Journal of Theoretical
and Applied Mechanics
Sofia 1992, Year XXIII, No 4

E. Vasseva

Nonlinear Analysis of Reinforced Concrete Frames under Seismic Excitations

Introduction

A method for a nonlinear study of reinforced concrete structures under strong ground motions is presented in this paper. The effect of the physical and geometrical nonlinearity on their behaviour is investigated. The cases when the predominant period of the seismic excitation is close to the one of the structure and when the difference between them is significant are discussed.

With the view to the maximal use of the bearing capacity of reinforced concrete structures under strong seismic excitations and the specific characteristics of reinforced concrete deformations, during the recent years the necessity of elaborating methods and programmes for nonlinear dynamical investigation wins recognition.

Quite a general and convenient matrix method for the investigation of one - dimensional deformable systems such as frames having a double non-linearity - physical and/or geometrical [1] is used. The numerical solution is realized by the accelerated methods for solving nonlinear systems of differential equations, suggested in [3] and a two-parametric method for step by step integration [2].

Numerical method and basic assumptions

The dynamic investigation of the frame reinforced concrete structures taking into account the two nonlinearities under earthquake excitations is a complex and labour - consuming problem, which necessitates the introduction of some basic assumptions. It is assumed that :

1. The deformations of the frame elements are subordinated to a hysteretic law, similar to Clough's model [4], [5] for moment-curvature relationship with the respective rules for unloading.
2. Bernoulli - Navier' hypothesis is adopted.

3. Evaluating flexibility characteristics of the members, deformations due to axial and shear forces are taken into consideration.

4. The zones in which inelastic deformations are expected are preliminary divided into a finite number of short elements in such a way that the difference of the member end flexural moments is relatively small. This permits their consideration with a sufficient accuracy at each step of loading as such with "instantaneous stiffness".

5. The structure free joints have three degrees of freedom.

6. Large displacements of structure joints are assumed.

7. The masses are assumed to be concentrated in the structure joints. Their rotational inertia is neglected.

8. The supports are submitted to a synchronous translation.

9. The earthquake excitation is given by a real or simulated accelerogram.

The two - parameter step method [2] is used for a step integration of the matrix nonlinear differential equations

$$\mathbb{M}\ddot{x} + \mathbb{C}\dot{x} + k(x) = F(t) \; ; \tag{1}$$

where $\mathbb{M}$ is mass matrix, $\mathbb{C}$ - damping matrix, $k(x)$ - internal resisting force vector, and $F(t)$ - external dynamic excitation.

The initial conditions are :

$$x_0 = x_s ; \quad \dot{x}_0 = 0 ;$$
$$\dot{x}_0 = -\mathbb{M}^{-1}\left[F(t_0) + \mathbb{K}_0 x_s \right] ;$$

where x_s is the vector of static displacements, present in the structure when the seismic action starts.

The unknown vectors x, $\dot{x}$, $\ddot{x}$ at $t_1 = t_o + h$, where h is the time step are determined by the modified equation according to [2]

$$\mathbb{M}\dot{x}_1 + \mathbb{C}\ddot{x}_1 + (1-\alpha)k(x_1) = F_1(t) - \alpha\, k(x_0) \tag{2}$$

where $x_1, \dot{x}_1, \ddot{x}_1$ are approximate values of $x(t_1)$, $\dot{x}(t_1)$ and $\ddot{x}(t_1)$ and α is a free parameter and the additional equations

$$x_1 - x_0 = \frac{h}{2}\left[(1-\alpha)\dot{x}_0 + (1+\alpha)\dot{x}_1\right]$$
$$x_1 - x_0 = \frac{h}{2}\left[(1-\alpha)\ddot{x}_0 + (1+\alpha)\ddot{x}_1\right] \tag{3}$$

For each step of time the solution of the discrete nonlinear equation is obtained by means of simple iterations. The conclusions, made in [2] and the following iteration process are used :

$$x_1^{(\upsilon+1)} = x_1^{(0)} + \mathbb{D}^{-1}\left[\mathbb{K}x^{(\upsilon)} - (1-\alpha)(1+\alpha+\zeta)^2\, h^2\, k(x_1^{(\upsilon)})\right] \tag{4}$$

where $x_1^{(0)} = \mathbb{D}^{-1}P$; $\quad \mathbb{D} = 4\mathbb{M} + \mathbb{K}$

$$P=(1+\alpha+\xi)^2 h^2 \left[F_1-\alpha\, k(x_0)\right] + 4\,\mathbb{M}\left\{x_0 + h\,\dot{x}_0 + \frac{1}{4}\left[1-(\alpha+\zeta)^2\right] h^2 \ddot{x}_0\right\}$$

ξ -- viscous damping, ζ- article viscous parameter

The solution by step by step loading procedure gives also a possibility for the more exact taking into account of the geometrical nonlinearity.

