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Dynamics of a plane locomotion robot model in the nonresonance case for the walk phase. Initial conditions

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The dynamics of a plane antropomorphic robot model is considered in earlier papers [1, 2]. The next matrix differential equation is received there

(1) $$A \,.\, q^{\cdot\cdot} + C \,.\, q = \acute{e}.(F + p \,.\, M), \quad p = \frac{I_1}{I_{12}},$$

where

(2) $$q = |\, q_1,\ q_2,\ q_3,\ q_4 \,|^T$$

is the vector of the generalized coordinates,

(3) $$M = |M_i(t)\,|^T = \Big|H_{i0} + \sum_{m=1}^{\infty} H_{im} \,.\, \sin b_m \,.\, (ht + h_i)\,|^T$$

is the vector of the applied control drive moments in the connect ions for realising the desired motion and

(4) $$F(q,\ q^{\cdot}) = |A_{i1} \,.\, \sin \breve{r}_1 + A_{i2} \,.\, \cos \breve{r}_1 + A_{i2} \,.\, \cos 3 \,.\, \breve{r}_1\,|^T$$

is a function of the generalized coordinates and velocities. The elements of the matrices A and C and constants A_{ij}, i, $j = 1, 2$ in (4) depend on mass, inertia, geometric and elastic characteristic of the model, $\acute{e}$ is a little parameter [3].
Let the origine decision of (1) have the form

(5) $$q_0 = |q_{01},\ q_{02},\ q_{03},\ q_{04}\,|^T,\ q_{0i} = f_i^{(1)} \,.\, a_1 \,.\, \cos \breve{r}_1, \quad \breve{r}_1 = p_1 \,.\, t + \acute{y}_1.$$

The origine decision means the decision of (1) when $\acute{e} = 0$ and p_1 is the settled frequency which is the root of the characteristic equation of the system

(6) $$D(p_s^2) = -A \,.\, p_s^2 + C = 0, \quad s = 1,\ 2,\ 3,\ 4,$$

where p_s is any own frequency. The expressions $f_i^{(s)}$, $i = 1, 2, 3, 4$ are natural normal forms (own forms) for the frequency p_s, which are received as nontrivial decisions of

(7) $$(-A \,.\, p_s^2 + C) \,.\, f^{(s)} = 0,\ f^{(s)} = |f_i^{(s)}\,|^T,$$

and the chosen forms for the frequency p_1 are of a type

(8) $$f^{(1)}=|f_i^{(1)}|^T=|d_{4i}(p_1^2)|^T,$$

where $d_{4i}(p_1^2)$, $i=1,2,3,4$ are the associated matrices of the fourth row elements of the matrix (6) for the frequency p_1. The decision q differs little from the origine decision q_0, i. e. it is close with one of the normal oscilations of described mechanical system.

The object of the presented paper is receiving of the motion law of the robot model in first approximation to render an account of the initial conditions, defined in [2], which are necessary and sufficient for existing of the approximate periodical decisions of (1). It is supposed that the region of analiticity of the function F after q and $q^{\cdot}$ includes the region of analiticity of the origine decision q_0. Let the decision of (1) have a presentation

(9) $$q(t,\ n_1,\ n_2 é)=q_0(t)+R^{(1)}.n_1+R^{(2)}.n_2+R^{(3)}.é,$$

where the components of the vectors $R^{(1)}$ and $R^{(2)}$ are defined in [2] as time functions of a type

(10) $$R_i^{(1)}=d_{3i}(p_1^2).\cos p_1.t,\quad R_i^{(2)}=d_{4i}(p_1^2).\frac{\sin p_1.t}{p_1},$$

where n_1 and n_2 are the differences of the initial values of the vectors q and $q^{\cdot}$ from q_0 and $q_0^{\cdot}$ respectively [2, 3].

For components of the vector $R^{(3)}$ which are time functions too, it is supposed they are represented by rows of a type

(11) $$R_i^{(3)}=v_{i0}(p_1)+v_{i1}(p_1).\sin \breve{r}_1+v_{i2}(p_1).\cos \breve{r}_1+v_{i3}(p_1).\cos 3.\breve{r}_1$$

$$+\sum_{m=1}^{\infty}(W_{im}(p_1).\cos b_m.h.t+V_{im}(p_1).\sin b_m.h.t).$$

The expression (9) is put in (1) render an account of (11) and that the phase $\breve{r}_1$ and the amplitude a_1 must satisfy differential equations

(12) $$\frac{da_1}{dt}=é.A_1(p_1),\qquad \frac{d\breve{r}_1}{dt}=p_1+é.B_1(p_1).$$

Here A_1 and B_1 are asked functions and $\breve{r}$ is the phase when the settled phase is p_1

(13) $$\breve{r}_1=p_1.t+\acute{y}_1$$

and $\acute{y}_1$ is the phase difference for the same frequency.

The amplitudes of the harmonic functions in (11) v_{i0}, v_{i3}, V_{im} and W_{im} could be defined from the algebraic systems received after the replacing (11) in (1). Those decisions are represented with the expressions

(14) $$v_{i0}=\frac{H_0.d_i(0)}{I_{12}.D(0)};\quad H_0=|H_{i0}|^T;\quad d_i(0)=|d_{ij}(0)|^T;$$

$$v_{i3}=-\frac{d_i(9.p_1^2)A_2}{D(9.p_1^2)}.a_1^3;\quad d_i(9.p_1^2)=|d_{ij}(9.p_1^2)|^T;\quad A_2=|A_{j2}|^T,$$

$$V_{im}(p_1)=\frac{d_i(b_m^2.h^2).H_m\cos h_1}{I_{12}.D(b_m^2.h^2)};\quad W_{im}=\frac{d_i(b_m^2.h^2).H_m.\sin h_1}{I_{12}.D(b_m^2.h^2)};$$

$$d_i(b_m^2 . h^2) = |d_{ij}(b_m^2 . h^2)|^T; \quad H_m = |H_{jm}|^T; \quad i, j = 1, 2, 3, 4, \quad m = 1, 2, 3.$$

