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Hydrodynamical Aspects of Melt Spinning

Continuum mechanics provides a sound basis for modelling technological processes. In their turn technologies create new fields of continuum mechanics, some of them being of true fundamental significance.

The objective of this paper is to discuss some basic aspects of melt spinning as a mechanical phenomenon. The discussion will be focussed on the possible elastic effects, their manifestations and theoretical description.

1. Consider isothermal fibre spinning of a liquid polymer melt which issues from a converging nozzle ('die') and after extension is taken up in a spinline by a take-up roll (Fig.1). Basic mechanics of spinning is governed by two well-known conservation laws for spinline, namely continuity and momentum equations,

$$f,_t + (fu),_x = 0 \tag{1.1}$$

$$\rho[(uf),_t + (fu^2),_x] = (\sigma f),_x \tag{1.2}$$

Here f(x,t) and u(x,t) are basic kinematic variables, the fluid filament cross-sectional area and axial velocity, σ is axial tensile stress. The drag force and surface tension contributions are neglected in the r.h.s. of momentum equation (1.2). Equations (1.1) and (1.2) are based on the assumption that the spinline is a rather thin liquid filament or jet of a slowly varying cross-section and strain rate **D** and stress **S** tensors are nearly diagonal,

$$\mathbf{D} = \mathrm{diag}[u,_x;\ -1/2u,_x;\ -1/2u,_x],\quad \mathbf{S} = \mathrm{diag}[\sigma,\ -p,\ -p] \tag{1.3}$$

Expressions (1.3) account for fluid incompressibility; the term p is lateral pressure at the surface of the jet which vanishes for the spinline.

Conservation laws (1.1)-(1.2) need to be implemented by a rheological constitutive equation which relates kinematic and dynamic variables. Having in mind the analysis of elastoviscous effects we assume the following simple constitutive equations of an elasto-viscous incompressible fluid. First, (finite) elastic strain tensor $\mathbf{B}^e$ is introduced which defines stresses in the fluid uniquely (up to a hydrostatic pressure),

$$\mathbf{S} = \mathbf{T} - p\delta \qquad (1.4)$$

$\mathbf{T}$ being the extra-stress tensor, an isotropic function of the elastic strain tensor $\mathbf{B}^e$. In particular, the neo-Hookean stress-strain relation will be used,

$$\mathbf{T}=2G\mathbf{B}^e,\ G=\mathrm{const} \qquad (1.5)$$

For spinline flow due to the obvious symmetries, elastic strain tensor $\mathbf{B}^e$ is diagonal with diagonal components (λ^2, $1/\lambda$, $1/\lambda$). Incompressibility of elastic strains is assumed. In elastic fluids the stresses may relax. A simple relaxation law is used which for anelongational flow in jet or spinline is expressed as

$$\lambda^{-1}d\lambda/dt=\lambda^{-1}(\lambda,_t+u\lambda,_x)=u,_x - GF(\lambda)/3\eta \qquad (1.6)$$

Here $F(\lambda)=2(\lambda^2-1/\lambda)$, η is fluid viscosity, $\eta=G\theta$, θ is fluid relaxation time. While θ, G and η may depend on strain and /or strain rate here they are taken to be constant. For simple case (1.5) relaxation equation (1.6) becomes

$$\lambda^{-1}(\lambda,_t+u\lambda,_x)=u,_x -2(\lambda^{-2}- 1/\lambda)/\theta, \qquad (1.7)$$

The rheology assumed corresponds to a variety of Maxwell fluid models; it is closely related to the model advanced by Leonov, but differs from it in the minute details. With the rheology assumed, the spinline flow is described by the following set of three first-order partial differential equations

$$f,_t+uf,_x+fu,_x=0$$

$$\rho fu,_t+\rho fuu,_x - GF(\lambda)f,_x - GfF'(\lambda)\lambda,_x=0 \qquad (1.8)$$

$$\lambda,_t-\lambda u,_x+u\lambda,_x=-\lambda F/(3\theta)$$

It is a hyperbolic system with three characteristic velocities $\xi_1=v$; $\xi_2= v - c$; $\xi_3= v + c$;

$$c=[(G/\rho)(\lambda F'(\lambda)-F)]^{1/2} \qquad (1.9)$$

If one assumes that elastic strain λ in spinline of initial cross-section area of f_o is momentarily augmented by a small increment of $\delta\lambda$, then $c=(P'(\lambda)/\rho f_o)^{1/2}=(P'(\dot{\lambda})/m_o)^{1/2}$, P being the total axial elastic force in the spinline, m_o is the mass per unit length of the spinline in the initial state. Velocity c is an analogue of sound velocity in a moving compressible fluid: so,it is referred further as 'sound velocity'. However, it has nothing to do with compressibility, being just the velocity of propagation of small elastic deformation perturbations relative to fluid. Its order of magnitude estimate is $(G/\rho)^{1/2}$, so it may prove to be as low as 0.1 - 10 m/s for polymeric fluids. It implies that both subsonic ($u<c$) and supersonic ($u>c$) flows may occur. This fact is of a key importance. Indeed, for supersonic flow all three characteristic velocities are positive, $\xi_i>0$, $i=1, 2, 3$; hence for all perturbations, all the information is transported just downstream from the die to the take-up roll. In the subsonic regime two characteristic velocities are positive, while the third one is negative. In this case both upstream and downstream propagation of perturbations is possible. Obviously the spinning is possible only if a subsonic regime persists over the total spinline or at least in the domain adjacent to the take-up end, so that a pulling force is exerted on the spinline by the take-up roll. Next point is that three boundary conditions must be prescribed at the ends of the spinline, one at the take-up roll and two at the die exit. The usual condition at the take-up roll consists in prescribing the take-up velocity,

