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DYNAMIC LOADINGS DUE TO TORSIONAL AND BENDING VIBRATIONS ON THE LINKS OF MACHINES WITH CARDAN TRANSMISSIONS

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1. Problem statement

Set in motion of many machines is carried out by cardan transmissions and more frequently the rotating moment is transmitted from the driver to the working machine by a double cardan transmission.

During the exploitation dynamic loadings occur in the links of the cardan transmissions due to torsional and bending moments.

These vibrations are investigated in the references [1] but independently from each other and without accounting for their relationship.

The aim of the paper is to examine the torsional and the cross-sectional vibrations in the links of machines with double cardan transmissions, thus determining the dependence of the cross-sectional vibrations on the torsional ones and deriving expressions for the amplitudes of the torsional and the bending moments which act on the rollers of the cardan transmission in stationary regime.

2. Dynamic model

Usually, machines with cardan transmissions are considered as two-mass machine aggregates. In the paper we dwell in more details on machines with a double cardan transmission which are used in practice most frequently.

The dynamic model is given in Fig. 1.

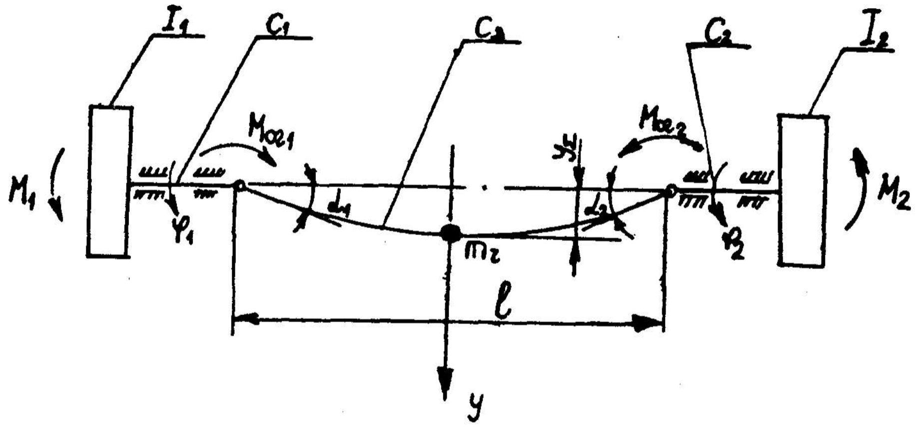

Fig. 1

The following denotations are introduced:

I_j $(j = 1, 2)$ are mass inertial moments of the driver and the working machine;

c_j $(j = 1, 2, 3)$ are elastic constants of torsion of the three rollers;

φ_j $(j = 1, 2)$ are angles at which the leading and the led rollers have been rotated;

α_j $(j = 1, 2)$ are angles of swing in both linkages;

l is the length of the intermediate roller;

M_j $(j = 1, 2)$ are torsional moments in the leading and the led rollers;

$M_{og\,j}$ $(j = 1, 2)$ are bending moments applied on the intermediate roller.

The intermediate roller is considered to be elastic, without mass and with a material point of mass $m_r = \frac{1}{2}m$ concentrated in its centre.

It is assumed as well that the mass inertial moments of the rotors I_1 and I_2 are constant.

3. Differential equations of motion

The differential equations of motion are composed by the Lagrange method and for the purpose we write down the expressions for the kinetic and the potential energy of the system

$$
\begin{aligned}
T &= \frac{1}{2}I_1\dot{\varphi}_1^2 + \frac{1}{2}I_2\dot{\varphi}_2^2 + \frac{1}{2}m_r\left[\dot{y}^2 + (y_0 + y_1 + y)^2 \cdot \dot{\varphi}_b^2\right], \\
\Pi &= \frac{1}{2}c_0(\varphi_1 - \varphi_2 + \eta)^2 + \frac{1}{2}c(y_1 + y)^2.
\end{aligned} \tag{1}
$$

The total suspension y_Σ in the middle of the intermediate roller is given by the expression:

(2) $$y_\Sigma = y_0 + y_1 + y,$$

where the single components are:

y_0 – assembling disbalance;

y_1 – suspensions in the middle of the roller, caused by the two bending moments;

y – dynamic component.

In agreement with "the method of distributed motions" [2] the following coordinates are introduced:

(3) $$I_1\varphi_1 + I_2\varphi_2 = (I_1 + I_2)\varphi, \qquad \varphi_1 - \varphi_2 = x.$$

The variable η is introduced too and it is given by the expression:

(4) $$\eta = \lambda\rho_0 \sin 2\varphi,$$

where $\lambda\rho_0 = \mathrm{tg}^2\frac{\alpha_1}{2} - \mathrm{tg}^2\frac{\alpha_2}{2}$ is called "kinematic factor".

