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Numerical Study of the Axisymmetrical Flow Past a Hollow Cylinder

The granulated catalysts are very important for many different chemical technologies. For such kind of catalysts the size of their surface is the main factor for their catalytic activity. For this reason the particles, in the shape of a tube piece (usually called Raschig rings) are often used in numerous chemical technologies. The general conclusions about the flow and the transport phenomena in the fluidized or semifluidized beds of catalyst particles can be obtained from the behavior of the single chemically active particle in the flow.

The purpose of the present investigation is to study the axisymmetrical flow past a hollow solid cylinder with a finite length and finite thickness of the wall (Fig.1). Due to the axial symmetry of the particle and the flow at infinity the problem is axisymmetrical. This makes the Navier-Stokes equations and the considered region to be two-dimensional. The existence of doubly connected region brings some difficulties in the considered problem because of the boundary conditions for the stream function. Such kind of difficulties are eliminated in [1]-[3] by- accepting the pressure to be a single valued function. In the present work the same condition is applied. To solve the other difficulty connected with the infinity, the so-called computational infinity is used. The latter is the contour placed far from the particle. The computational infinity is based on the assumption that far from the particle the stream becomes uniform. Finally the boundary of the computational region is $G = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4$ as shown in Fig. 2.

The two-dimensional Navier-Stokes in (ψ,ζ) - formulation is used:

(1) $$\frac{\partial \zeta}{\partial t} + \frac{2}{r^2}\frac{\partial \psi}{\partial z}\zeta + \frac{1}{r}\left[\frac{\partial \psi}{\partial r}\frac{\partial \zeta}{\partial z} - \frac{\partial \psi}{\partial z}\frac{\partial \zeta}{\partial r}\right] = \frac{1}{Re} D^2(\zeta)$$

(2) $$D^2\psi = \frac{\partial^2 \psi}{\partial r^2} + \frac{\partial^2 \psi}{\partial z^2} - \frac{1}{r}\frac{\partial \psi}{\partial r} = \zeta$$

The equations (1), (2) are written in a dimensionless form using the following formulas:

$$[r]=[z]=L,\ [\psi]=U_0L^2,\ [\zeta]=U_0,\ [p]=U_0\rho,\ Re=U_0L/\upsilon,\ [C_f]=U_0^2\rho F,\ F=2\pi(R_1^2 - R_2^2)$$

where ψ is stream function, ζ - the vorticity, L - character length (thickness of the particle wall), U_0 - velocity at infinity, ρ - fluid density, υ kinematic viscosity, Re - Reynolds number, C_f - drag coefficient, F - surface of the particle cross-section, R_1 and R_2 are outer and inner radius of the particle.

In the equation (1) a new term $\partial\zeta/\partial t$ is taken where t is fictitious time which is used as an iteration parameter. This makes the application of theADI method possible.

The dimensionless boundary conditions are:

(3)
$$\begin{aligned} \psi &= 0 && \text{over } \Gamma_4 \\ \psi &= r^2/2 && \text{over } \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \\ \psi &= \text{const.} && \text{over } \Gamma \\ \zeta &= 0 && \text{over } \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4 \end{aligned}$$

In the system of boundary conditions two conditions are missing - the conditions for ψ and ζ over Γ (rigid surface). In order to obtain ψ over the particle surface, the condition for the pressure to be a single valued function is used [3]. It has the following form:

(4) $$\oint_{\Delta} dp = 0,$$

where Δ is a contour, which includes Γ and p is pressure.

The boundary condition for ζ is Tom's condition, combined with the relaxation procedure.