For the solution of the formulated problem it is very important to establish a relation between the curvatures, the relative deformations of the elements axes and the displacements at their ends.

From the above assumptions and some results obtained in [1] it becomes clear that each finite element can be considered as a subject of flexibility at a simultaneous action of an axial force. From the solution of the known linear differential equation and using Livesley's functions, the following matrix equation is obtained :

$$\text{(5)}\qquad \overset{\kappa^j}{\begin{bmatrix}\kappa_i\\ \kappa_k\\ \varepsilon_o\end{bmatrix}} = \frac{1}{l}\ \overset{\mathbb{R}^j}{\begin{bmatrix}4\,\Phi_3 & 2\,\Phi_4 & \frac{6}{l}\,\Phi_2 & 0\\ 2\,\Phi_4 & 4\,\Phi_3 & \frac{6}{l}\,\Phi_2 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}}\ \overset{V^j}{\begin{bmatrix}\varphi_i\\ \varphi_k\\ v\\ u\end{bmatrix}}$$

or

$$\text{(6)}\qquad \kappa^j = \frac{1}{l}\,\mathbb{R}^j\, V^j$$

where κ_i and κ_k are the curvatures of the element axis, in the left and right ends respectively, $\Delta u = u_k - u_i$, $\Delta v = v_k - v_i$, Φ are Livesley's functions [6].

To make a connection between the vector of deformations and the vectors of joints displacements $Z_i=(x_i, y_i, \varphi_i)$ and $Z_k=(x_k, y_k, \varphi_k)$ at the two ends of the element we should pass to a global coordinate system. After suitable transformations in (1) the following relationship is obtained:

$$\text{(7)}\qquad V^j = a_i^j\, Z_i + a_k^j\, Z_k$$

where a_i^j and a_k^j are matrixies including the indicated cosines of the angle α, which the element under consideration has with the X axis.

After substituting (7) in (6), a matrix equation is obtained for determining the vector of deformations in the finite element according to given structural joint displacements.

(8)
$$\kappa^j = \frac{1}{l_j} \mathbb{R}^j \left(a_i^j Z_i + a_k^j Z_k\right), \qquad \varepsilon_o = \frac{\Delta u}{l}$$

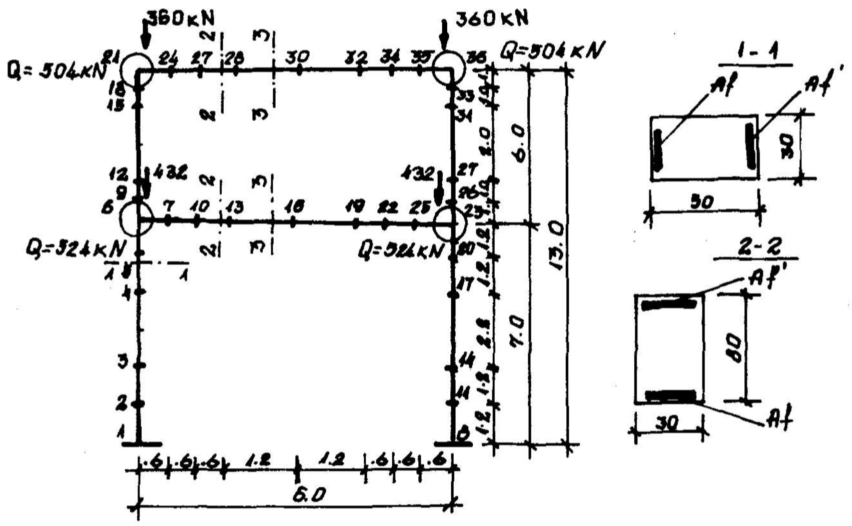

Fig.1

Section 1 - 1 $A_f = A_{f'} = 9.42\ cm^2$

Section 2 - 2 $A_{f'} = 24.13\ cm^2$, $A_f = 20.36\ cm^2$

Section 3 - 3 $A_{f'} = 7.60\ cm^2$, $A_f = 30.54\ cm^2$

B 40
St A-III

The internal forces are determined using the diagrammme curvature-moment "κ - M " made for each element at different normal force N [4], [5]. By it, for a found curvature κ and normal force N the respective bending moment is taken into account. On its part N is determined from the found ε_0 using the reached stiffness of longitudinal deformation.