$$u=u_L, \qquad x=L \tag{1.10}$$

To prescribe boundary conditions at the die exit ($x=0$) one must account for the flow in the die itself. Usually it is a converging flow from a large reservoir through a short die of length $l \ll L$, and its description must be two-dimensional. However, an over simplified 1D description will be used here to account for the gross elastic effect. Assuming the flow uniform across the die section and the drag negligible, one finds for the flow in the die the conservation laws

$$\rho f u = q(t)$$
$$(\rho f u),_t + (\rho f u^2),_x = (\sigma f),_x + p f,_x , \qquad -l \le x \le 0, \tag{1.11}$$

complemented by rheological equations (1.4), (1.5), (1.7).

In these equations of die flow $f=f(x)$ is a prescribed die profile (shape), while $u(x,t)$, $p(x,t)$, $\lambda(x,t)$, $\sigma(\lambda)$ are to be determined. Let first the total flowrate q be a known constant $q=q_0$. Then $u(x,t)$ is calculated immediately while Eqs. (1.11) and (1.4), (1.5), (1.7) serve to find $p(x,t)$, $\lambda(x,t)$ using the respective boundary conditions. Assuming that at the die entrance $x=-l$ the fluid is in the stress-free state, one of the boundary conditions is evident,

$$\lambda(-l, t)=\lambda_0=1 \tag{1.12}$$

The second boundary condition serves to fix pressure at a point of the die. It seems natural to put exit pressure equal to the ambient one, or $p(0,t)=0$. It appears however valid only for a supersonic flow if exit velocity exceeds local "sound" velocity c in the jet, $u>c$ ($x=0$). For the subsonic flow the pressure boundary condition appears to be more intricate.

Indeed, the die exit $x=0$ is a point of change of the type of flow, namely, the tube flow in the die turns into a jet flow in the spinline. This transition involves as well an alteration of the set of dependent variables, so $x=0$ is a priopi a discontinuity boundary at which all variables may undergo a discontinuous change. However matching conditions must be fulfilled.

$$[fu]=0; \ [\rho fu^2-\sigma f]=0; \ [\lambda f]=0 \tag{1.13}$$

Two of them are mass- and momentum conservation laws, the third condition expresses the characteristic property of Maxwellian elastic fluid, namely that instantaneous deformation with jumplike transition is of pure elastic nature and does not involve any relaxation. Here brackets denote a change of the respective quantities across the jump, $[y] = y^+ - y^- = y(+0) - y(-0)$.

In the supersonic regime no restrictions are imposed on the 'plus'-variables as no information is transformed upflow from the spinline region. So matching conditions (1.13) allow for a trivial solution: $f^+=f^-$; $u^+=u^-$; $\sigma^+=\sigma^-$.

These equations imply as well $p=p^+(=0)$. For a subsonic flow the information transported from the distant end of the spinline imposes an additional restriction on the 'plus'-variables, so that relationships (1.13) are not valid anymore. An analysis shows that the exit pressure in the die p^- is positive, $p^->0$ and it can not be calculated independently but must be found in the course of the solution of the complete conjugated problem. The accompanying cross-sectional area jump at the die exit $f^- < f^+$ is a manifestation of an elastic recoil and is interpreted as a die swell. The complete problem of the steady-state spinning of elastic fluid is solved numerically. One of the important features of the solutions is displayed by a 'discharge mastercurve', i.e. a relationship between flowrate q and entrance pressure P. It appears to have a characteristic non-monotonic shape (Fig.2). The sharp kink corresponding to the minimum entrance pressure marks transition from subsonic (left part) to supersonic (right part) flow in the die. The decreasing part of the master curve which is somewhat unexpected consequence of the mathematical statement of the problem may serve as a source of the self-excited oscillations of the flowrate if the working regime corresponds to a point of the decreasing part of the mastercurve. The respective analysis is reported elsewhere both for free jet flow and spinline flow. The analysis uses a description of the feeding (or pumping) system as a second-order dynamical system having a kind of inertia and a storing element. The instability range has been predicted using linear small-amplitude analysis and some transient regimes and development of self-sustained oscillations has been followed numerically. Some typical results are shown at Fig.3. They correspond respectively to small and large supercriticality or to weak and strong instability.

The weak instability manifests itself in the small-amplitude near the harmonic oscillations of the flowrate in the subsonic range (and consequently to the periodic variations of the diameter of the fiber formed). The strong instability leads to large amplitude relaxation oscillations which include temporal excursions into supersonic regimes. For free jet flows this corresponds to anintermittent jet flow. For a spinning process it may lead to total breakdown of the spinning. It may be conjectured as a guess that this type of instability corresponds to the phenomenon of the melt fracture. However this correspondence remains to be an open problem.

