Journal of Theoretical and Applied Mechanics
Sofia, 1992, Year XXIII, No 2

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Influence of Surface Tension Minimum on Thermocapillary Flow in Thin Liquid Layer

1. Introduction

Surface tension-driven problems are among the most interesting ones not only due to their intrinsic academic value but also because of their technological importance. Capillary convection plays a significant role in technology today, especially when it is the only cause of motion, in the absence of gravity forces.

Imposed thermal gradients at liquid-gas interface induce surface movements which generate flows in the adjoining phases. The surface tension σ at a liquid/gas interface is generally a monotoneously decreasing function of temperature. This is a good model for many liquids but is not appropriate for describing surface tension effects in fatty alcohol and in some particular ionic, non volatile surfactants. V o c h t e n e t a l. [1] notice that a better model is provided by a parabolic function of the form

$$\sigma = \sigma_M + \frac{b}{2}(T-T_M)^2,$$

where σ_M is the surface tension minimum at a given temperature T_M corresponding to the minimum of the function $\sigma(T)$. Here, b stands for

$$b=(\partial^2\sigma/\partial T^2)>0,$$

and is assumed to be a positive quantity. This minimum is about 40°C, b is of the order of 10^{-6} N/m°C^2 while σ_M takes values ranging from 3.10^{-2} to 7.10^{-2} N/m if a water-n-heptanol 5.10^{-3} mol/l solution is taken into consideration. Extrema in surface tension also exist at higher temperatures in liquid alloys such as Ag-Pb and Al-Sn.

Evidently, the Marangoni number (proportional to $\partial\sigma/\partial T$) is very small or even vanishes in the neighbourhood of T_M. Thus when the motor of convection is the thermocapillary effect alone, it is expected that at temperatures arround the minimum, convection pattern will be deeply modified and surface velocities could be slowed. That kind of system is extensively studied by V o c h t e n & P e t r e [1], V i l l e r s & P l a t e n [2, 5, 7], L e g r o e t a l. [3, 4]. The anomalous thermocapillary effect is also discussed in [6, 8, 9].

2. Formulation of the problem

Consider an infinite horizontal layer of thickness d, placed at a rigid wall at z=0 and opened to the ambient gas, and subjected to a heating from above. The liquid is assumed to be viscous and with constant physical properties exept for the density ρ and the surface tension σ which depend on the temperature. The former is the linear function of the temperature and the latter is a quadratic one. Moreover, the deviations from the constant density are only included into the buoyancy term in the equations of motion, e. g. the equation are used in the Boussinesq approximation. Besides, the energy dissipation is not taken into account. Restricting ourselves to the case when the Fourier law of heat conduction is taken for granted. Free liquid surface is deformable and it is necessary to be determined.

Under these assumptions, the governing non-steady equations are the equations of mass, momentum, and energy diffusion in the form:

$$\nabla \cdot V = 0,$$

(1) $$\frac{\partial V}{\partial t} + (V \cdot \nabla) V = -\frac{1}{\rho} \nabla P + \upsilon \nabla^2 V,$$

$$\frac{\partial T}{\partial t} + (V \cdot \nabla T) = \chi \nabla^2 T,$$

where V(U,V,W) is the fluid velocity , P – the presure, T – the temperature, χ – the thermal diffusivity, ∇ – the gradient operator, ∇^2 – the Laplacian. Because of the axisymmetric behaviour of the heater the cylindrical coordinate system (R, φ, Z) is applied and it is not dependendent on the φ coordinate.

It is assumed that the liquid layer is initially at rest with temperature T_0 and heat flux Q(R,t) to be imposed instantaneously at $t=0^+$. Let us also suppose the motion of the gas above to be negligible and its temperature to be given by $\tilde{T}(R,t)$. So, the initial and the boundary conditions are:

(2) $U=0,\ W=0,\ T=T_0,\ H=d$ at $t=0$

(3) $$U=0,\ W=0,\ T=T_0 \qquad \text{at } Z=0,\ t>0$$

(4) $$\mu(\tilde{U}_N+\tilde{W}_S)=\sigma_S,\ P=P_0+2\mu W_N-\sigma/K,\ -\lambda\frac{\partial T}{\partial Z}=\kappa(T-\tilde{T})-Q \quad \text{at } Z=H,\ t>0$$

$$-U\frac{\partial H}{\partial R}+W=\frac{\partial H}{\partial t}$$

where $Z=H(t,R)$ defines the free liquid surface, (S,N) is the coordinate system located at this surface, $\tilde{U}$ and $\tilde{W}$ are the velocity components in the S and N directions, respectively, P_0 is the atmospheric pressure, and K is the surface curvatire radius, λ is the thermal conductivity, κ is the heat transfer coefficient. The temperature boundary conditions express that the lower boundary is perfect heat conductor and the upper one is conductivelly heated. The very last condition is the kinematic one.

The coordinates R, and Z, the velocity components U, and V, the time, the pressure and the temperature are related to L, d, U, U.d/L, ϑ, $[P-P_0-\rho g(H-Z)]/(\mu UL/d^2)$, ΔT. The same notations as in the dimensional case but in small letters are reserved for the nondimensional variables, only the nondimensional temperature is denoted by θ and nondimensional time by - τ.

The following dimensionless parameters appear in the equations and the boundary conditions : the Reynolds number $Re=UL/\upsilon$, the Prandtl number $Pr=\upsilon/\chi$, the Capillary number $Ca=\sigma[b(\Delta T)^2]^{-1}$, the Grashof number $Gr=g\beta\Delta Td^3/\upsilon^2$, the modified Bond number $Bo=\rho gd^2/b(\Delta T)^2$, the dynamic Bond number $Bd=Gr/Re$ and the Biot number $Bi=d\kappa/\lambda$. The characteristic velocity U is obtained from tangential force balance at the free surface and is equal to $\varepsilon b(\Delta T)^2/\mu$. Considering a thin layer ($\varepsilon^2=d^2/L^2 \ll 1$) it is assumed that ε^2Re, $\varepsilon^2Re.Pr$ and ε^2Ca tend to zero with ε^2. These conditions typically are satisfied by a layer of silicon with d=0.5 cm, and $\Delta T=5^oC$. Under these restrictions the governing equation and the boundary conditions become:

(5) $$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\partial w}{\partial z}=0,\quad \frac{d^2}{\upsilon\vartheta}\frac{\partial u}{\partial\tau}=-\frac{\partial^2 u}{\partial z^2}-\frac{\partial p}{\partial r}-Bo\frac{\partial h}{\partial r},$$

$$-\frac{\partial p}{\partial z} = Bd\,\theta\,, \quad Pr\,\frac{d^2}{\upsilon\vartheta}\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial z^2}\,.$$

(6) $u=0,\ w=0,\ \theta=0,\ h=1$ at $\tau=0$

(7) $u=0,\ w=0,\ \theta=0$ at $z=0,\ \tau>0$

(8) $\frac{\partial u}{\partial z} = (\theta-\theta_M)\frac{\partial\theta}{\partial r}\,,\quad p=0,\quad \frac{\partial\theta}{\partial z} = -Bi(\theta-\tilde{\theta})+q$ at $z=h,\ \tau>0.$

The continuity equation can be combined with the kinematic surface boundary condition and the no-slip condition at $z=0$ to give

(9)
$$\left(\frac{L}{U\,\theta}\right) h_\tau + \frac{1}{r}\frac{\partial}{\partial r}\left(\int_0^h u\,dz\right) = 0.$$

On account of governing equations and boundary conditions one can conclude that three time scale exist. The first one $\vartheta_1=d^2/\upsilon$ is connected with flow development, the second one $\vartheta_2=L/U$ with free surface deformability and the last one $\vartheta_3=d^2.Pr/\upsilon$ with the temperature field.

3. Solution of the problem

In this report let us pay attention only to the investigation of the free surface. It remains flat in limit of the small time scale ϑ_1 owing to be initially flat and equation (9) to be restricted to $h_\tau=0$. In frame of the large time scale ϑ_2 the free surface is obtained from solving of equation

(10)
$$\frac{\partial h}{\partial\tau} = \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{Bo}{3}\left[1-\frac{29}{40}\frac{Bd}{Bo}\theta^*\right]rh^3\frac{\partial h}{\partial r} - \frac{1}{2}rh\frac{\partial\theta}{\partial r}\left[(\theta^*-\theta_M)-\frac{11}{60}Bdh^2\right]\right]$$

with

(11) $\frac{\partial h}{\partial r}=0$ at $r=0$, $h=1$ at $r\Rightarrow\infty$.