The "instantaneous" stiffness of the finite elements, that have obtained plastic deformations, are approximately determined as mean values

$$(9) \quad D_{ik} = \frac{1}{2}\left(\frac{\Delta M_{ik}}{\Delta \kappa_i} + \frac{\Delta M_{ki}}{\Delta \kappa_i}\right); \quad D_{ik} = EJ \text{ - flexural stiffness;}$$

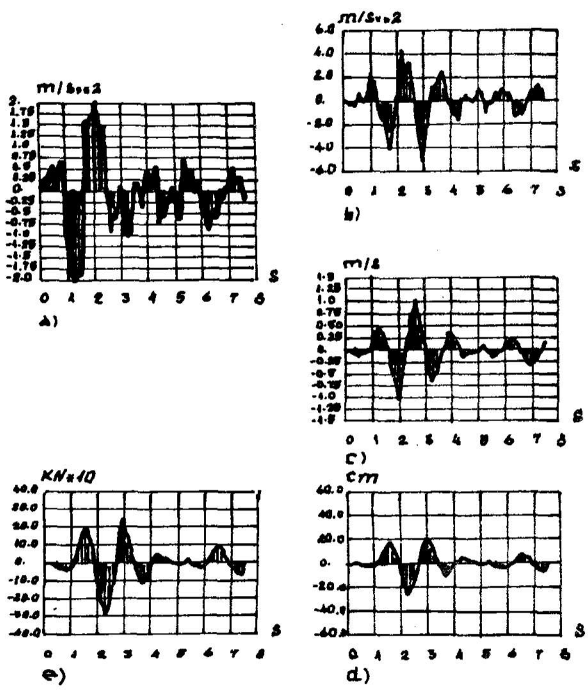

Fig.2 Investigation with Accelerogram "Vranchea Earthquake" Two - storey frame T=1.3s Elastic solution
a) Accelerogram b) Accelerations
c) Velocities d) Displacements
e) Seismic forces

By means of these stiffness, Livesley's functions for different elements are calculated. The mean "instantaneous" stiffness of longitudinal deformation of the element is determined by the relationship :

$$D'_{ik} = \frac{N_{ik}}{\varepsilon_o} ; \qquad D'_{ik} = EF \text{ - axial stiffness;} \tag{10}$$

$$\varepsilon_o = \text{axial deformation;}$$

By introducing Livesley's functions in $\mathbb{R}^j$ matrix, part of the geometric nonlinearity of the system is taken into account as well as the decrease of the elements member stiffness and of the structure as a whole, as a consequence of the axial compressive forces action. Besides, correcting the indicated cosines of the elements, the change of the structure geometry, obtained from the large displacements of the structure joints is taken into consideration.

On the basis of the indicated assumptions and relationships a procedure with simultaneous iterations for the physical and geometrical nonlinearity is used for each step of the seismic loading. This procedure is in the basis of the algorithm of the elaborated computer programme.

