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MODELLING AND OPTIMIZATION OF NONLINEAR FRICTION LAWS WITH RESPECT TO FORCED VIBRATIONS DAMPING*

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Introduction

The arising vibrations spoil accuracy and speed characteristics of the most widespread machines and mechanisms. The attempts to apply traditional methods for vibrations damping in practice usually fail because of the complexity of the system kinematic structure. In such systems energy exchange processes are of great importance. This fact leads to the necessity of modelling the energy dissipation and the level of oscillation energy in the kinematic structure.

In the paper a new energy criterion is proposed for evaluating the damping of the forced vibrations of mechanical dissipative systems. This performance index has plain physical interpretation: the level of the average power dissipated for a period of the forced vibrations motion. It allows simple modelling and optimization of various friction laws from unified point of view.

This approach is illustrated on the model of the dissipative oscillator with various friction laws: Coulomb or dry, linear or damp, quadratic, etc. It is shown that optimal dry friction law "provides" maximum level of the dissipated power compared to the other ones.

The approach proposed is connected with adequate modelling and optimization of friction laws and energy exchange processes among elements of the system [2, 4].

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According to this approach, the forced vibrations are damping by using adjustable dampers. In this case external energy sources are not necessary and vibration control is realized by proper choice of friction characteristics of the dampers.

Consider the model of a mechanical system performing forced vibrations around a stable equilibrium position defined by certain goal configuration. Suppose that this system is supplied by absorbers consisting of a spring and a damper. With the help of such absorbers the elastic and dissipative forces and moments are modelled [1]. If the dampers have speedy characteristics realizing various friction laws, then the forced vibrations of the system are described by the equation [4]:

$$A\ddot{\varphi} + H(\dot{\varphi}, \mu) + C\varphi = f(\tilde{\varphi}_0,\ U,\ \Omega,\ \delta;\ t) \tag{1}$$

Here the following notations are used

A – $(n \times n)$ matrix of the system mass and inertia characteristics;

H – the matrix of the dissipative terms h_{ij}:

$$h_{ii} = h_i \mid \dot{\varphi}_i \mid^{\mu_i} sgn(\dot{\varphi}_i), \quad h_i > 0; \quad h_{ij} = 0 \quad (i \neq j).$$

$$sgn(\dot{\varphi}_i) = \begin{cases} 1 & \dot{\varphi}_i > 0 \\ 0 & \dot{\varphi}_i = 0 \\ -1 & \dot{\varphi}_i < 0 \end{cases}$$

h_i – the coefficients of the friction (Coulomb or dry for $\mu_i = 0$, linear or damp for $\mu_i = 1$ and quadratic for $\mu_i = 2$); $\mu = (\mu_1, \mu_2, \ldots, \mu_n)^T$;

C – the matrix of elasticity;

φ – n-dimensional vector of generalized coordinates φ_i; dot denotes differentiation with respect to the time t;

$\tilde{\varphi}_0$ – n-dimensional vector defining the system goal position;

U – the amplitude, Ω – the frequency, δ – the initial phase of the forced vibrations;

f – n-dimensional vector-function of the system goal configuration and excitation characteristics; when $f \equiv 0$ equation (1) describes system free vibrations.

The generalized dissipative function of the system (1) can be presented as follows:

$$\Phi = \sum_{i=1}^{n} \frac{h_i}{\mu_i + 1} \mid \dot{\varphi}_i \mid^{\mu_i + 1} \tag{2}$$

In the case of linear friction (or if the dissipative terms are substituted by linear ones) (1) can be replaced with the matrix equation of linear mechanical systems. With respect to such systems a new performance index is proposed and the corresponding optimization problem is formulated and solved.

Optimal forced vibrations damping

Forced vibrations of linear mechanical systems are described by the matrix-vector equation

$$A\ddot{\varphi} + B\dot{\varphi} + C\varphi = f(t). \tag{3}$$

Formally this equation can be derived from (1) by substituting all non linear elements of matrix H with equivalent linear ones (with respect to the vector of the generalized velocities). Usually the harmonic balance method is used for this purpose.

The value of the average power N dissipated for one period T of the forced vibrations is estimated with the help of the function (2) as follows [4]:

$$N = \frac{1}{T}\int_0^T \Phi(t)dt. \tag{4}$$

This functional is proposed as a performance index to evaluate the stationary forced vibrations process described by (3).

State the problem for optimal adjusting of the damper with regard to maximization of the functional (4). In other words, it is necessary to determine optimal coefficients h providing maximum values for the function N on the established solutions of (1) with a limited amplitude.

Consider the solution of this problem assuming that dissipative terms can be linearized by the harmonic balance method. The equation of motion can be derived from (1) for $n = 1$ and $f = f_0\Omega^2\cos(\Omega t + \delta)$, $f_0 \neq 0$ where all variables are scalar ones. In this case the function Φ coincides with the generalized dissipative function of Lurye [6].

