Journal of Theoretical
and Applied Mechanics
Sofia 1993, Year XXIV No 2

CONTINUUM MECHANICS

A LINEAR COMPLEMENTARITY APPROACH TO SEISMIC FRICTIONAL INTERACTION BETWEEN ADJACENT BUILDINGS UNDER SECOND-ORDER GEOMETRIC EFFECTS

A. Liolios

1. Introduction

Lessons learned from recent earthquakes, e.g. Bertero [2] ,recognize that interaction among adjacent buildings is often the main cause for damages in seismically active regions. This problem is particularly common in the commercial centers of cities, where, due to various socio-economic factors and land usage requirements, the building codes permit full or partial contact between neighboring building systems. Thus a numerical estimation of the interaction effects to earthquake response of such buildings is significant for their earthquake resistant design, construction and repair.

Certainly the problem is very difficult from many aspects. Mathematically this problem of pounding of structures belongs to the inequality problems of mechanics, where the governing conditions are equalities as well as inequalities. These so-called unilateral problems can be treated mathematically by the variational inequality concept, see e.g. Panagiotopoulos [10].

As regards to the numerical treatment of such inequality problems, in Earthquake Mechanics, some numerical approaches have already been presented for multi-degree of freedom systems by Liolios [5]–[8]. On the other hand, interesting results of parametric studies based on simplified models of single- degree-of-freedom systems and a trial-and-error approach have been also reported [1], [14].

In this paper a special case of seismic building interaction is treated numerically. This case concerns the unilateral elastoplastic-softening contact with Coulomb's law of friction between adjacent buildings, initially designed and constructed as seismically symmetric and latter modified to asymmetric ones, when P-Delta effects are taken into account. The problem usually arises when industrial or housing blocks are constructed in series, where the same constituent parts, seismically symmetric (i.e. with zero eccentricity), are repeatedly constructed in contact. Very often, during (or after) the construction, some modifications are made by changing the place of, or by adding arbitrarily, various non-structural elements, e.g. infill walls. But these modifications can cause serious eccentricities in some buildings. So, although in initially symmetric buildings should appear transitional lateral vibrations only without any interaction, in modified buildings appear transitional as well as torsional lateral vibrations [3]. Therefore, interaction can take place and the real building seismic response can be different from that of the design. The quantitative estimation of these interaction effects can be calculated by the next presented procedure, which constitutes an extension of the procedures of [15] to the 3D frictional contact and of [16] to taking into account second-order geometric effects. The approach is further applied to and tested in a civil engineering example with remarkable numerical results. Finally, some concluding remarks useful for the praxis are discussed.

2. Method of analysis

The numerical procedure is based on a double discretization and a piece-wise linearization of the nonconvex and in general nonmonotone constitutive laws for the contact interface and the modificating elements (infill walls). In details, the discretization consists of a spatial one by the finite element method and a temporal one by the finite difference method. Instead of using a trial-and-error method as in other proposed procedures, which may not guarantee convergence to the exact solution, in each time step here a nonconvex linear complementarity problem is solved by extending the procedure of Liolios [8]. For simplicity, a system of only two adjacent linearly elastic buildings (A) and (B) is considered here.

(i) Uncoupled system analysis

First the system of the two initial buildings (A) and (B) considered as uncoupled is discretized by the finite element method. Assuming no interaction and no P-Delta effects, the matrix equations of dynamic equilibrium are

$$M_L\ddot{u}_L + C_L\dot{u}_L + K_L u_L = -M_L\ddot{U}_g, \quad (L = A, B), \tag{1}$$

where M_L, C_L, K_L are the mass, damping and stiffness matrices, respectively; $u(t)$ is the sought node displacement (relative to ground) vector corresponding to given ground earthquake excitation $\ddot{u}_g(t)$ and appropriate initial conditions; and dots over symbols indicate time derivatives. Problem (1) can be solved by anyone of the wellknown methods of Structural Dynamics.