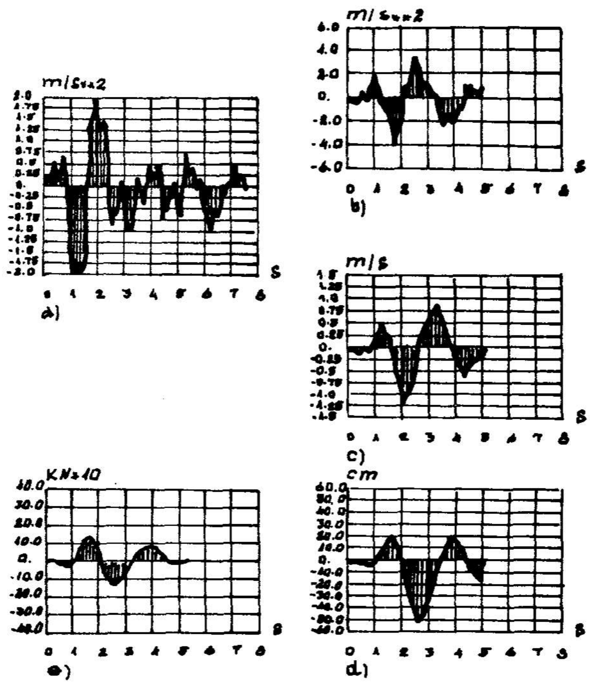

Fig.3 Investigation with Accelerogram "Vranchea Earthquake" Two - storey frame T=1.3s Solution at physical nonlinearity
a) Accelerogram b) Accelerations
c) Velocities d) Displacements
e) Seismic forces

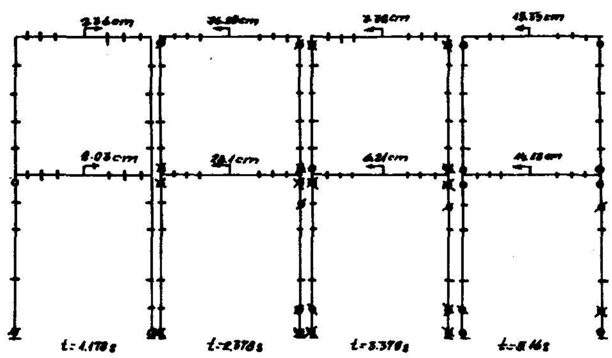

Fig.4

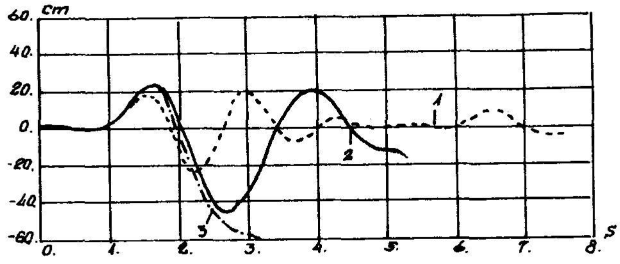

Fig.5 Displacments

1 - elastic solution

2 - solution at physical nonlinearity

3 - solution at physical and geometrical nonlinearity

By this programme a study of a two-storey one-span frame is made. Its calculation scheme with geometric dimensions, numbering of structure joints, cross sections and the corresponding type of reinforcement as well as external loading, is shown in Fig.1. The frame with a natural period of vibrations T=1.3s is studied with the prediction for linear work of the material at physical and double nonlinearity (physical and geometrical). The real stiffness

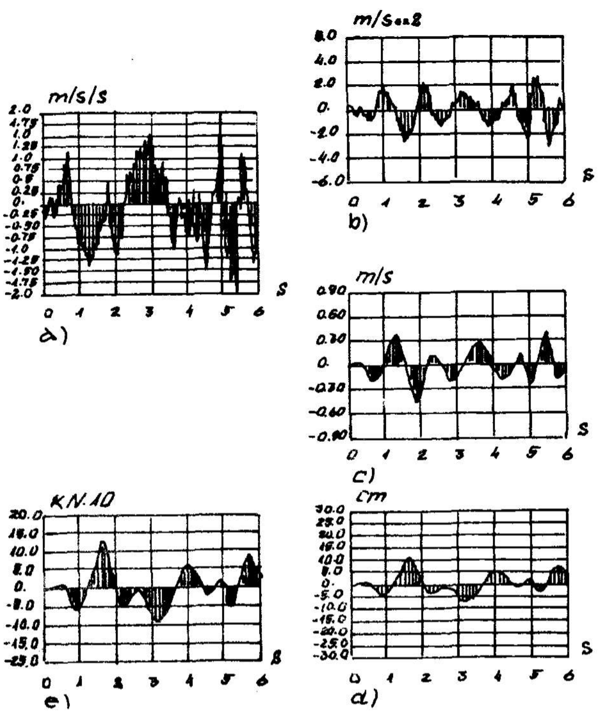

Fig.6 Investigation with Accelerogram "Cherna Gora Earthquake" Two - storey frame T=1.3s. Elastic solution

a) Accelerogram b) Accelerations
c) Velocities d) Displacements
e) Seismic forces

of the reinforced concrete frame is determined at the existing vertical loading. The seismic investigation is carried out using two accelerograms of earthquake in Vranchea and Cherna Gora with a different frequency content, with the view to comparing the results being reduced to the same peak acceleration a_{max} = 0.2g.

