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Reliability Prognosis of Mechatronic Systems Using Analytical-Simulation Method

The synthesis of mechanics, electronics and informatics led to the appearance of a new trend of machine building and machine building mechatronics, it's object being mechatronic systems (MS). The special cases of MS are: robot-technical complexes, manipulator systems, flexible manufacturing systems, flexible manufacturing modules. MS consists of a hardware, realised by mechanic and electronic components, as a programme-drive support for hardware control.

The questions of reliability prognosis at the stage of design of such systems have not been still sufficiently elucidated, but they are very important, because they have a wide application in practice.

1. Mechatronic system formalization

It may happen a not inspected (functional) failure in any component of MS. The number of functional failure varieties is the number of singled out components n_k .

The second reason for failures is connected with the unstableness of the determinative parameters, therefore these failures are called parametric failures. The number of varieties of the parametric failures is the number of the singled out parameters n_p .

Operative MS is subjected to technical maintenance, plan and preventive repair, and classified by one of the n_r types. Cycle and order of operations are defined for every type of maintenance.

Operative MS is subjected to a retuning, connected with the transfer to the output of new types of production.

As said above, MS may be formalized as a combination of elements united into four groups: 1) elements, subjected to functional failures; 2) elements, subjected to parametric failures; 3) elements, subjected to breaks in the plan-preventive repair; 4) an element, subjected to retuning. The total number of the elements is $n=n_k+n_p+n_r+1$. In the simplest case the elements are combined successively (element failure leads to a system failure), but net structure is possible.

The reestablishment after element failure is carried out in several phases. Two phases of reestablishment are discussed in the paper: repair organization and repair itself. It is assumed that distribution functions of time-to-failure $F_o^i(t)$ and reestablishment times on phases $G_1^i(t)$, $G_2^i(t)$ are known for every element.

MS particularity is the dependence between elements, when the failure of the element has an influence on the probable characteristics of the others. It makes it difficult to use the classical methods of reliability prognosis. In this paper prognosis is meant to estimate and investigate reliability factors at the stage of the system design. To solve the prognosis problem is offered an analytical-simulation method. Creation and realization of this approach are the aims of the paper.

2. MS reliability factors

A random marked pointed process $\{(T_k,Z_k),\ k>0\}$ is chosen as a mathematical model, describing the MS behaviour, where T_k is the moment of time when the system changes it's state; Z_k is the system state at time T_k [1].

If one element has a failure in the successive scheme then the others stop to operate during the reestablishment time, when the number of states is n+1.

Let's introduce consequence

$$\{(X_m,Y_m),m>1\}\ , \tag{1}$$

X_m is system time in capacity to operate state, Y_m is system time in uncapacity to operate state. If variables X_m, Y_m are independent and have distributions independent of m, then process (1) is called alternate [1].

Let $F_0(t)$, $G(t)$ are distribution functions of X and Y; $f_o(t)=F_o'(t)$; $\bar{F}_0(t)=1-F_0(t)$; $\bar{x}_o={}_0\int^{\infty} F_0(t)dt$; $g(t)=G'(t)$; $\bar{y}={}_0\int^{\infty}(1-G(t))dt$.

For the reestablishment system, described through the alternate process by the following reliability factors, are offered [1], [2]:

1) parameter of reestablishment flow

$$\omega(t)=f_c(t)+\int_0^t \omega(z)f_c(t-z)dz\ , \tag{2}$$

where $f_o(t) = \int_0^\infty f_o(t)g(t-z)dz$;

2) active function

(3) $$K_\tau(t) = \bar{F}_o(t+\tau) + \int_0^\tau \bar{F}_o(t+\tau-z)\omega(z)dz \ .$$

At $\tau=0$ $K_o(t)$ is called the function of availability.

3) coefficient of active readiness

(4) $$k_\tau = \lim_{t \infty} K_\tau(t) = \frac{k_o}{\bar{x}_o} \int_\tau^\infty \bar{F}_o(t)dt \ .$$

At t=0 readiness coefficient

(5) $$k_o = \frac{\bar{x}_o}{\bar{x}_o + \bar{y}} \ ;$$

4) fulfillment task probability at time T

(6) $$P = F_z(T)_z,$$

where $F_z(t)$ is distribution function of fulfillment task time.

For the reliability prognosis of MS it is necessary:

1) to base one's arguments that process (1) is alternate on facts ;

2) to work out the estimation method of functions $f_c(t)$ and $f_o(t)$;

3) to automate the receiving process of factors (2)-(6).

For solving of these tasks an analytical-simulation procedure (ASP) is worked out. It is realized by means of Pascal on PC/AT-286 and includes three parts: FORMIR, NATALY, POKIN.