The main characteristics of the problem are the drag coefficient and the flow through the particle cavity. These two characteristics are defined as follows:

(5)
$$C_f = \frac{1}{3Re}\left[4\int_{l_1}\frac{\partial\zeta}{\partial r}dz - 2\int_{l_2}\frac{\partial\zeta}{\partial r}dz - 4\int_{l_1}\zeta\, dz + 4\int_{l_2}\zeta\, dz - 2\int_{l_3} r\frac{\partial\zeta}{\partial r}dr + 2\int_{l_4} r\frac{\partial\zeta}{\partial r}dr\right]$$

(6)
$$Q = \int_0^{2\pi} d\varphi \int_0^{k_1}\frac{\partial\psi}{\partial r}dr = 2\pi C$$

where l_1, l_2, l_3 and l_4 are the four parts of the contour of particle cross-section (Fig. 2), k_1 is the radius of the particle cavity and C is unknown constant from (3).

The boundary-value problem (1)-(3) is solved by using both the ADI and over-relaxation methods [4]. The value of the unknown constant is obtained from (4) using the iterative procedure.

The results for values of Reynolds number 1, 10 and 40 are obtained in the present investigation.

In Fig. 3 the flow around the particle with thickness of the wall 1, length 1.8 and radius of the particle cavity 1 is represented. Stokes' character of the flow is well expressed. The value of the constant C is 0.0577

In Fig. 4 the streamlines pattern for $Re = 10$ for the same geometrical configuration is shown. Due to the increase of the intensity of the flow, one can see that the stagnation zone is arising behind the particle. In this case C = 0.1889.

The flow around the particle with the same size as in the above cases for $Re = 40$ is illustrated in Fig. 5. In this case the stagnation zone behind the particle is well seen. It contains two vorticities restricted by the isoline $\psi = C = 0.3439$.

The drag force inducted on the particle from the flow is represented in Fig.6.

R e f e r e n c e s

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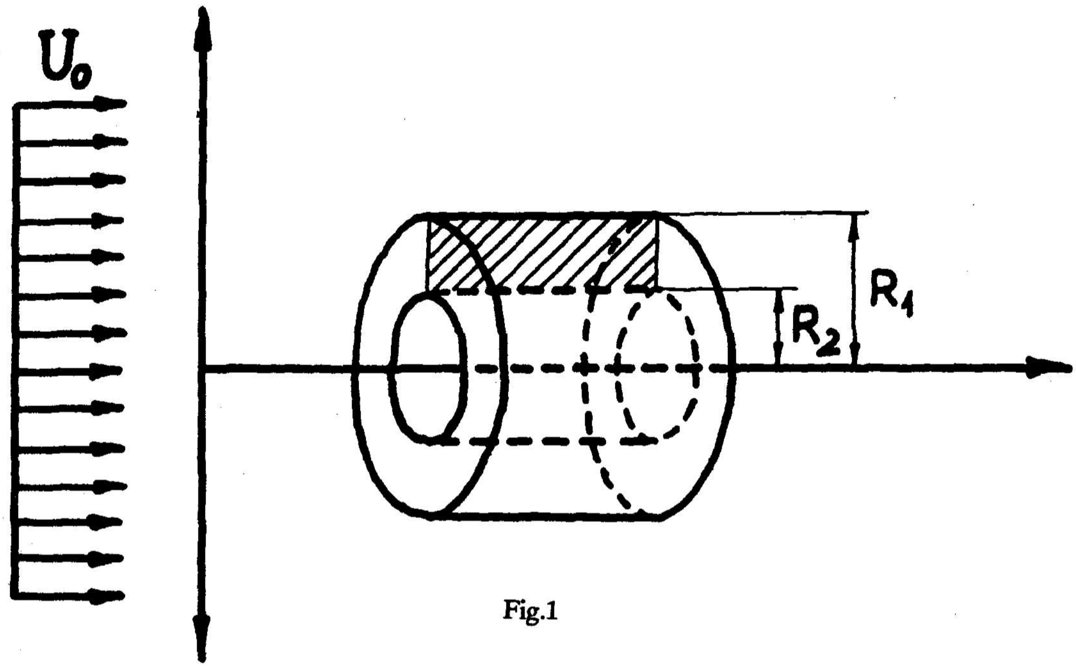