I. Investigation under seismic excitation with the main period close to that of the structure

1. Physical nonlinear response

The results of the linear and respectively nonlinear response of the frame at seismic investigation with accelerogram Vranchea (record 1977) shown in Figs. 2 and 3, are given with a time history of the accelerations, velocities, displacements and seismic forces on the second floor level.

Comparing the graphs of displacements it is established that when taking into account the physical nonlinearity there is an increase about two times and moving of the peaks to the right, i.e. extension of the period approximately with 48%. These changes are due to the developed inelastic deformations, which is taken under consideration, and leads to a change in stiffness of the structure under consideration. Under their influence the change of signs of the seismic forces themselves occurs more slowly and the value of these forces also decreases sensitively (with about 33%). The consecutiveness of the zones development with inelastic deformations is shown in Fig.4 where the different degree of the developed inelastic deformations of the different sections are reflected: o - yielding of the reinforced steel; o - one-sided reaching of the ultimate strain in the extreme compression concrete fiber; o - two-sides reaching of the ultimate concrete strains in a given cross section; o - the respective formation of a plastic hinge. When the two-sided establishment of the ultimate concrete strains becomes **L** - times, it means that a plastic hinge in the section has been formed (**L** is an input parameter). Since the elements of the frame are preliminary divided into a finite number of short elements, with length similar to the depth of their cross section, than the formation of the above mentioned "plastic hinge" in two adjoining sections is accepted only as one.

It should be noted that schemes up to 1.178s are not drawn, indicating the zones in which cracks have occurred, but the latter are taken into account, when determining the stiffness of the different reinforced concrete sections. At t=1.178s in three sections of the columns, shown in Fig.4, the first yielding of the reinforced steel occurs. The figure indicates the corresponding displacements of the floors, as well as the relative floor displacements. Because of the small stiffness of the columns in comparison with that of the beams, "plastic" zones are mainly formed in the bottom and top ends of each column. The most intensive inelastic deformations are developed during the second peak of the accelerogram "Vranchea" at t=2.378s, in six sections occurs the single two-sided crushing of the concrete. As a result of developed inelastic deformations, the stiffness, especially on the first story, significantly decreases and that is why a large relative displacement is obtained at the level of the first beam (1/18H). At t=3.379s the first "plastic hinge" if obtained. Step-by-step such hinges appear in the bottom and top end of each column and are a serious danger for the structure, which at the t =5.16s turns into a mechanism. This manner of plastification only of the columns is quite undesirable. When considering the frame by taking into account the nonlinear relationship stress - strain, which is characteristic for the reinforced concrete, in the process of the seismic excitation, the stiffness significantly decreases (in a definite moment in some sections of the columns the decrease comes to 45 - 55 %).

2. Physical and geometrical nonlinear response.

As a result of the general stiffness decrease of the frame, large displacements appear and in this case it is imperative to take into account the influence of the axial forces with a verification of the loss of general stability as well. On the basis of the methods in (1) an investigation has been carried out, taking into account the geometric nonlinearity too. The frame response in this case is significantly changed. The structural instability occurs and the frame is destroyed in t=2.635s before the bearing capacity of the material in a sufficient number of sections is exhausted (Fig.5). This specific example confirms the necessity in the seismic investigation of each structure, its stressed and strained states to be determined besides by taking into account the real properties of the material, also by considering the displacements. The verification of the general stability in this case should be obligatory for each dynamic analysis.

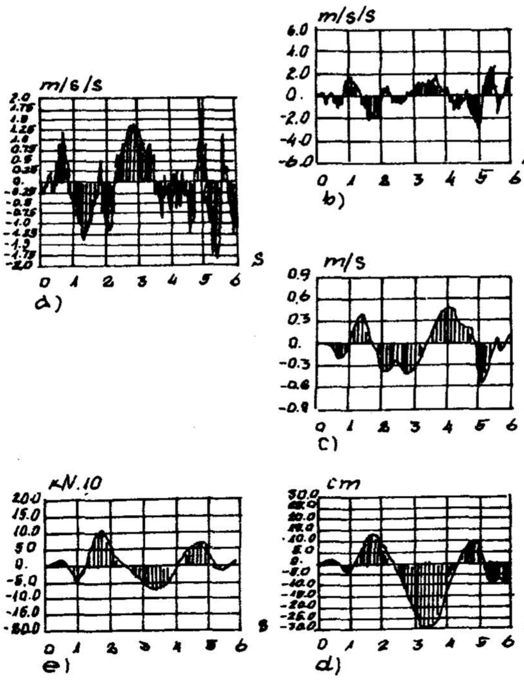

Fig.7 Investigation with Accelerogram "Cherna Gora Earthquake" Two - storey frame T=1.3s Solution atphysical nonlinearity

a) Accelerogram b) Accelerations
c) Velocities d) Displacements
e) Seismic forces