In the limit $\tau \Rightarrow 0$, i. e. $\tau = O(\varepsilon^2 Re)$, the surface is flat. So, the initial condition is

(12) $$h=1 \text{ at } \tau=0.$$

Here with θ^* is denoted the temperature, obtained from (5-8) at the free liquid surface. The temperature θ_* depends on h only as a parameter. A predictor-corrector technique is used to solve (10-12) in transformed, in respect to r and τ, domein [0,1] using

(13) $$r^* = r/(1+r), \ \tau^* = \tau/(1+\tau).$$

4. Results

Numerical results demonstrated in this report are obtained for a Gaussian surface temperature distribution

(13) $$\theta^* = \exp(-Cr^2).$$

The numerous computations have been performed for varios values of the constant C, the minimum temperature T_M, modified Bond number and dynamic Bond number.

The nontrivial steady-state solution of (10-12) is obtained in an explicit analytic form. Here, for the sake of brevity, only the function h, provided the buoyancy forces are excluded is given by

(14) $$h^2 = 1 + \frac{1.5}{Bo}\left[(\theta^* - 2\theta_M)\theta^*\right].$$

When the surface tension σ as a monotically decreasing function of temperature is taken into consideration, the free liquid surface is described by (see [10],[11])

(15) $$\tilde{h}^2 = 1 - \frac{3}{\tilde{Bo}}\theta^*,$$

where $\tilde{B}o = \rho g d^2 / [-\partial\sigma/\partial T(\Delta T)]$.

Decreasing of the modified Bond number leads to an increase of the thermocapillary effects. This property is presented in Fig. 1, where two different values of Bo are considered. The curves, labelled by *1* and *3* are for Bo=5 and *2* and *4* for Bo=10. Here the curves *1* and *2* are determined from (15) and the curves *3* and *4* - from (14) at θ_M equals to 0.5.

The influence of θ_M on the behaviour of the free liquid surface is given in Fig. 2. In this figure the curve, obtained from (15) is labelled by *1*, and curves, determined from (14) and θ_M equals to 0.9, 0.5 and 0.1 by *2*, *3* and *4* respectively. For all presented curves the modified Bond number is equal to 5.

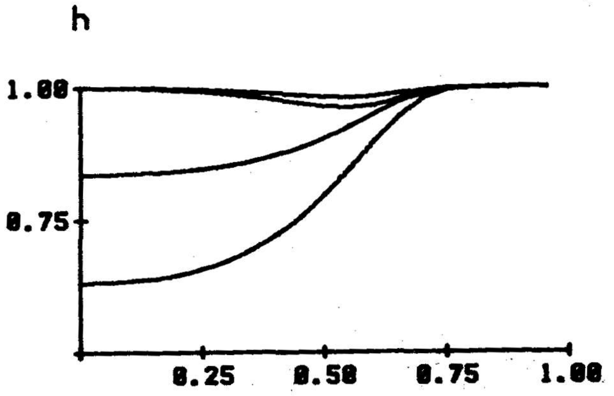

Fig.1. Free liquid surface in dependence on the Bo number.

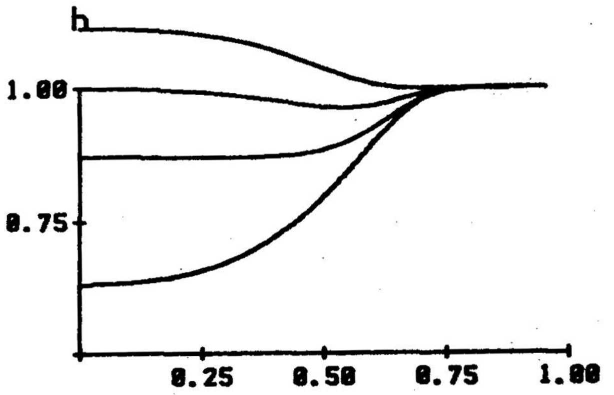

Fig.2. Free liquid surface in dependence on θ_M.

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