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Modelling Technological Process of Plastics Moulding under Pressure

The method of plastic moulding under pressure is widely used in industry. Melting polymer is supplied in the cavity of the necessary form, fills it out and then the polymer is exposed for some time until final solidification and tension relaxation. We can consider the filling process and the process of cooling and condensation independently. Mathematical models structure of heat and mass transfer processes has been developed in all basic stages of the plastic moulding process.

Simplified quasi-three-dimensional mathematical models were proposed for plastic moulding of thin-walled constructions. This models are based on the approximation of the thin layer. They include two-dimensional elliptical or parabolic pressure equations at the middle surface, simplified three-dimensional equations of motion and continuity, equation of thermal conductivity with regard to phase transition. A feature of the filling stage is the free (unknown) boundary of the liquid polymer. In the condensation and cooling stages the principal question is to take into account compressibility of the polymer that is considered as a reologically complex liquid. We have developed calculating algorithms that realize quasi-two-dimensional and quasi-three-dimensional models of polymer motion in thin-walled cavities. In this paper we use the method of decomposition (separation) to simple shells in simulation of complex shell constructions. Corresponding exchange boundary conditions on the common boundary are proposed.

We have developed the system **PLASTIC** of the automation of projection of casts from plastic. It includes standard means of computer graphics, data bases of thermoplastic casting and casts equipment. The system **PLASTIC** allows to optimize the parameters of the technological process, runner systems. The first version is based on application of quasi-two-dimensional mathematical models of the process of filling of the cavities.

1. Mathematical model

For the simulation of movement of the melting polymer in thin-walled cavities we use models that are based on the approximation of the thin layer. These models are analogous to the approximation of the boundary layers in liquid and gas mechanics [1]. For cavities with nonplanar middle surface, coordinates that are connected with the shell, are discussed. Transition to such shells does not lead to substantial complications from the point of view of mathematical models. This complications are similar to those which appear in the transition from orthogonal coordinates to not orthogonal ones.

The peculiarity of the polymer currents in thin cavities considered is taken into account most perfectly using the pressure equation as a determining one. This approach is well known (see, for instance, [2]). In our paper we get pressure equation in the most general case (curvilinear shell, not isothermal current of reologically complex liquid) on the basis of the procedure of cavities thickness averaging of the input equation. It is convenient to illustrate the mathematical model and the used calculating algorithms by the problem of the cavity filling, cavity being thin chink of variable section. Middle chink surface is supposed plane (a plane chink).

We adduce basic equations in the thin layer approximation, i.e. the quantity $\varepsilon=h/L$ is supposed to be small. Here h is the cross chink measure, L - characteristic linear measure along the chink.

Ordinary Cartesian coordinates are used for the considered plane chink. Let, for the clearness, z is the cross coordinate ($z=0$ middle surface) and x,y are longitudinal coordinates. Let us write out the corresponding simplified system of equations in a thin layer approximation ($\varepsilon<<1$). The continuity equation has the form

(1) $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0.$$

Motion equations are taken in the form:

(2) $$\frac{\partial p}{\partial x}=\frac{\partial}{\partial z}(\mu\frac{\partial u}{\partial z}),$$

(3) $$\frac{\partial p}{\partial y}=\frac{\partial}{\partial z}(\mu\frac{\partial v}{\partial z}),$$

(4) $$\frac{\partial p}{\partial x}=0.$$

Thermal conductivity equation in the thin layer approximation is

(5) $$c\rho\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right]=\frac{\partial}{\partial z}(k\frac{\partial T}{\partial z})+\mu\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right]$$

Here u,v,w are corresponding velocity v components, p - pressure, μ- viscosity, ρ - density, c - thermal capacity, k - coefficient of thermal conductivity and T - temperature. The solution of the boundary value problem for the system of equations (1) - (5) can be based on the solution of elliptical equation for pressure $p = p(x, y, t)$. It is obtained by an integration of the continuity equation (1) with respect to z (perpendicularly to the middle surface). Let the width of the chink in the point $(x, y, 0)$ of the middle surface is characterized by the relation: $f_-(x, y)<z<f_+(x, y)$, i.e. the local width of the chink $h(x,y)$ is determined by the expression $h(x, y) = f_+(x, y) - f_-(x, y)$.