3. Analytical-simulation procedure description

FORMIR part initializes the ASP w.r.t the MS structure, strategies of maintenance, distribution of operations and phases of reestablishment for each element, parameters validity of these distributions, providing the defined values of numerical characteristics.

Twelve distributions are used in ASP, i.e.: Weibull (W), Gamma (G), cutted down normal distribution (UN), inverse Gaussian (OG), Saunders - Birnbaum (SB) and others. The number of distributions may be increased.

For the operation of elements of the first group, IFR-distributions or its superpositions are used.

Distributions connected with the changing process of parameters are recommended for the operation of elements of the second group. If this process is described by the fan function, then an α-distribution [2] or a U-distribution [3] are recommended. If the process is described by Viner's process with displacement, then a OG-distribution is added [1]. If the failure results from a distribution connected with the accumulation of a great number of micro distructions, then a SB-distribution [1] is advisable.

It is shown in paper [3], that if organization losses are realized and equipment stopping is connected with the limited number of repair-workers, then this time is described by the distribution

$$F(t)=q\lambda(t)+(1-q)(1-e^{-lt})/(1-e^{-lt_m}),\ 0\leq t\leq t_m,$$

This distribution is called UPD-distribution, where $\lambda(t)$ is the isolated function, q is probability of expectation time equal to zero, λ is a parameter.

Module NATALY simulates the behaviour of the system elements taking into consideration their interaction, strategies of maintenance and operation. And it forms the random marked pointed process $\{(T_k,Z_k),\ k>0\}$, classifying states Z_k, forming process (1). It accumulates and analyzes working up and statistical information.

The checking of the hypothesis about the homogeneity of process (1) is carried out by the H-statistics of Kraskel-Waliss

(7) $$H=\frac{12}{N(N+1)}\sum_{j=1}^{m}\frac{R_j}{N_j}-3(N+1),$$

where m is sample size, N_j is volume of the j-sample, $N=\sum_{j=1}^{m}N_j$, R_j is ranges sum of the j-sample. By $H<\chi^2(m-1,\alpha)$ the homogeneity hypothesis is proved.

The checking of the hypothesis on the independence of X and Y is conducted by the C-statistics

(8) $$C=N[\sum_{i=1}^{m_x}N_{i}^{-1}\sum_{j=1}^{m_y}(N_{.j}^{-1}N_{ij}^{2})-1],$$

where N_{ij} are the frequencies of the values (X_i, Y_j); m_x, m_y are the numbers of the intervals in the histogramme; $N_{i.}=\sum_{j=1}^{m_y}N_{ij}$; $N_{.j}=\sum_{i=1}^{m_x}N_{ij}$.

By $C<\chi^2[(m_x-1)(m_y-1),\alpha]$ the hypothesis is proved.

Confidence interval (k_l, k_h) for readiness coefficient (5)

$$k_l = \tilde{k}_o - \delta; \; k_h = \tilde{k}_o + \delta; \quad \delta = z(\frac{\alpha}{2}) S(\tilde{k}_o),$$

where $z(\frac{\alpha}{2})$ is quantity of normal distribution at confidence probability $1-\alpha$; $\tilde{k}_o$ estimates coefficient k_o, $S(\tilde{k}_o)$ is standard deviation estimation $\tilde{k}_o$.

$$k_o = \sum_{j=1}^{M} X_j / \sum_{j=1}^{M} Z_j; \quad Z_j = X_j + Y_j,$$

$$S(\tilde{k}_o) = \{[\sum_{j=1}^{M} X_j^2 - 2\tilde{k}_o \sum_{j=1}^{M} X_j Z_j + \tilde{k}_o^2 \sum_{j=1}^{M} Z_j^2]/[\frac{M-1}{M}(\sum_{j=1}^{M} Z_j^2)]\}^{1/2};$$

where M is number of intervals; Z_j — length of the j-interval; X_j — time of the operation state of the system within the j-interval. In paper [4] it is pointed put that the approximation of the relation by normal distribution is possible if the following conditions are fulfilled: 1) $M>30$; 2) variance coefficients of the average X and Z must be less then 10%. In module NATALY these conditions are checked.

Confidence interval $[p_l, p_h]$ for task fulfillment probability is defined from Klopper-Pirson equation. Probability estimation

$$\tilde{p} = (L_z - L_n)/L_z,$$

where L_z is common number of tasks; L_n — non-executed tasks in the time because of failures. Let D_k be fulfillment duration of the k-task, ΔD_k — the increase because of MS failures. So if

$$\Delta D_k / D_k > \varepsilon_g,$$

k-task will not be fulfilled, ε_g being permissible increase for technology.