II.Investigation under high frequency seismic excitation

The response of the frame under consideration is analyzed as well as the accelerogram of Cherna Gora earthquake record - 1979. Since in this accelerogram predominate the high frequencies ($T_o \sim 0.24s$), and the frame under consideration has a relatively large period of natural vibrations ($T=1.3s$), then with this external loading it shows less ability for the development of plastic deformations, thus it is not destroyed. That is why the difference between the parameters of response at linear and nonlinear analysis is much less. This is also seen from the comparison of the graphs in Fig.6 and 7 where the accelerations, the velocities, the displacements and seismic forces changes in time are drawn. The percent of decrease of the seismic forces is less too - averagely by 25 %. The process of development of the plastic deformations itself is similar to the one when investigating with accelerogram "Vranchea". This is reflected on the schemes of the frame with the marked plastic zones in Fig.8. The first plastic hinge is formed only at t=4.54s. Basically the plastic deformations are concentrated in the columns, and "plastic hinge" are formed only in columns from the first story. As a consequence of this manner of plastification the frame stiffness in the first story significantly decreases which leads to a big increase of displacements, at that in the relative floor displacements on the level of the soft story. At t=3.46s they reach up to 1/24H.

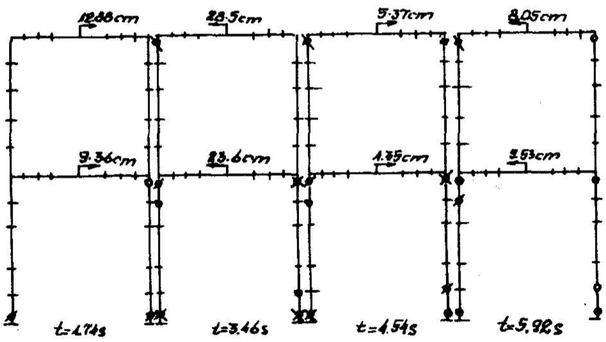

Fig.8

It is clear that a real evaluation of the behaviour of such type of structures with less stiffness of the columns in comparison with the one of the beams, in which the development of the plastic hinges in the columns endangers their safety, can be obtained only by means of a dynamical analysis in time and taking into account their nonlinear deformability.

Conclusions

1. The calculated results obtained from the investigations made, confirm that the response of each structure depends on the concrete excitation, on its specific characteristics and its ability to develop plastic strains.

2. The obtained results show the change of the structure stiffness in the process of seismic excitation and the decrease of the seismic forces when taking into account the physical nonlinearity.

3. On the basis of the investigations carried out, the important role of the geometrical nonlinearity was proved in the process of decrease of the stiffness and its not taking into consideration leads to an error which might be dangerous for the structure.

References:

1. S i m e o n o v, S. On the nonlinear static research of one dimensional frame systems. — Theor. Appl. Mech.,No 1, BAS, 1987.
2. S i m e o n o v, S. On one numerical method in dynamic of elastic systems. — Theor. Appl. Mech., No 1, BAS, 1988.
3. S i m e o n o v, S. Two methods for solution of nonlinear equations of potential type. — Papers, BAS, **39**, 1986, No 11, 33.
4. V a s s e v a E., Investigation of Reinforced Concrete Frame Structures to Seismic Excitation Taking into Account Physical and Geometrical Nonlinearities, Ph.D. Thesis, Higher Institute for Architecture and Civil Engineering, Sofia, 1985 (in Bulgarian).
5. P a r k. R., I. P a u l a y. Reinforced Concrete Structures. University of Conterbury, New Zeland, 1975.
6. L i v e s l e y, R. K., Matrix Methods of Structural Analysis, Moscow, 1980.

Received on May 27, 1992