Let us find the function R as the solution of the following boundary value problems for any point (x, y) belongs to the middle surface:

(6)
$$\frac{\partial}{\partial z}\left(\mu \frac{\partial R}{\partial z}\right) = 1,$$

(7)
$$R = 0, \; z = f_{\pm}(x, y).$$

For the longitudinal velocities we have the representations:

(8)
$$u = R\frac{\partial p}{\partial x}, \quad v = R\frac{\partial p}{\partial y},$$

i.e. $\mathbf{v} = R \operatorname{grad} p + w$. Let us define

(9)
$$S = \int_{f_-(x, y)}^{f_+(x, y)} R(x, y, z, t)\,dz$$

and obtain the pressure equation as follows:

(10)
$$\frac{\partial}{\partial x}\left(S\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(S\frac{\partial p}{\partial y}\right) = 0,$$

where the coefficient S is determined according to (6) - (9).

Now let us discuss the boundary conditions for the equation (10), for which there is a free boundary problem. On rigid walls $\mathbf{v}.\mathbf{n} = 0$ and therefore

(11)
$$\frac{\partial p}{\partial n} = 0.$$

Continuity of normal components of forces on the free boundary (dynamic condition) leads to a condition of the first kind

(12)
$$p = p_0,$$

if p_0 is the pressure in the unfilled part of the cavity. The motion of the boundary (kinematic condition on the free boundary) is determined by the condition

(13)
$$\mathrm{v}_n = S\frac{\partial p}{\partial n},$$

where v_n is normal velocity of the melted polymer front. This velocity is naturally connected with the average velocity

$$V_n = \frac{1}{h}\int_{f_-}^{f_+} \mathrm{v}_n \, dz$$

with regard to (8)

$$v_n = R\frac{\partial p}{\partial n}$$

and taking into account (9) we obtain the condition (13).

Thus, we come to the problem of the free (unknown) boundary of the elliptical equation of the second order [3] with two boundary conditions (12), (13).

Input from the cavity boundary is modelled by introduction of the input velocity on the boundary, i.e. by the not uniform condition from the second kind:

(14) $$S\frac{\partial p}{\partial n} = q_{ing}.$$

For input from the side surface of the cavity (point input) we use a not uniform pressure equation. Let (x_k, y_k) is input point and q_k is a polymer expenditure in this input point. The pressure equation has the form

(15) $$\frac{\partial}{\partial x}(S\frac{\partial p}{\partial x}) + \frac{\partial}{\partial y}(S\frac{\partial p}{\partial y}) = \sum_k \delta(x-x_k,y-y_k)\, q_k,$$

where $\delta(x, y)$ is two dimensional δ-function.

The mathematical model that we use relatesto the class of quasi-three-dimensional models. This characteristic is determined using the two-dimensional pressure equation. In this case the thin structure of the melted polymer boundary with respect to cross coordinate is not taken into account (the front is supposed to be plane. For the determination of velocity and temperature fields we use, though simplified, the three dimensional models. We can also note that the processes considered here are substantially unstable. All this determines the complexity of the mathematical model and the produced specifications to the computers being used.

2. Main features of the computing algorithm

For the computing projection of complex products, systems of projection automation are usually used. These systems are based on the construction of products from simpler blocks. In particular, this technology is used in popular system **AutoCAD**, that is the professional universal system for projection automation of the company **Autodesk**. Such a synthesis of complex products allows us to raise the design works efficiency substantially. This also relates to the projecting of thin-walled constructions which are produced by means of plastics moulding under pressure. The curvilinear shell can be considered in a view of elementary components capacity of thin-walled constructions.

Mathematical simulation of physical - chemical processes which take place in the cavity during its filling by melted polymer can be realized in component constructions on the basis of decomposition of the computed domain to simple subdomains [4]. In these methods complex original computed domain is decomposed to simple subdomains, that can be not-overlapping one another (constituent bodies). Such an approach is naturally connected with the automated projection of complex products on the basis of simple blocks.

The subdomains themselves are sufficiently complex. In our case of casting of thin-walled constructions we can take curvilinear shells with arbitrary boundaries, inside cuts and other peculiarities in the capacity of subdomains.

References

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