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M. Grois,
L. Dimitrov

Redistribution of the Power Input in the Gearbox with General Branching of Flow

A very interesting task has occurred in the practice of the "slide rule boys". It belongs to the branch of classical mechanics and it has been applied in the course of designing a rather complex drive. There is no need to discuss the importance and the demanding character of the task — it is concerned with the design of the distribution of the flow of power input in the auxiliary power unit, designed for operation on board of a turbo-prop aeroplane. The unit is fitted with its own internal combustion turbine, used for starting the engines, for air-conditioning of the cabins and actuating a number of sets, but first and foremost it is used for generating power of accurate frequency, needed for high-quality radio communication. In general it is an independent and economic power source for use on the ground and also during flight.

The structure of the distribution is better visible from an axonometric representation (Fig. 1). It is obvious that the flow of the power input is branched off with the help of pinion *1* at the very beginning, into two unequal currents, connected only partially on the shaft of electric generator **EG**, through wheel *4*. Part of the power input from both branches of different rigidity is transferred by pinion *3'* to intermediate wheel *5*, where the power input flow branches off again:

— both through the gearing of intermediate wheel *9* to pinion *10* and through shaft **ES**, to the starter of the main engines, through the electric clutches;

— and through shaft 56 and wheels *7* and *8* to the shaft of hydrogenerators **HG**, respectively to the shaft of oil pump **OP**.

We can conclude that it is a gearing whit kinetically, generally and specially branched power input flow. Of special conception is the branching of the flow on wheel *3'* in gear meshing at the same time with wheels *4* and *5*. It is obvious that part of the power input flow to the electric generator will be distributed evenly by both branches. The remaining part of the power input flow from pinion *1* to toothed wheel *5* should be divided non-subjectively. It is so because during transfer through the upper secondary branch appears the bending compliances of the teeth of gearings *3''4''* and *43'*, and the losses due to the friction between them. At the same time we should not overlook the torsional compliance of shafts *2'3'* and *2''3''*.

The distribution of the power input flow on wheel *3'* to wheels *4* and *5* is a statistically one-time indeterminate. It can be solved in the classical way. The solution has been divided into the analysis of the equilibrium of a selected subgroup and to the analysis of the equilibrium of the individial members of the subgroup. The momentary equilibrium in the section between pinion *1* and wheel *5* can be described around the axis of the rotation of pinion *3'* in the following way:

$$\Sigma M_{x3'} = M_{2'} + M_4' - M_5 = 0,$$

where

$$M_5 = (M_6 + M_9 u_{59} \eta_{59}) \eta_{3'5},$$

resp.

$$M_4 = M_{3'} u_{34} \eta_{3'4},$$

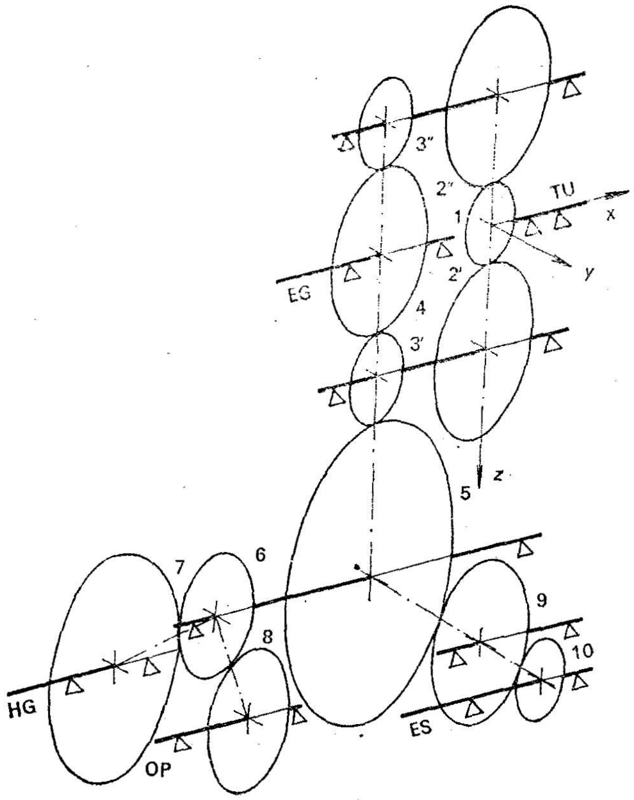
where

$$M_{3'} = M_1' u_{12} \eta_{12'}$$

and

$$M_{2'} = M_1' u_{12} \eta_{12}.$$

The equilibrium of the forces acting on pinion *3'* will be reached through the reaction of its housing.



The following geometrical conditions of the deformation of teed and shafts of the subgroup can be more distinct after the imaginary immobility of the disk of the pinion rim *3'*. Anyhow, it holds that

$$\varphi'_{13'} = \varphi''_{13'},$$

where

$\varphi'_{13'}$ — is the angle of turning pinion *1* against pinion *3'* due to internal static effects of the external load of bottom branch *12'3'*,

$\varphi''_{13'}$ — ditto for upper branch *12''3''43'*.

