БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ
ТЕОРЕТИЧНА И ПРИЛОЖНА МЕХАНИКА
София, 1988, год. XIX, № 4

BULGARIAN ACADEMY OF SCIENCES
THEORETICAL AND APPLIED MECHANICS
Sofia, 1988, Year XIX, No 4

# The Influence of Nonlinear Mass Transfer on the Laminar Boundary Layer

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## 1. Introduction

The properties of laminar boundary layers depend considerably on the form of the velocity profile. One aspect of this is the possibility to control the boundary layer — that is to produce boundary layers of prescribed properties by generating appropriate velocity profiles.

One important feature of the laminar boundary layer is its stability in respect to separation. The more stable the layer is, the less is its friction coefficient. One way to stabilize a boundary layer is to suck it [1].

Another important property of a laminar boundary layer is its heat transfer coefficient towards the solid surface. Blowing into the layer results in a decrease of this coefficient which is particularly important when flying vehicles enter the dense layers of the atmosphere. To decrease the local flux in such cases one often applies ablation (sublimation from a solid surface).

The methods for control of the laminar boundary layers are characterized by the fact that the normal component of the velocity at the solid — gas (liquid) interface is not zero

$$v_n \neq 0 \tag{1}$$

and this can be realized by a number of techniques [1].

In some recent papers [2, 3] it was shown that in the presence of intensive mass transfer the high mass fluxes initiate secondary flows due to the relation between the normal velocity and the concentration gradient at the phase interface [3], which has the form

$$v_n = -\frac{MD}{\rho_0}\frac{\partial c}{\partial n}, \tag{2}$$

where $M$ [kg/kgmol] is the molecular mass of the diffusing species, $D$ [$m^2$/s] is its diffusivity, $n$ is the outward normal to the solid surface, $\rho_0$ [kg/$m^3$] is the mass gravity of the basic flow and $c$ [kgmol/$m^3$] is the concentration of the diffusing substance.

This effect is negligible in the limits of the linear theory of mass transfer but in the presence of intensive interphase transfer it might become substantial and is accounted for by the nonlinear theory [2].

The secondary flow described by Eq. (2) is similar to a Stephan flux induced flow but it differs from the latter because it can be initiated without a phase change. A secondary flow can result from blowing or sucking but it will differ from the case considered here because $v_n$ is not prescribed but it is obtainable depending on the rate of mass transfer and the laminar boundary layer flow, respectively.

As shown in [1] the normal velocity at the phase interface is a major controlling parameter. In the presence of intensive mass transfer it depends on the rate and the direction of the mass transfer. Purpose of this study is to investigate this relation as a preliminary step to boundary layer control through intensive mass transfer.

## 2. The Mathematical Model and Its Solution

Earlier [4] it was shown that nonlinear mass transfer in boundary layers can be described by following system of nonlinear partial differential equations:

(3a) $$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2};\quad x>0,\ y>0,$$

(3b) $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\quad x>0,\ y>0,$$

(3c) $$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2};\quad x>0,\ y>0,$$

(3d) $$u=u_0;\quad x=0,\ y>0,$$

(3e) $$c=c_0;\quad x=0,\ y>0,$$

(3f) $$u=0;\quad x>0,\ y=0,$$

(3g) $$c=c^*;\quad x>0,\ y=0,$$

(3h) $$v=-\frac{MD}{\rho_0}\frac{\partial c}{\partial y};\quad x>0,\ y=0,$$

(3i) $$u=u_0;\quad x>0,\ y=\infty,$$

(3j) $$c=c_0;\quad x>0,\ y=\infty,$$

where $x$ [m] and $y$ [m] are the Cartesien coordinates for the problem considered, the $x$-axis is directed downflow, the $y$-axis is perpendicular to the solid surface; $u$ [m/s] and $v$ [m/s] are the velocity components along $x$ and $y$, respectively; $\nu$ [m²/s] is the kinematic viscosity of the basic flow; subscript "0" denotes the initial value and superscript "*" — the equilibrium value.

Introducing the similarity variables

(4a) $$u=0.5\,u_0\varepsilon\Phi,$$

(4b) $$v=0.5\left(\frac{u_0\nu}{x}\right)^{0.5}(\eta\Phi'-\Phi),$$

(4c) $$c=c_0+(c^*-c_0)\Psi,$$

where $\eta$ is the new independent variable, $\Phi(\eta)$ and $\Psi(\eta)$ are the dependent ones, transforms the boundary value problem, Eqs. (3), into a two-point boundary value problem for a system of nonlinear ordinary differential equations, namely:

(5a) $$\Phi'''+\varepsilon^{-1}\Phi\Phi''=0,\quad \eta>0,$$

(5b) $$\Psi''+\varepsilon\Phi\Psi'=0,\quad \eta>0,$$

(5c) $$\Phi(0)=\theta\Psi'(0),\quad \eta=0,$$

(5d) $$\Phi'(0)=0,\quad \eta=0,$$

(5e) $$\Psi(0)=1,\ \eta=0,$$

(5f) $$\Phi'(\infty)=2\,\varepsilon^{-1},\ \eta=\infty,$$

(5g) $$\Psi(\infty)=0,\ \eta=0,$$

where "′" denotes differentiation in $\eta$ and

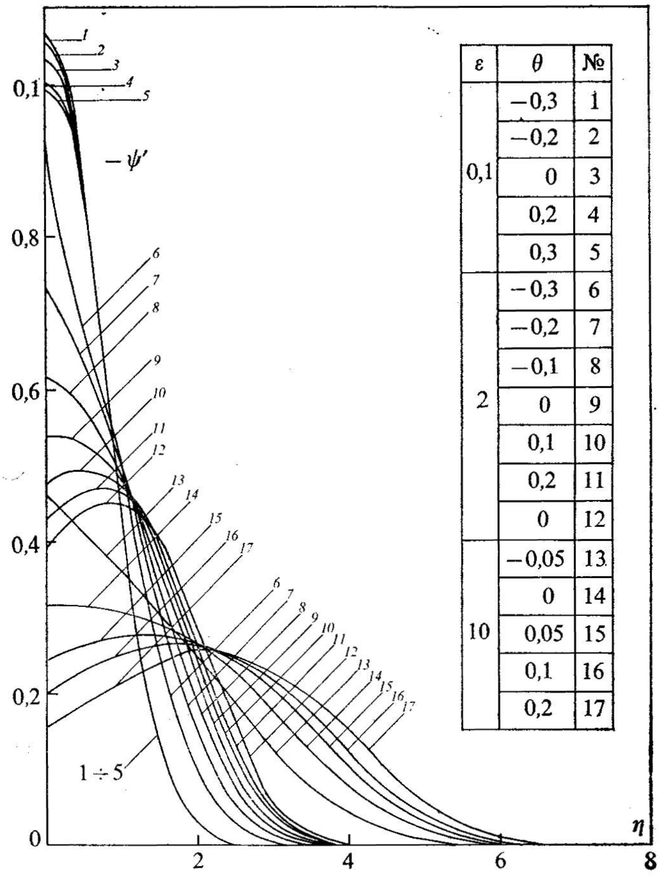


Fig. 1

(6a) $$\eta=y\left(\frac{u_0}{4Dx}\right)^{0.5},$$

(6b) $$\varepsilon=\mathrm{Sc}^{0.5},$$

(6c) $$\mathrm{Sc}=\frac{\nu}{D},$$

(6d) $$\theta=\frac{M(c^*-c_0)}{\rho_0\varepsilon}.$$

In Eq. (6c) Sc is the Schmidt number. From Eqs. (4) it follows that

(7) $$v_n=v(x,\,0)=-0.5\left(\frac{u_0\nu}{x}\right)^{0.5}\Phi(0),$$

where Φ(0) is obtainable from the integration of the boundary value problem, Eqs. (5).

Equations (5) were integrated numerically. A continuation shooting procedure [5] was utilized. Computed solutions were plotted on Figs 1-4, while the missing boundary conditions at $\eta = 0$ are listed in Table 1. The initial value problems which are the core