(ii) Modificating elements analysis

Next we consider that the system is modified by adding arbitrarily (or changing the place of) some infill walls. Each wall is simulated by a couple of equivalent diagonal struts [4], [12]. Here these struts are considered as transmitting compressive forces only and appearing an elastoplastic-softening-fracturing (or with remaining resistence) behaviour, as e.g. that one of Fig. 1c. The behaviour of the i-th strut is expressed mathematically by the relation [7]

$$r_{iD}(d_i) \in \partial R_i(d_i), \tag{2}$$

where r_{iD} and d_i are the stress (compressive force) and deformation (shortening), respectively; ∂ is the generalized gradient of Clarke; and $R_i(\cdot)$ is the superpotential function, see Panagiotopoulos [10]. By definition, rel. (2) is equivalent to the following hemivariational inequality, where $R\uparrow$ denotes subderivative and e_i virtual deformation:

$$R_i \uparrow (d_i, e_i - d_i) \geq r_{iD}(d_i).(e_i - d_i). \tag{3}$$

For the numerical treatment of the problem, rel. (2) is piecewise linearized and written as follows:

$$\text{if } d_i' \leq g_i' \text{ then } r_{iD} = 0, \tag{4a}$$

$$\text{if } d_i' > g_i' \text{ then } r_{iD} > 0, \tag{4b}$$

where d_i' is the relative approach displacement of the two nodes connected by the i-th strut and $g_i' = g_i'(t)$ is the total gap existing at time t between these nodes and the ends of the strut.

(iii) Interaction simulation

Due to above modifications, seismic interaction between the two buildings can occur, as in the Introduction was explained, and eqs. (1) do not hold more. Let j_A and j_B be two associated nodes on the interface (joint), where unilateral frictional contact can take place during an earthquake. These nodes are considered [8]. as connected by two fictive unilateral constraints, normal to interface the first and tangential the second one. The corresponding force-reactions and retirement relative displacements are denoted by r_{jN}, z_{jN} and r_{jT}, z_{jT}, respectively, and satisfy in general nonconvex and nonmonotone constitutive relations of the type (2), expressing the unilateral elastoplastic softening contact with friction. By piecewise linearizing these relations as in Liolios [8] and Maier [9] we obtain the following linear complementarity conditions:

$$r_{jN} = p_{jN}(z_{jN} - g_j + w_j) + c_j \dot{z}_{jN}, \tag{5a}$$

$$(5b,c,d) \qquad w_j \geq 0, \; r_{jN} \leq 0, \; w_j.r_{jN} = 0,$$

$$(6a,d) \qquad |r_{jN} \leq f_j|r_{jN}|, \; \dot{z}_{jT}.r_{jT} \leq 0$$

$$(6c) \qquad \dot{z}_{jT}.(|r_{jT}| - f_j|r_{jN}|) = 0.$$

In (5a) c_j is damping coefficient, p_{jN} the reaction function for the normal unilateral constraint, g_j the existing normal gap and w_j a non-negative multiplier; in (6) f_j is the Coulomb's friction coefficient. So, rels. (5) and (6) impose that friction phenomenae (slip or adhesion) can take place only when unilateral contact occurs, i.e. when the compressive contact force r_{jN} appears.

(iv) Coupled system conditions

Taking into account, now, on the one hand the influence of the modifications and on the other hand the interaction effects, we write the dynamic equilibrium conditions for the coupled system of the interacting buildings (A) and (B):

$$(7a) \qquad M_A\ddot{u}_A + C_A\dot{u}_A + (K_A + G_A).u_A = -M_A\ddot{u}_g + r_{DA} + r,$$

$$(7b) \qquad M_B\ddot{u}_B + C_B\dot{u}_B + (K_B + G_B).u_B = -M_B\ddot{u}_g + r_{DB} + r,$$

$$(7c) \qquad r = r_N + r_T$$

Here r_{DA}, r_{DA} are the vectors of the strut forces, G_A and G_B are the geometric stiffness matrices, by which P-Delta effects are taken into account [11], and r is the coupling vector of the normal and tangential interaction forces. To conditions (7) are adjoined the initial conditions, and so the problem consists in finding the time-dependent vectors u_A, u_B, g, g', r, r_{DA} and r_{DB} which satisfy rels. (2)-(7) for the given earthquake excitation $u_g(t)$. Obviously this inequality problem is a nonlinear one.