Fig.1

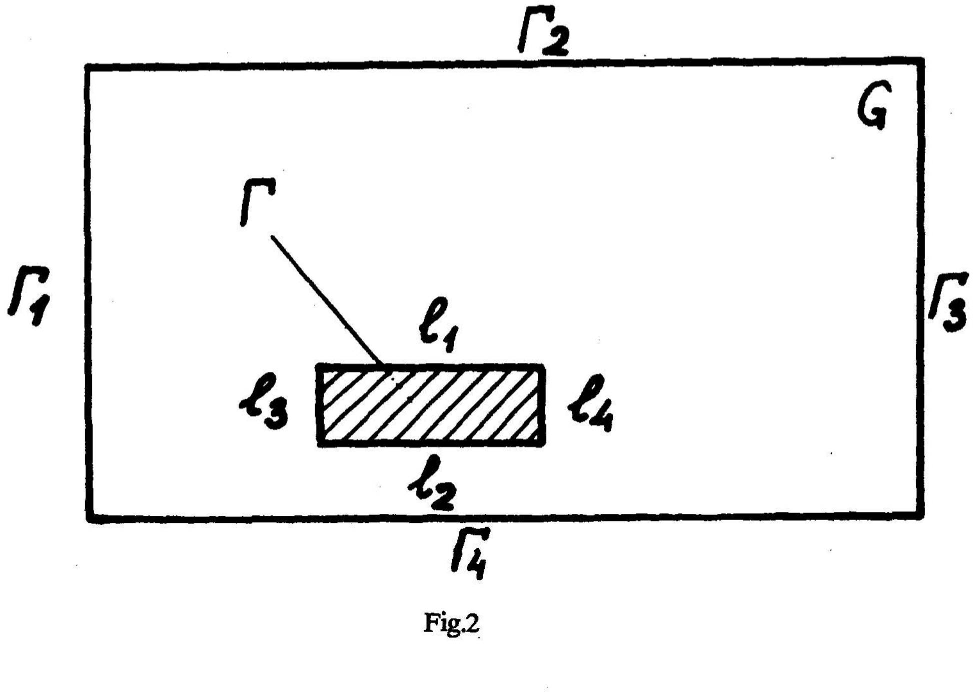

Fig.2

Stream Function for Re=1

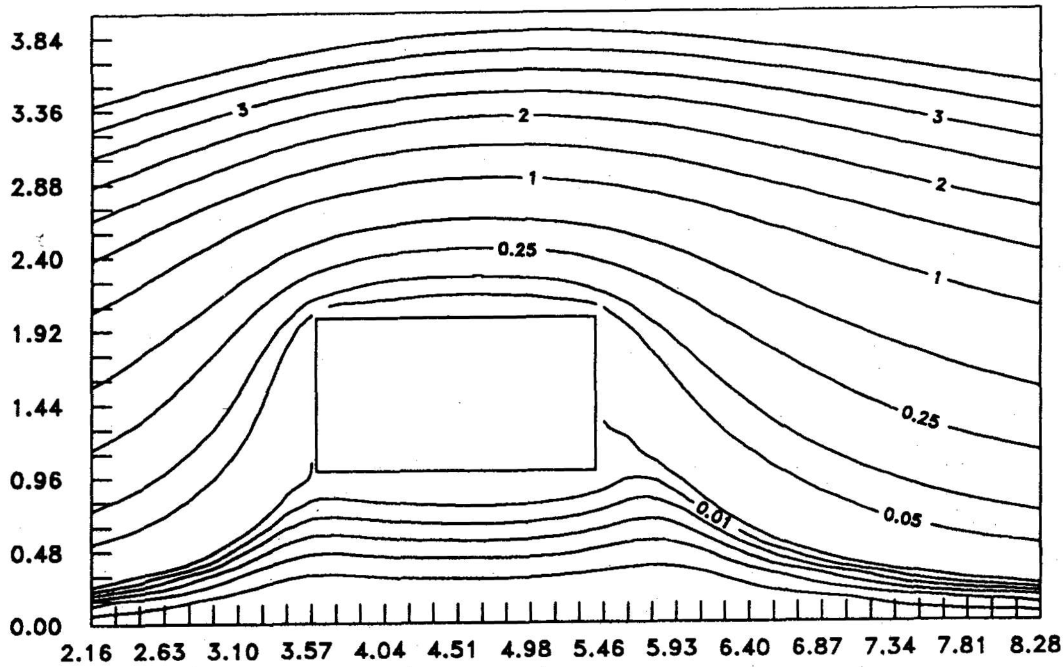

Fig. 3

Stream Function for Re=10

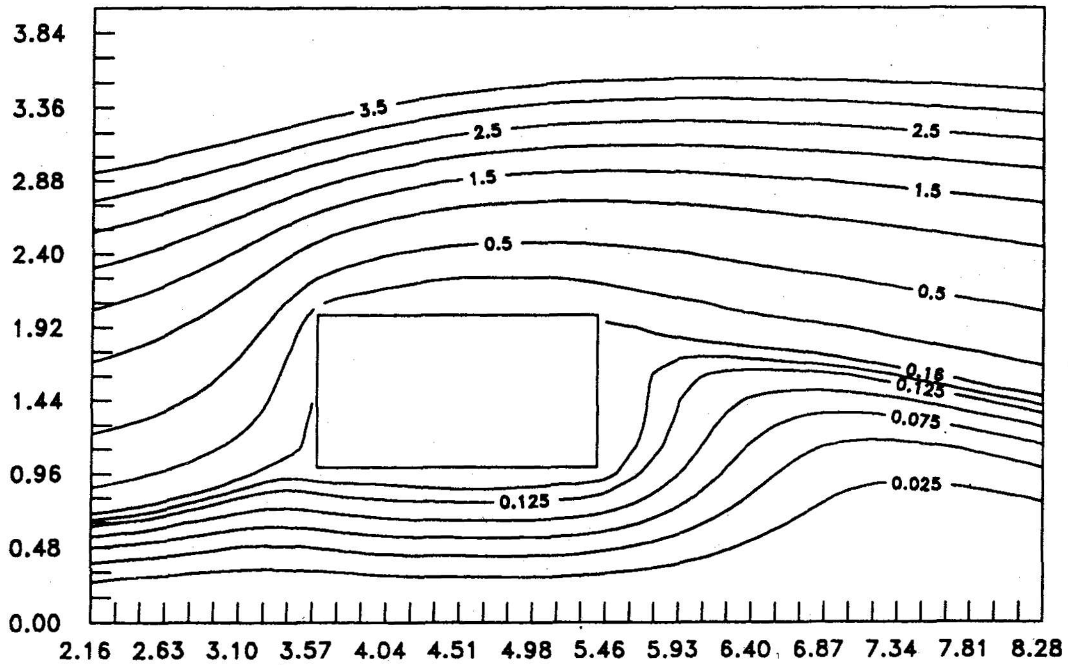

Fig. 4