It may be assumed that $\dot\varphi \approx \dot\varphi_b$, where $\dot\varphi_b$ is the angular velocity of the intermediate roller.

Since the mass of the intermediate roller is too small in comparison with the others it may be assumed that $\dot\varphi \approx \omega$ =const.

Using well-known dependences we derive for the suspension y_1:

(5) $$y_1 = hc_0(x+\eta), \quad h = \frac{l^2}{8EJ}\left(\frac{\mathrm{tg}\,\alpha_1 + \mathrm{tg}\,\alpha_2}{2}\right).$$

In the expression for h we have denoted the axis inertial moment of the intermediate roller by J.

Using (3), after respective transformations the expressions for the kinetic and the potential energy take the form:

(6) $$\begin{aligned} T &= \frac{1}{2}I(\omega^2 + q_1q_2\dot x^2) + \frac{1}{2}m_r\left\{\dot y^2 + [y_0 + hc_0(x+\eta) + y]^2\omega^2\right\}, \\ \Pi &= \frac{c_0}{2}(x+\eta)^2 + \frac{c}{2}\left[hc_0(x+\eta) + y\right]^2. \end{aligned}$$

In the above expressions we have put down:

(7) $$I = I_1 + I_2, \quad q_j = \frac{I_j}{I} \ (j = 1, 2).$$

In a stationary regime it may be assumed with sufficient practical accuracy that $M_1 \approx M_2 = \overline{M}$ [1].

The total torsional moment in the intermediate roller is given by the expression:

$$L = c_0(x + \eta), \tag{8}$$

where

$$\frac{1}{c_0} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}. \tag{9}$$

Thus, the differential equations of motion take the form:

$$\begin{aligned} &k_1^{-2}\ddot{L} + (1 + \bar{c}h^2c_0)L + \bar{c}hc_0y = M_0 + H_1 \sin 2\omega t, \\ &m_r\ddot{y} + \bar{c}hL + \bar{c}y = N_0. \end{aligned} \tag{10}$$

Here we have put down:

$$\begin{aligned} &m_r\omega^2hc_0y_0 + \overline{M} = M_0, \quad m_r\omega^2y_0 = N_0, \\ &\frac{I_1I_2}{Ic_0} = k_1^{-2} \qquad\qquad c - m_r\omega^2 = \bar{c}. \end{aligned} \tag{11}$$

After solving the homogeneous system of differential equations (10) we derive the following expressions for the proper frequencies:

$$k_1^2 = \frac{Ic_0}{I_1I_2}, \quad k_2^2 = \frac{c}{m_r} - \omega^2. \tag{12}$$

In case of resonance absence ($2\omega \neq k_1$, $2\omega \neq k_2$) we seek the solution of the non-homogeneous system in the form:

$$L = B_1 + A_1 \sin 2\omega t, \quad y = B_2 + A_2 \sin 2\omega t. \tag{13}$$

Substituting (13) in (10) and solving the derived algebraic system we obtain the following expressions for the unknowns B_j and A_j $(j = 1, 2)$:

$$\begin{aligned} &B_1 = \overline{M}, \qquad\qquad B_2 = \frac{m_r\omega^2y_0}{\bar{c}} - h\overline{M}, \\ &A_1 = \frac{4\omega^2\lambda\rho_0c_0}{4(\omega^2 - k_1^2)}, \quad A_2 = \frac{4\omega^2\lambda\rho_0c_0}{m_r} \cdot \frac{(c - m_r\omega^2).h}{(4\omega^2 - k_1^2).(4\omega^2 - k_2^2)}. \end{aligned} \tag{14}$$

For the amplitude values of L and y we get:

$$L_{\max} = B_1 + A_1, \quad y_{\max} = B_2 + A_2. \tag{15}$$

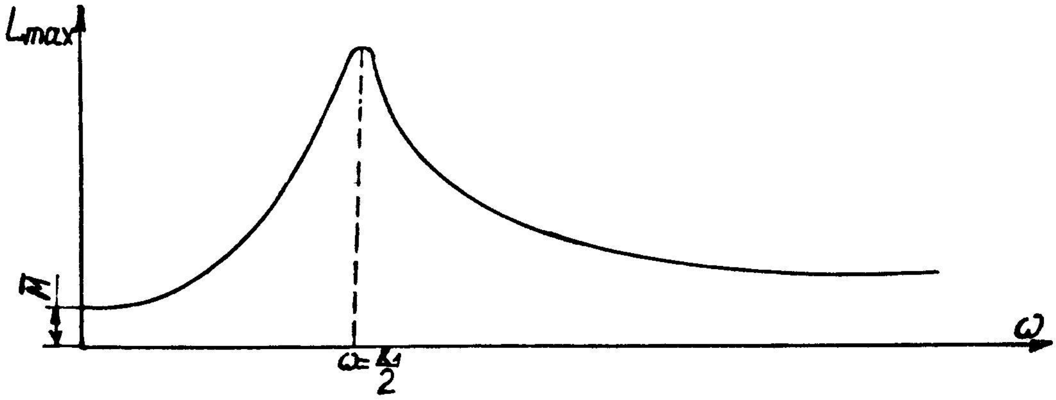

Fig. 2

For the maximum bending moment acting in the middle of the intermediate roller we derive the expression:

$$\max M_{og} = \frac{(y_{\max} + y_1)cl}{4}. \tag{16}$$

We write down the final expressions for the amplitude values of the torsional and the bending moments in the following form:

$$\begin{aligned} L_{\max} &= \overline{M} + \frac{4\omega^2 \lambda \rho_0 c_0}{4\omega^2 - k_1^2} \\ \max M_{og} &= \frac{cl}{4}\left[\frac{4\omega^2 \lambda \rho_0 c_0 h}{(4\omega^2 - k_1^2)} + \frac{\omega^2 y_0}{(k^2 - \omega^2)} + \frac{4\omega^2 \lambda \rho_0 c_0 h.(k^2 - \omega^2)}{(4\omega^2 - k_1^2).(4\omega^2 - k_2^2)}\right] \end{aligned} \tag{17}$$

Here we have put down $k^2 = \dfrac{c}{m}$, where c is the elastic constant of bending of the roller.

4. Analysis of the derived theoretical results

The amplitude-frequency characteristics (AFC) of the torsional and the bending moment are given in Fig. 2 and Fig. 3.

By the derived AFC we conclude that the torsional vibrations generate one resonance which occurs when the angular velocity of the roller is equal to the half of the first proper frequency.

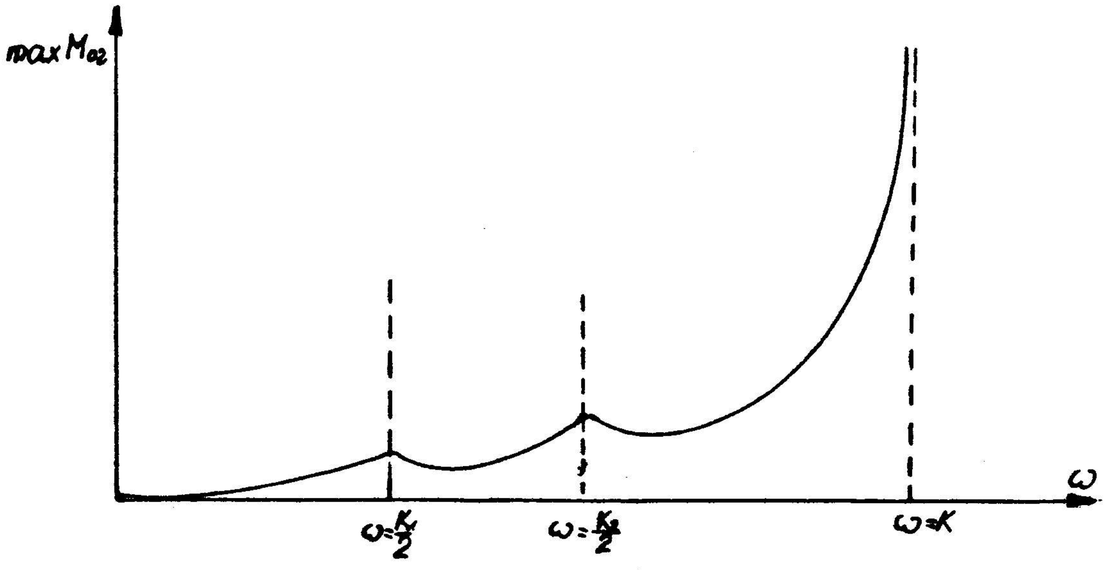

Fig. 3

The resonances are three in cross-sectional vibrations. The first of them coincides with the resonance of the torsional vibrations because the bending moments depend on the torsional ones. The second one is generated by the cross-sectional vibrations, while the third – by the assembling disbalance y_0.

The values of both moments are not constant but depend on the angular velocity and at definite values of ω they reach their resonance values.

The zones round the resonances must be avoided in defining the working velocities. The velocities for which the torsional and the bending moment have minimum values are preferred.

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