As another important conclusion of the formal analysis of the developed mathematical model it is worthwhile to mention that besides pure subsonic spinning regimes (Fig.4a) and purely supersonic jet flow regimes (Fig.4b) mixed regimes may occur when upstream supersonic flow coexists with downstream subsonic flow. The two flow regions are matched by means of an unloading shock wave (Fig.4c). This type of flow may occur both in free jet and in spinline; it may be put into correspondence with the phenomenon of the delayed die swell discussed by Joseph et al [3].

In analysis of this type of flow, stability of the shock-wave plays an important role. For a discontinuity to be stable an extra family of characteristics must arrive to it. It means that the upstream flow is supersonic, $v^- > c^-$ while the downstream flow is subsonic, $v^+ < c^+$. Thus for the neo-Hookean elastic strain-stress relationship (1.5) occurs only in the unloading shock wave with $\lambda^+ < \lambda^-$, $f^+ > f^-$ as in Fig.4c

However with other elastic laws, quite an opposite behaviour is possible, namely supersonic-subsonic transition may occur in a loading shock-wave with $\lambda^+ > \lambda^-$, $f^+ < f^-$ (while the upstream flow is supersonic and the downstream flow is the subsonic one). In this case the spinline profile would be of the type shown in Fig.5. It resembles the spinline profiles which are typical for the necking in high-speed melt spinning [4-6]. Whether this resemblance is just coincidental or it has good physical reason remains to be investigated.

References

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Fig.1 Melt spinning defining sketch

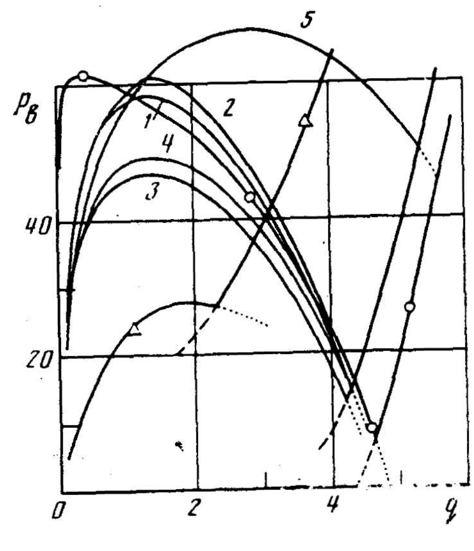

Fig.2 Discharge mastercurve for a convergent die of length l for different values of spinline length L, take-up velocity u_L and relaxation time θ (circle - 10^3, no mark - 10^2, triangle - 10)

L/l \ u_L	0.5	1.5	2.4
10^2	2	1	
10	5	4	3

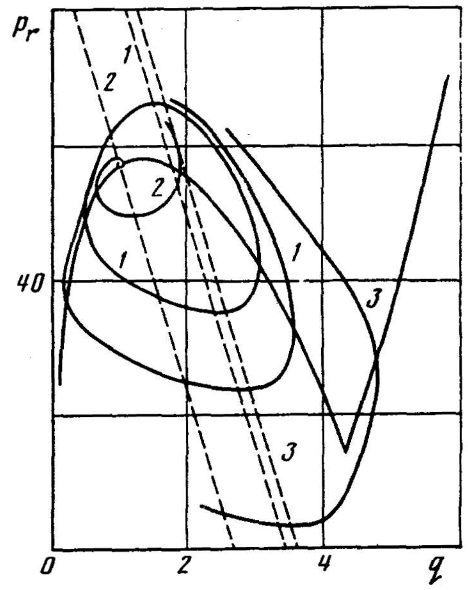

Fig.3 Transient regimes of spinning.
1 - weak supercriticality . Limit cycle of smallamplitude.
2 - subcritical case. Damping oscillations.
3 - strong supercriticality. Spinning breakdown.

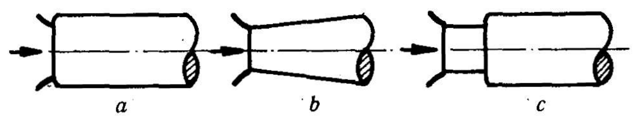

Fig.4 Sub- and supersonic regimes of flow at die exit.

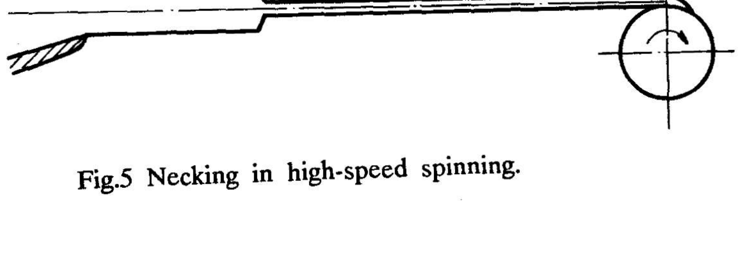

Fig.5 Necking in high-speed spinning.