The main purpose of module POKIN is to define probability factors (2)-(5) using numerical methods. Densities $f_o(t)$, $f_c(t)$ are approximized by bit-linear functions such as

$$\tilde{f}_0(t)=\begin{cases} 0 & t\leq 0 \\ a_1, & 0<t\leq \Delta t/2 \\ a_j+\dfrac{(a_{j+1}-a_j)(t-T_j)}{\Delta t}, & T<t\leq T_{j+1} \\ 0, & t>T_{m+1} \end{cases}$$

where T_j are centers of histogramme intervals with length Δt, $T_j=\Delta t/2+(j-1)\Delta t$, m is number of intervals, $a_j=N_j/(N\Delta t)$; N_j are frequencies, $N=\sum_{j=1}^{m} Nj$.

By the solution of integral equation (2) the method of successive approaches of initial approach $\tilde{f}_c(t)$ is used.

For readiness function $K_o(t)$ is defined maximum deviation from the etalon function $\check{K}_o(t)$, when production and reestablishment time have an exponent distribution

$$R=\max_t|K_o(t)-\check{K}_o(t)|,$$

where $$\check{K}_o(t)=k_o+(1-k_o)\exp\{(1/\bar{x}_o+1/\bar{y})t\}.$$

4. Investigation of reliability factors

Six factors have been defined as basic: coefficients k_t and k_o, task fulfillment probability P; mean production $\bar{x}_o$ and reestablishment $\bar{y}$ times; maximum deviation R.

The following variables have been studied as regulated: A is distribution low for the first group of elements, B is strategy of technical maintenance for the first group of elements; C is distribution low of production time for the second group of element; D is strategy of technical maintenance of the second group of elements; E is strategy of operation for the fourth group of elements; F is distribution low of repair time.

Using fractional 2 design with 4 replications, the validity of the chosen

Table 1

N	1	2	3	4	5	6	7	8
H	2.379	2.283	1.744	1.545	5.234	3.377	3.270	1.543
C	187.700	191.040	205.420	190.730	267.600	184.960	250.570	204.520
k_i	0.817	0.736	0.733	0.801	0.725	0.799	0.801	0.713
$\tilde{k}_0$	0.826	0.747	0.740	0.810	0.737	0.809	0.812	0.721
k_h	0.834	0.757	0.747	0.820	0.749	0.818	0.824	0.730
$\tilde{k}_\tau$	0.560	0.469	0.458	0.548	0.439	0.537	0.538	0.427
p_i	0.920	0.726	0.680	0.922	0.707	0.920	0.846	0.604
$\tilde{p}$	0.977	0.768	0.725	0.977	0.752	0.977	0.932	0.652
p_h	1.000	0.810	0.770	1.000	0.794	1.000	1.000	0.701
$\tilde{x}_o$	23.702	16.375	16.368	22.885	15.072	21.393	21.091	14.829
$\tilde{y}$	5.008	5.555	5.759	5.352	5.376	5.056	4.867	5.736
R	0.023	0.037	0.032	0.036	0.035	0.026	0.023	0.050

Table 2

Q\P	k	k_t	p	$\bar{x}_o$	$\bar{y}$	R
A	-	-	-	-	-	-
B	+	+	+	+	+	-
C	+	+	+	+	-	-
D	+	+	+	+	+	-
E	+	+	+	+	+	+
F	+	-	-	-	+	+

variables has been checked for main reliability factors.

The results of calculations for the 8 variants are shown in Table 1

H-statistics (7) is calculated, the number of samples m = 10, then $H_{cr} = c^2(9; 0.05) = 16.9$; C-statistics (8) is calculated, the number of intervals $m_x = m_y = 16$, then $C_{cr} = \chi^2(225; 0.05) = 286.0$. Regarding the analysis of the results, let us conclude, that for formalized MS the process (1) is alternate, therefore factors (2)-(5) may be used. The validity of chosen variables (Q) regarding factors (P), obtained as a result of the factor analysis, are shown in Table 2 (- is unmeaning, + is meaning).

The analysis of Table 2 gives us grounds to conclude:

1) variable A is unmeaning for all factors. We think, it is connected with the fact that only three components have been defined and they all have high reliability as suggested ;

2) variables B, C, D, E are meaning practically for all stationary factors (k_o, k_t, p, $\bar{x}_o$, $\bar{y}$). They show strategies validity of technical maintenance and operation;

3) for the unstable factor R variables E and F are meaning;

4) variable E connected with the periodical reestablishments is meaning for all main reliability factors.

Investigating $P(\varepsilon_q)$, it has turned out, that this probability is not increasing at the increment of ε_q, and the rate of function verification is first increasing and then decreasing. When increasing the time between reestablishments the interval of rate verification is diminished .

As a result, it must be concluded that the desired analytical-simulation procedure allows to prognosticate system reliability, which is suitable for the suggested formalization and must be used by designers.

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