(v) Time discretization and problem solution

Further the problem of rels. (2)-(7) is discretized in time. Because this problem is nonlinear due to inequalities the mode superposition method cannot be applied. Thus direct time-integration methods have to be used. As it has been proved in numerical experiments, the time-step, required for a physically meaningful and in details satisfactory and convergent solution, is very small. On the other hand, the interaction forces and the stresses of the modificating elements depend nonlinearly on the displacements. All these quantities are unknown at the same time moment and if we use the known unconditionally stable time integration methods, e.g. Newmark, Houbolt, Wilson etc., then we will have to solve in each time-step a high nonlinear inequality problem.

For all the above reasons, the simple central difference method is preferred to other implicit schemes, which use relatively larger time-steps in comparison to central difference method. In each time-step we assume that the unilateral constraints remain either active or unactive by adjusting suitably the time-step. To compute what is happening, the procedure of Liolios [8] is applied taking into account discontinuity of the velocities [17] due to contact-impact. So, a nonconvex linear complementarity problem of the following form is solved:

$$v \geq 0, \; Av + a \leq 0, \; v^{\mathsf{T}}.(Av + a) = 0. \tag{8}$$

For most practical applications in structural mechanics, matrix A is a P-matrix [9], [18] and thus a unique solution of problem (8) can be assured.

3. Numerical example

The system of the two one-storey buildings (A) and (B) of Fig. 1a,b had been initially designed and constructed without the infill walls of masonry brick W_x and W_y. The buildings were exactly the same ones, seismically symmetric, of reinforced concrete with elasticity modulus $E_b = 3 \cdot 10^7 \; KN/m^2$, slab thickness 0.25 m, beams 25/75 cm connecting the columns tops perimetrically, columns section 30/30 cm and damping ratio 5%. The system is subjected to the horizontal ground seismic excitation:

$$u_g(t) = u_0 e^{-2t} \sin(4\pi t), \; u_0 = 10 \; mm,$$

along the axis $x - x$. In the absence of the infill walls in the buildings appear transitional vibrations only along axis $x - x$, without interaction on the joint $J - J$.

For functional reasons, after the construction, in the building (A) the two infill walls W_x and W_y have been added. So the building (A) has become an asymmetric one from the seismic response point of view. The infill walls are simulated by two couples of diagonal struts with the stress-deformation law depicted in Fig. 1c. The system is subjected to the same ground seismic excitation, but now interaction takes place between the two buildings. The stress-deformation law for the unilateral constraints normal to the interface $J - J$ is estimated by experimental results to be given as in Fig. 1d and the friction coefficient for the tangential constraints is estimated as $f_j = 0.50$.