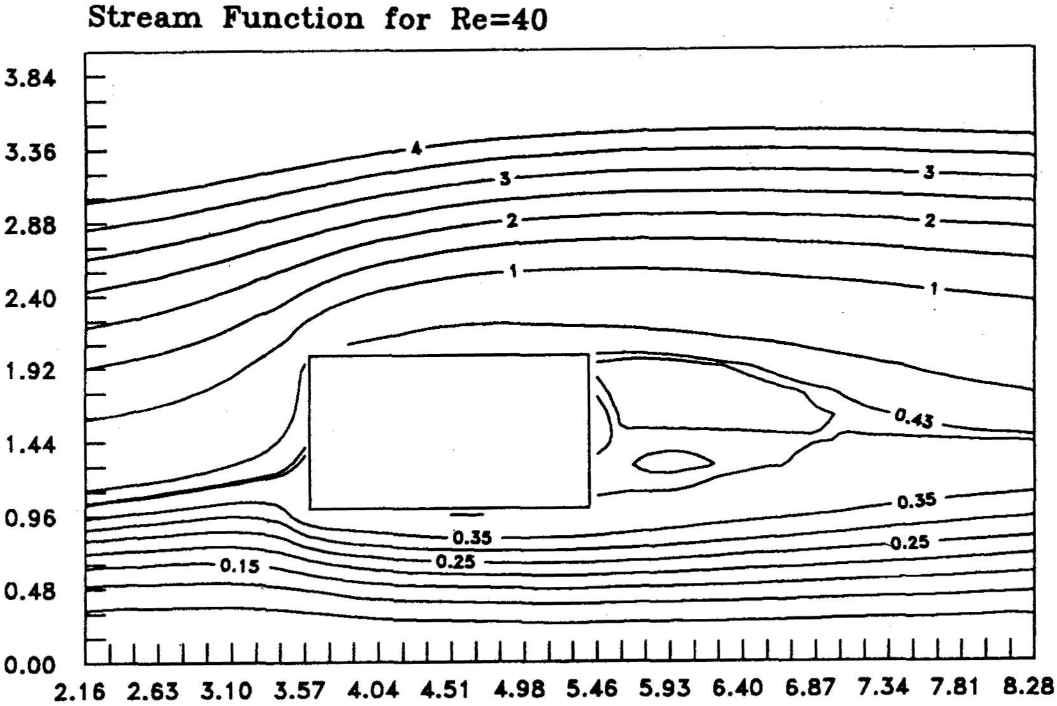
Stream Function for Re=40
3.84
3.36
2.88
2.40
1.92
1.44
0.96
0.48
0.00
2.16 2.63 3.10 3.57 4.04 4.51 4.98 5.46 5.93 6.40 6.87 7.34 7.81 8.28
4
3
2
1
0.43
0.35
0.25
0.15

Fig. 5