By the help of substitutions

$$\Omega_0^2 = \frac{C}{A}, \quad \tau = \Omega_0 t, \quad \omega = \frac{\Omega}{\Omega_0}, \quad q = \frac{A\varphi}{f_0} \tag{5}$$

$$q' = \frac{dq}{d\tau}, \quad h_\mu = \frac{f_0^{\mu-1}\Omega_0^{\mu-1}}{A^\mu}h \tag{6}$$

we transform the equation of the forced vibrations to the form

$$q'' + h_\mu | q' |^\mu sgn(q') + q = \omega^2 cos(\omega\tau + \delta). \tag{7}$$

According to the harmonic balance method, this equation is replaced by the linear one [5]:

$$q'' + k_\mu q' + q = \omega^2 cos(\omega\tau + \delta), \tag{8}$$

where $q = U_\mu \cos(\omega\tau + \delta)$ and U_μ is the amplitude of forced vibrations for a given friction law ($\mu = 0, 1, 2, \ldots$). The coefficient k_μ is defined according to [5]:

$$k_\mu = \frac{h_\mu}{\pi\omega U_\mu} \int_0^{2\pi} [| - \omega U_\mu \eta|^\mu sgn(-\omega U_\mu \eta)]\, d\psi, \quad \eta = \sin\psi. \tag{9}$$

With the help of the substitution

$$v_\mu = \int_0^{2\pi} |\sin\psi|^{\mu+1}\, d\psi \tag{10}$$

rewrite (9) in the form

$$k_\mu = \frac{v_\mu \omega^{\mu-1} U_\mu^{\mu-1}}{\pi} h_\mu. \tag{11}$$

It follows from (3) – (8) that

$$U_\mu = U(\omega,\ k_\mu) = \frac{\omega^2}{\sqrt{(1-\omega^2)^2 + \omega^2 k_\mu^2}} \tag{12}$$

and, with regard to (11) – (12) the amplitude U_μ has to satisfy the following algebraic equation;

$$\frac{v_\mu \omega^{2\mu} h_\mu^2}{\pi^2} U_\mu^{2\mu} + (1-\omega^2)^2 U_\mu^2 - \omega^4 = 0. \tag{13}$$

If this equation has a positive solution then the coefficient h is presented by the formula

$$h = \frac{\pi A^\mu f_0^{1-\mu} \Omega_0^{2-\mu}}{v_\mu \omega^{\mu-1} U_\mu^{\mu-1}} k_\mu. \tag{14}$$

The corresponding value of the average power dissipated for one period of the forced vibrations is determined with regard to (4), (5), (7), (12) and (13)

$$N_\mu = \frac{\Omega f_0^2 \Omega_0^2 \omega}{2A(\mu+1)} N_l(\omega, k_\mu) \tag{15}$$

$$N_l(\omega, k_\mu) = k_\mu U^2(\omega, k_\mu) = \frac{\omega^4 k_\mu}{(1-\omega^2)^2 + \omega^2 k_\mu^2}. \tag{16}$$

Hence, the following optimal problem can be formulated as follows:

For a given value $\omega > 0$ determine the optimal coefficient $k_\mu = k_\mu^{(0)}$ that provides a maximum for the functional (4) if the amplitude U_μ is limited $U(\omega, k_\mu) \leq a$.

For any $\omega \neq 1$ the function N_l has a unique point of maximum

$$k_\mu^{(0)} = \frac{|1-\omega^2|}{\omega}. \tag{17}$$

According to (12), (16) and (17) we have

$$U^{(0)} = \frac{\omega^2}{\sqrt{2} - |1-\omega^2|} \tag{18}$$

$$N_l^{(0)} = \frac{\omega^3}{2|1-\omega^2|}. \tag{19}$$

It follows from (19) that $\lim_{\omega\to 1} N_l(\omega) = \infty$ and, hence, this expression has no sense for $\omega \to 1$. In a neighbourhood $[\omega_{1a}, \omega_{2a}]$ of the unit frequency for the system (8) the optimal coefficient is defined from the condition $U(\omega, k_\mu) = a$. The variables ω_{1a} and ω_{2a} are functions of the maximal amplitude a and optimal parameters are defined by [4]:

$$k_\mu^{(0)} = \frac{1}{\omega a}\sqrt{\omega^4 - a^2(1-\omega)^2}, \quad U^{(0)} = a \tag{20}$$

$$N_l^{(0)} = \frac{a|1-\omega^2|}{\omega}\sqrt{\left[\frac{\omega^2}{1-\omega^2}\right]^2 - a^2}. \tag{21}$$

The solution of the problem is given by the relations (17) – (19) for $\omega \notin [\omega_{1a}, \omega_{2a}]$ and (20) – (21) for $\omega \in [\omega_{1a}, \omega_{2a}]$ [4].

The maximum of the average power dissipated for one period of the forced vibrations is

$$N_\mu^{(0)} = \frac{\Omega f_0^2 \Omega_0^2 \omega}{2A(\mu+1)} N_l^{(0)}. \tag{22}$$

This expression allows to estimate critical possibilities of the dampers with speedy characteristics. It follows from (22) that the damper with dry friction provides maximum level of energy dissipation compared to the dampers with linear or quadratic friction.

Conclusions

The proposed approach can be applied for optimal vibroisolation of objects with rotational axes, in particular, for optimal damping of free and forced vibrations of articulated manipulators.

The obtained results can be directly used for optimal adjusting of the passive dampers, installed at the joints of articulated manipulators. They may also be applied for optimal active damping of the excited vibrations in case that the manipulator drivers perform corresponding friction laws. For this purpose additional actuators can be used in order to reach high accuracy at the stage of final motion.

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