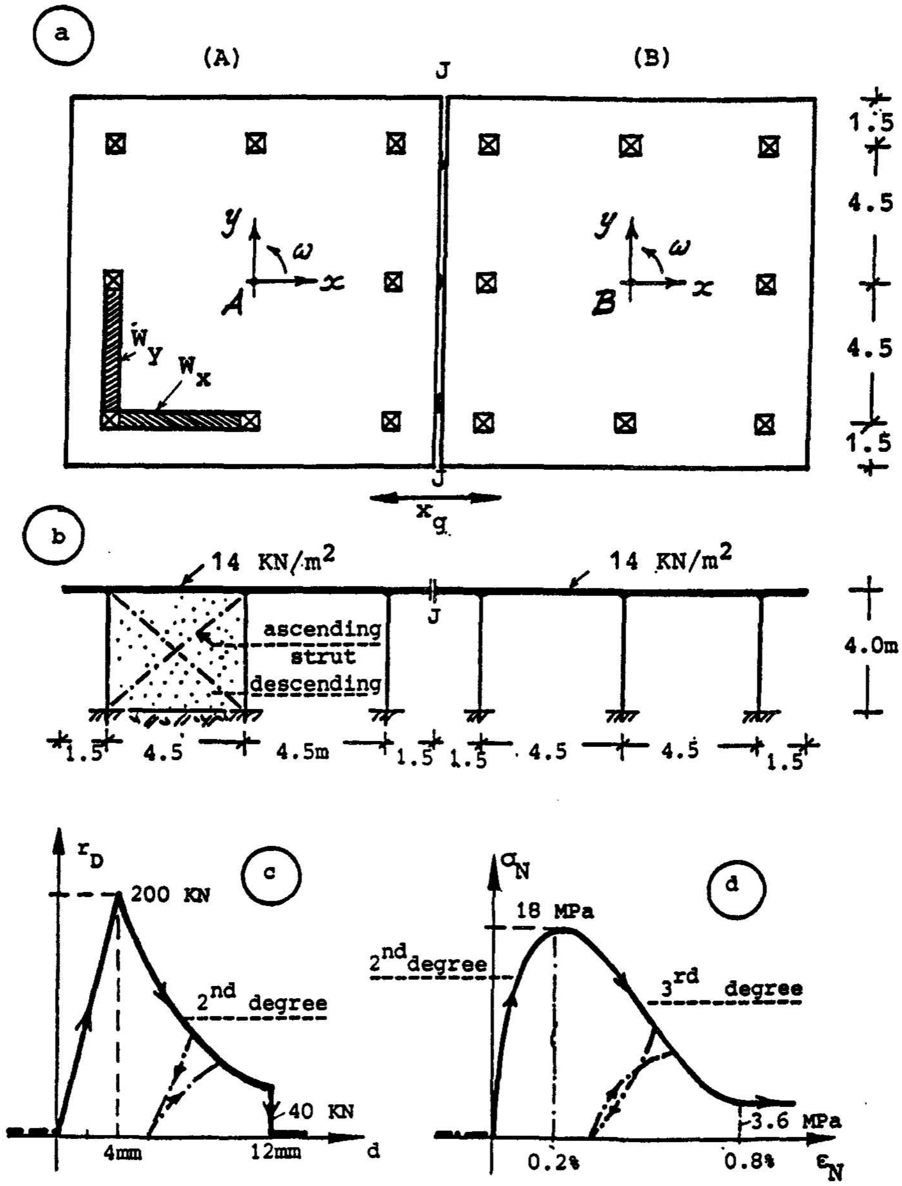

Fig.1 Numerical example:
a. Plan view of the system of buildings (A) and (B)
b. Vertical section view
c. Stress-deformation law for the diagonal struts
d. Stress-deformation law for the unilateral constraints normal to the interface (joint) $J - J$

Table 1. COMPARISON OF SOME RESPONSE QUANTITIES			
BUILDING	QUANTITY	INITIAL SYSTEM	MODIFIED SYSTEM
	H_x	600.0 KN	−653.0 KN
(A)	H_y	0	−92.8 KN
	M_z	0	−437.8 KNm
	H_x	600.0 KN	578.4 KN
(B)	H_y	0	39.7 KN
	M_z	0	−408.4 KNm

In Table 1. the absolutely extremum values of some response quantities occured during the earthquake excitation are shown indicatively. These quantities are the horizontal forces (base shear forces) H_x and H_y and the torsional moment M_z in the mass centers of the buildings (A) and (B), first without modifications (initial system, no interaction) and second when the interaction due to modifications is taken into account (coupled system). The interaction effects in the second case are remarkable as regards to H_y and M_z. On the other hand, for the ascending strut of the infill wall W_x the permanent deformation results in 10.43 mm and for the descending strut in 20.87 mm. Therefore, from Fig. 1c it is concluded that the ascending strut has been cracked and the descending strut has been fractured.

4. Concluding remarks

As in the numerical example has been shown, arbitrary modifications, which are not taken into account in the usual initial design, can change significantly the earthquake response of one and the same seismically symmnetric buildings in series. A numerical estimation of the so caused seismic interaction effects can be obtained by the herein presented approach. This numerical procedure is realized by using available computer codes of the finite element method and the nonlinear mathematical programming (optimization algorithmes). For simplicity, a system of only two buildings has been considered here, for the problem formulation and solution as well as in numerical example. This poses certainly no restrictions, and the extension to systems with more than two buildings in the case of nonlinear elastic structures can be done in a straightforward way.