For the amplitudes $v_{i1}(p_1)$ and $v_{i2}(p_1)$ are received the systems

(15)
$$\sum_{r=1}^{4} a_i . f^{(r)} . c_r . (p_r^2 - p_1^2) = A_{i1} . a_1 + 2 . a_1 . A_1 . (v_r^2 - p_1^2),$$

$$\sum_{r=1}^{4} a_i . f^{(r)} . c'_r . (p_r^2 - p_1^2) = A_{i2} . a_1^3 + 2 . p_1 . a_1 . B_1 . (p_r^2 - p_1^2),$$

$$a_i = |a_{ij}|^T, \quad f^{(r)} = |d_{4j}(p_r^2)|^T, \quad i, j = 1, 2, 3, 4,$$

and $v_{i1}(p_1)$ and $v_{i2}(p_1)$ are supposed of a type

(16)
$$v_{i1}(p_1) = \sum_{r=1}^{4} c_r . f_i^{(r)}, \quad v_{i2}(p_1) = \sum_{r=1}^{4} c'_r . f_i^{(r)},$$

where $c = |c_r|^T$ and $c' = |c'_r|^T$ are vector-constants which have to be defined. For its definition each of the equations (15) is multiplied with $f_i^{(r)}$, $i = 1, 2, 3. 4$, where the index i is the number of the equation of (15). After, it is summed after i. The received equations are resolved for the components of c and c'. Taking an account of the condition for limited disturbed oscilations, the amplitudes v_{i1} and v_{i2} are

(17)
$$v_{i1}(p_1) = \sum_{r=2}^{4} f_i^{(r)} . \frac{a_1 . A'_1 . f^{(r)} + 2 . m_r . p_1 . A_1}{m_r . (p_r^2 - p_1^2)}, \quad A'_1 = |A_{i1}|^T,$$

$$v_{i2}(p_1) = \sum_{r=1}^{4} f_i^{(r)} . \frac{a_1^3 . A_2 . f^{(r)} + 2 . m_r . p_1 . a_1 . B_1}{m_r . (p_r^2 - p_1^2)},$$

and A_1 and B_1

(18)
$$A_1 = -\frac{A'_1 . f^{(1)}}{2 . m_1 . p_1} . a_1, \quad B_1 = -\frac{A_2 . f^{(1)}}{2 . m_1 . p_1}.$$

Now it is possible to define the system decision render an account of (18) for a_1 and $\breve{r}_1$

(19)
$$\frac{da_1}{dt} = -\frac{I_{12}}{I_1} . L . a_1, \quad \frac{d\breve{r}_1}{dt} = p_1 - \frac{I_{12}}{I_1} . S . a_1^2,$$

where

(20)
$$L = (2 . p_1 . m_1)^{-1} . A'_1 . f^{(1)}, \quad S = (2 . p_1 . m_1)^{-1} . A_2 . f^{(1)}.$$

Then the amplitude a_1 and the phase $\breve{r}_1$ as time functions are given with the expressions

(21)
$$a_1 = e^{1 - p.L.t}, \quad \breve{r}_1 = p_1 . t + \breve{y}_1 - \frac{1}{2} . Q\left(1 - e^{2.\left(1 - \frac{L I_{12} t}{I_1}\right)}\right),$$

where $Q = S . L^{-1}$ and for initial values of a_1 and $\hat{r}_1$ are taken 0 and $\breve{y}_1$ respectively. The generalized coordinates vector q render an account of the components of the vectors $R^{(k)}$, $k=1, 2, 3$, (10), (14), (15) and (16) in first row approximation could be represented by the expression

$$(22)\quad q=|q_i(t)|^T=|a_1 . f_i^{(1)} . \cos \breve{r}_1+\frac{H_0 . d_i(0)}{D(0)}+\sum_{m=1}^{\infty}\left(\frac{d_i(b_m^2 . h^2) . H_m}{D(b_m^2 . h^2)} . \sin h_1 . \cos b_m . h . t\right.$$

$$\left.+\frac{d_i(b_m^2 . h^2) . H_m . \cos h_1}{D(b_m^2 . h^2)} . \sin b_m . h . t\right)+\frac{I_{12}}{I_1} .$$

$$. \left(\frac{d_i(9 . p_1^2) . A_2 . a_1^3}{D(9 . p_1^2)} . \cos \breve{r}_1 . 3+\sin \breve{r}_1 . \sum_{r=2}^{4} f_i^{(r)} \frac{a_1 . A_1' f^{(r)}+2m_r . p_1 . A_1}{m_r . (p_r^2-p_1^2)}\right.$$

$$\left.+\cos \breve{r}_1 . \sum_{r=2}^{4} f_i^{(r)} \frac{a_1^3 . A_2 . f_i^{(r)}+2 . m_r . p_1 . a_1 . B_1}{m_r . (p_r^2-p_1^2)}\right)+d_{3i}(p_1^2) . \cos p_1 . t . n_{1i}$$

$$+d_{4i}(p_1^2) . \frac{\sin p_1 . t}{p_1} . n_{2i}|^T,$$

where

$$(23)\qquad m_r=\sum_{s=1}^{4} \sum_{j=1}^{4} a_{js} . f_j^{(r)} . f_s^{(r)}.$$

The received motion law (22) could be applied in the further investigations for example for definition of the dynamics reactions in the support leg, in the description of the motion of the connection point of the legs with the body and so on. The application of (22) for the computation procedure leads to congruent compute processes.

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