Let the specific bend compliance of one pair of teeth c^{-1} be given by the proportion of microshift *s*, measured along pitch circle, and with mean specific load of the teeth *w* in the following way

$$c^{-1} = s/w \quad (\mu m \, . \, mm \, . \, N^{-1} = 10^{-3} \; mm.^2 . \, N^{-1}).$$

In fact we have in mind a microshift composed of the effect of the bending and contact stress of the teeth.

To the analytic expression of the mean specific rigidity of teeth of gearing we should calculate the total gear mesh coefficient, the virtual number the teeth and height shift of the basic profile. The microshifts between pitch circles can be expressed with relation

$$s = F/b_w c_\gamma = M/0.5 d b_w c_\gamma \quad (10^{-3} \, mm),$$

where F — circumferential force,

M — moment of force couple,

b_w — the used tooth width,

d_w — diameter of pitch circle.

The earlier indicated deformation condition should be put in the following way

$$\varphi'_{13'} = \varphi_{12'} + \varphi_{2'3'} = \left| \begin{array}{l} \varphi_{12'} = s_{12}/0.5 d_1 = M'_1/0.25 b_{w12} c_{\gamma 12} d_1^2 \\ \varphi_{2'3'} = M'_2 \, l_{23}/G J_{p23} \left| \begin{array}{l} M'_2 = M'_1 u_{12} \eta_{12'} = M_3 \\ J_{p23} = 0.1(d_{23}^4 - d_{023}^4) \end{array} \right. \end{array} \right.$$

$$= M'_1 \{(0.25 b_{w12} c_{\gamma 12} d_1^2)^{-1} + [u_{12} \eta_{12'} l_{23}/0.1 G(d_{23}^4 - d_{023}^4)\} = M'_1/c'_{13'},$$

$$\begin{array}{ll} \varphi''_{13} = \varphi_{12'} + \varphi_{2''3''} + \varphi_{3''4} + \varphi''_{43'} & \left| \; \varphi_{12''} + \varphi_{2''3} = \varphi'_{13''} \quad \text{because} \quad \eta_{12''} \cong \eta_{12'} \right. \\ = \varphi'_{13'} \qquad\qquad\quad + \varphi_{3''4} + \varphi''_{43'} & \left| \; \varphi_{3''4} = M''_3/0.25 b_{w34} c_{\gamma 34} d_3^2 \right. \\ & \left| \; \varphi''_{43'} = M''_4/0.25 b_{w34} c_{\gamma 34} d_4^2 \right. \end{array}$$

$$= M''_1 \{c'_{13'} + (u_{12} \eta''_{12}/0.25 b_{w34} c_{\gamma 34} d_3^2) + (u_{14} \eta_{12} \eta_{34}/0.25 b_{w34} c_{\gamma 34} d_4^2)\} = M''_1/c''_{13'}.$$

Since

$$M'_1/c'_{13'} = M''_1/c''_{13'}, \; \therefore \; M''_1 = M'_1 \, c''_{13'}/c'_{13'}$$

and while

$$M'_1 + M''_1 = M_1$$

is

$$M_1'(1+c_{13'}''/c_{13'}')=M_1$$

$$M_1'=M_1c_{13'}'/(c_{13'}'+c_{13'}'')$$

and following the evaluation

$$M_1'=0.707M_1, \text{ when } c_{13'}'=44.25 \text{ mm. N, resp. } c_{13'}''=18.35 \text{ N. mm.}$$

The determined result can be expressed verbally as follows: "under the given elasto-mechanical conditions of the bottom branches *12'3'* 71 % of power input will flow to the hydrogenerator, oil pump and starter." The remaining 29 % will flow through the upper, in this sense secondary branch *12"3'43'*.

The above result has been determined on the basis of carefully evaluated mean specific rigidity of teeth [2], according to the following dimensionless geometrical indicators

$$c_\gamma=(\varepsilon_\alpha+\varepsilon_\beta)/(4.72_{-2}+1.56_{-1}z_{vp}^{-1}+2.58_{-1}z_{vk}^{-1}-6.35_{-3}x_p-1.93_{-1}x_k^+$$
$$-1.17_{-1}x_pz_{vp}^{-1}-2.42_{-1}x_kz_{vk}^{-1}+5.3_{-3}x_p^2+1.8_{-3}x_k^2)$$

where ε_α — coefficient of profile gearing,
ε_β — pace coefficient,
x — height shift of the profiie,
z_v — virtual number of teeth.

The final note on the above described calculating procedure says, that although the need of a similar non-subjective flow of the power input flow under the given conditions is a matter of fact, the authors of the paper have not come across the solution or a similar solution in the literature, with the exception of [3] — nevertheless, they admit that similar solutions may exist.

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