On the other hand, besides of the structure-to-structure interaction case studied here, the presented numerical approach can be also applied and extended to other inequality problems in earthquake engineering. Some appropriate slight modifications are needed for this purpose. So, the soil-structure interaction [13] can be treated numerically as an inequality problem by such modifications of the herein procedure, see e.g. Liolios [6]. Certainly the most complicated task in all the above cases is the realistic simulation of the unilateral contact. To overcome this difficulty, experimental results can be used for rational estimation of parameters involved to simulate the interface behaviour between adjacent space structures.

REFERENCES

[1] ANAGNOSTOPOULOS, S.A. Pounding of buildings in series during earthquakes. *Earthq. Engin. Struct. Dynamics*, **16** (1988), 443-456.

[2] BERTERO, V.V. Observations on structural pounding. Proc. Intern. Conf. "The Mexico Earthquakes", ASCE, 264-278, 1987.

[3] ERDIK, M. Torsional response of earthquake excited structures. In Earthquake Resistant Construction and Design (ed. S. Savidis), Balkema, Rotterdam, 1991, 469-488.

[4] KLINGNER, E.R., BERTERO, V.V. Earthquake resistence of infilled frames. *J. Str. Div., Proc. ASCE*, **104** (1973) 973-989.

[5] LIOLIOS, A.A. A finite-element central-difference approach to the dynamic problem of nonconvex unilateral contact between structures. In Numerical Methods and Applications (eds. B. Sendov, R. Lazarov and P. Vasilevski), Bulgarian Acad. Sciences, Sofia, 1984, 394-401.

[6] LIOLIOS, A. A. A numerical approach to the dynamic nonconvex unilateral problem of soil-pile interaction. In Differential equations, (eds. Dafermos, G. Ladas and G. Papanicolaou), Marcel Dekker, Inc., Basel, 1989, 437-443.

[7] LIOLIOS, A. A. A unilateral contact approach for numerically estimating the influence of additional infill elements to the seismic response of buildings. In Scientific-Technical Union of Bulgaria (ed.), Construction in seismic regions. Sofia, 1988.

[8] LIOLIOS, A. A. A linear coplementarity approach to the nonconvex dynamic problem of unilateral contact with friction between adjacent structures. *J. Appl. Math. Mech.*, **69**, *5* (1989), 420-422.

[9] MAIER, G. Mathematical programming methods in structural analysis. In Variational methods in engineering (eds. C. Brebbia and H. Tottenham), University Press, Southampton, 1973, 8/1-8/32.

[10] PANAGIOTOPOULOS, P. D. Inequality problems in mechanics and applications. Convex and nonconvex energy functions. Birkäuser Verlag, Basel, 1985.

[11] CHEN, W. F., LUI, E. M. Structural Stability-Theory and Implementation. Elsevier, New York, 1981.

[12] STAFFORD SMITH, B., CARTER, C. A method of analysis of infilled frames. *Proceedings of the Institution of Civil Engineers*, **104** (1969), 31-48.

[13] WOLF, J. P. Soil-Structure-Interaction Analysis in Time Domain. Prentice-Hall, Englewood Cliffs, N. J., 1988.

[14] WOLF, J.P., SKRIKERUD, P.E. Mutual pounding of adjacent structures during earthquakes. *Nuclear Engin. Design.* **57** (1980), 253-275.

[15] PAPADRAKAKIS, M., APOSTOLOPOULOU, K., BITZARAKIS, S., ZACHAROPOULOS, A. A 3D model for the analysis of building pounding during earthquakes. In Structural Dynamics-Eurodyn'93, (eds. Moan, T. et al.), Balkema, Rotterdam, 1993, 85-92.

[16] LIOLIOS, A. A numerical estimation for the influence of modifications to seismic interaction between adjacent structures. In Earthquake Resistant Construction and Design, (ed. S. Savidis), Balkema, Rotterdam, 1991,461-467.

[17] ZIENKIEWICZ, O., WOOD, W. L., TAYLOR, R. L. An alternative single-step algorithm for dynamic problems. *Earthq. Eng. Struct. Dyn.*, **8** (1980), 31-40.

[18] MURTY, K.G. Linear Complementarity, Linear and Nonlinear Programming. Heldermann Verlag, Berlin, 1988.

Presented at the 7th Congress on Theoretical and Applied Mechanics, Sofia, August 30 – September 1, 1993
section: Solid Mechanics