction of the seismic excitation is taken into account in (3). For minimizing the computations we propose the "most dangerous" directions to be analyzed computed from the condition for maximum of the floor inertia force F_j

$$\tan\varphi_j = \frac{\sum_{i=1}^{k} \Phi_{(3i-1)j}M_i}{\sum_{i=1}^{k} \Phi_{(3i-2)j}M_i}, \text{ where } k \text{ is the number of floors.} \tag{26}$$

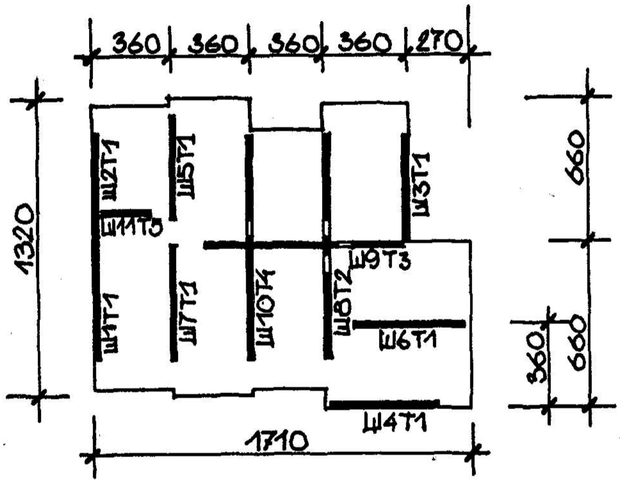

Fig. 2

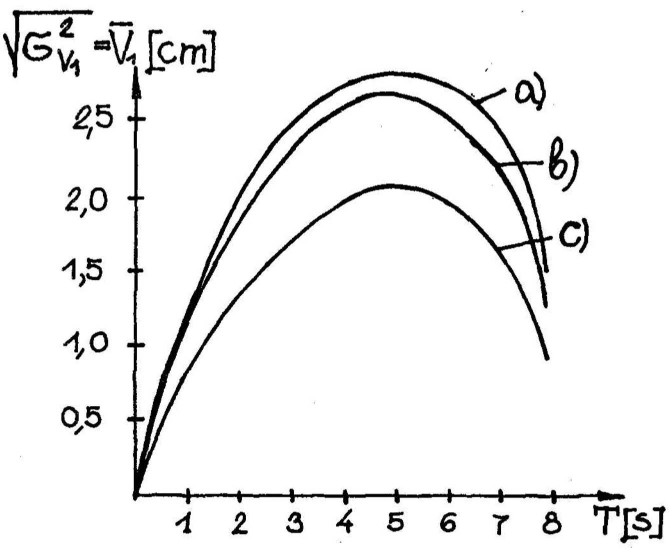

Fig. 3

Numerical results

A 9-story shearwall R/C building with 11 shearwalls (Fig.2) is analyzed. The floor gravity is 3200kN and the upper slab gravity is 2050kN. The corresponding gravity inertia moments are 115854kNm and 74219kNm. The first three eigen frequencies are: a $\omega_1 = 8.103s$, $\omega_2 = 9.353s$ and $\omega_3 = 13.048s$. The input spectral density function (see (1)) has $\omega_g = 25s$, $\xi_g = 0.24$ and $S_0 = 108.2cm/s$. The earthquake duration is $T = 8s$, the time-step – 0.04s and $\xi_j = 5$. The first 6 eigen modes are taken into consideration. The upper horizontal displacement variance in X-direction (Fig.3 – expected value v_1) and its upcrossing probability for a barrier $\bar{a} = 3.4cm$ (Fig.4) are plotted for the first three "dangerous directions": a) $\varphi_1 = -5.59°$, b) $\varphi_2 = 12.02°$ and c) $\varphi_3 = -88.97°$, computed from (26).

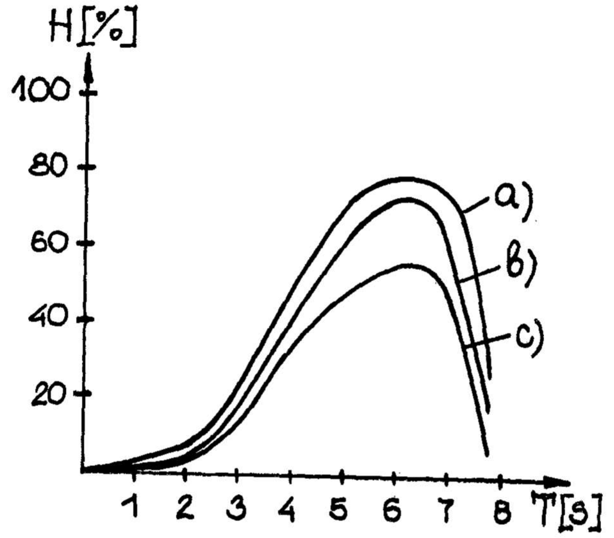

Fig. 4

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