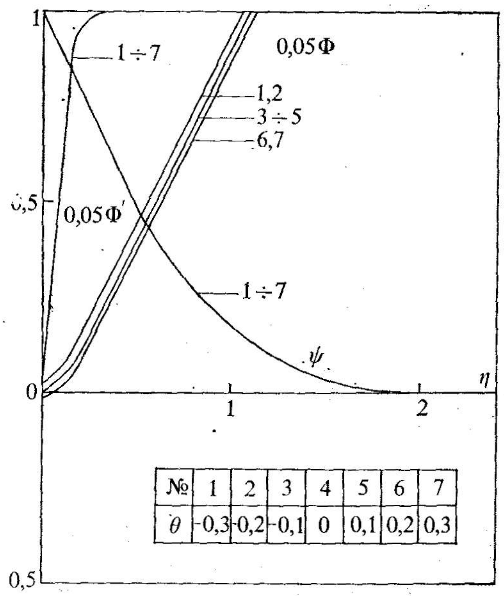


Fig. 2

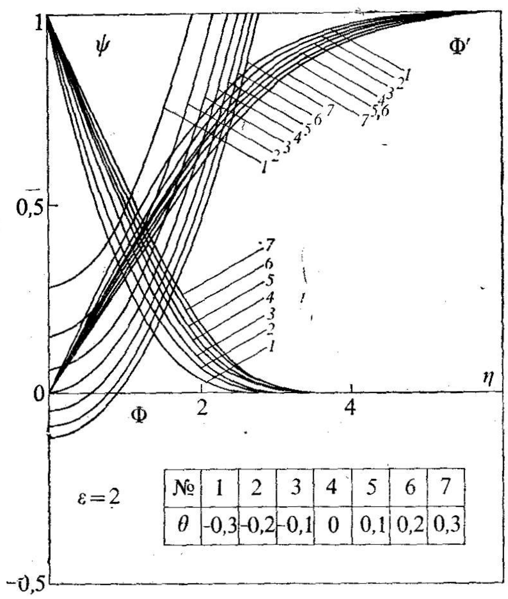


Fig. 3

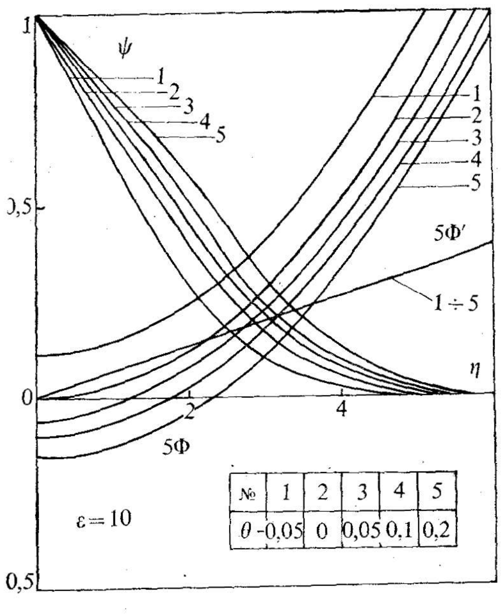


Fig. 4

Table 1

| θ | ε=0.1 | | ε=2.0 | | ε=10.0 | |
|---|---|---|---|---|---|---|
| | Φ''(0) | Ψ'(0) | Φ''(0) | Ψ'(0) | Φ''(0) | Ψ'(0) |
| 0.0 | 132.8 | —1.032 | 0.3321 | —0.5348 | 0.01328 | —0.3144 |
| 0.05 | 125.5 | —1.025 | 0.3230 | —0.5029 | 0.01309 | —0.2475 |
| —0.05 | 140.4 | —1.038 | 0.3424 | —0.5720 | 0.01359 | —0.4591 |
| 0.10 | 118.3 | —1.020 | 0.3150 | —0.4753 | 0.01298 | —0.2074 |
| —0.10 | 148.3 | —1.044 | 0.3545 | —0.6161 | | |
| 0.20 | 104.5 | —1.007 | 0.3013 | —0.5813 | 0.01281 | —0.1598 |
| —0.20 | 164.1 | —1.055 | 0.3863 | —0.7360 | | |
| 0.30 | 91.42 | —0.9947 | 0.2901 | —0.3928 | | |
| —0.30 | 180.8 | —1.066 | 0.4371 | —0.9374 | | |

of the numerical procedure [5] were integrated in double precision arithmetics on the ES 10-33 computer (machine epsilon — $10^{-15}$) with $10^{-5}$ accuracy. For large values of ε and θ the boundary value problem, Eqs (5), cannot be integrated numerically with the method and in the computing environment discussed in the above because either its solution becomes too stiff or the corresponding effects cannot be described adequately by the nonlinear theory of the diffusion boundary layer.

## 3. Conclusions

The results from the numerical experiments indicate that when the Schmidt number (ε) increases the effect of the concentration gradient (θ) decreases and for liquids ($\varepsilon > 10$) it can be observed in the presence of very high gradients only. This means that if the laminar boundary layer is controlled by means of mass consumption or generation at the solid phase boundary, the controllability of the layer decreases with the increase of ε.

From Table 1 it can be seen that when the layer is being "sucked" ($\theta < 0$) the friction coefficient [1] increases

$$c_{f_\infty} = \frac{2\nu}{u_0^2}\left(\frac{\partial u}{\partial y}\right)_{y=0} = K\Phi''(0), \tag{8}$$

where $K$ is proportionality constant. The layer becomes thinner thus increasing its stability — the critical value of the separation Reynolds number increases.

From Fig. 1 it can be observed that "blowing" into the layer ($\theta > 0$) decreases $\Psi'(0)$ which is a measure for the dimensionless rate of mass transfer [4]. The velocity distribution corresponding to this case would result to an analogical decrease of the heat transfer rate. This effect is similar to ablation, but in this particular case there is no phase change at the phase interface.

These results illustrate the possibility to control laminar boundary layers by nonlinear mass transfer effects.

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*Received 13. 05. 1987*