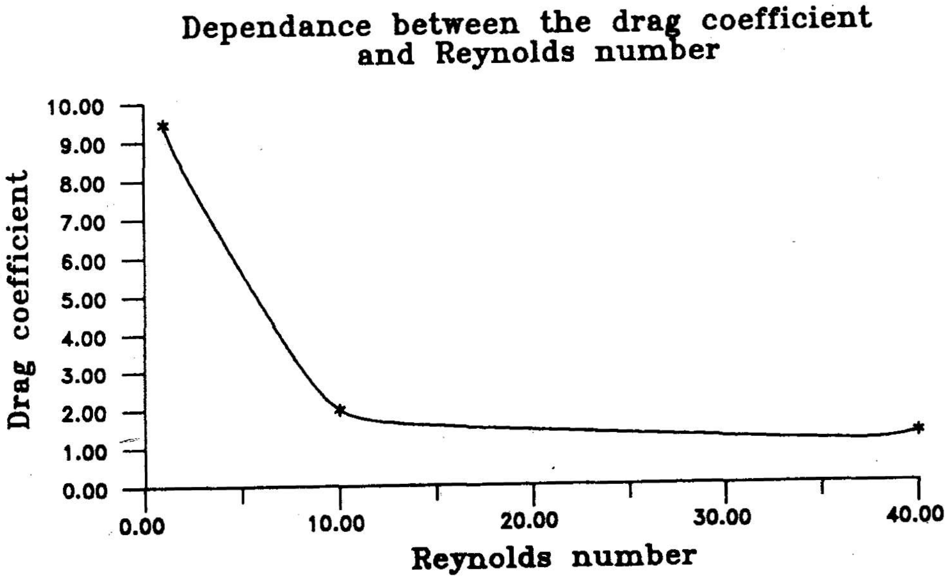
Dependance between the drag coefficient
and Reynolds number
Drag coefficient
10.00
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
0.00
10.00
20.00
30.00
40.00
Reynolds number

Fig. 6