БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ
ТЕОРЕТИЧНА И ПРИЛОЖНА МЕХАНИКА
София, 1987, год. XVIII, № 4

BULGARIAN ACADEMY OF SCIENCES
THEORETICAL AND APPLIED MECHANICS
Sofia, 1987, Year XVIII, No 4

Numerical Experiments on the Process of Drop Formation in a Liquid Jets

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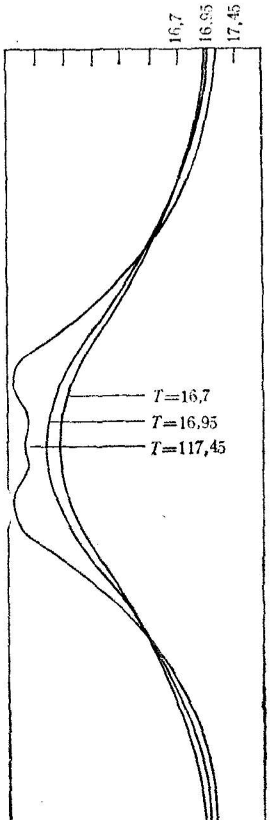
1. Introduction. There are lots of authors treating different aspects of the problem of the capillary jets instability. For description of the works in this domain the reader can be referred to McCarthy & Molloy (1974) and D. B. Bogy (1979) [1], [2]. Our work is an extension of the numerical investigation of Shokoohi (1976) [3], who studies the infinite-jet and finite-jet problems.



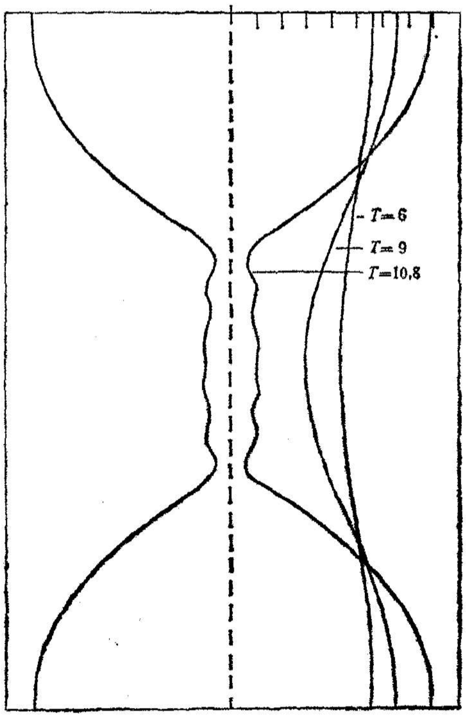


Fig. 2. $\lambda=8R_N$, $\varepsilon=0.02$, $\sigma=72.5$ erg/cm^2, $\mu=0.01$ g/cm . s

Fig. 1. $\lambda=6.6\ R_N$, $\varepsilon=0.02$, $\sigma=72.5$ erg/cm^2, $\mu=0.01$ g/cm . s

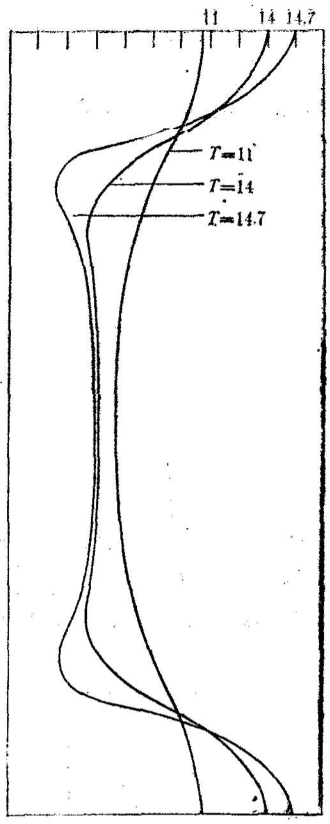


Fig. 3. $\lambda=16\,R_N$, $\varepsilon=0.02$, $\sigma=72.5$ erg/cm², $\mu=0.01$ g/cm . s

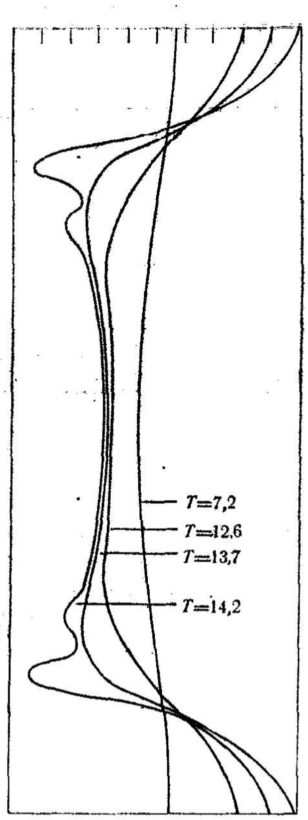


Fig. 4. $\lambda=16\,R_N$, $\varepsilon=0.02$, $\sigma=72.5$ erg/cm², $\mu=0.0001$ g/cm . s

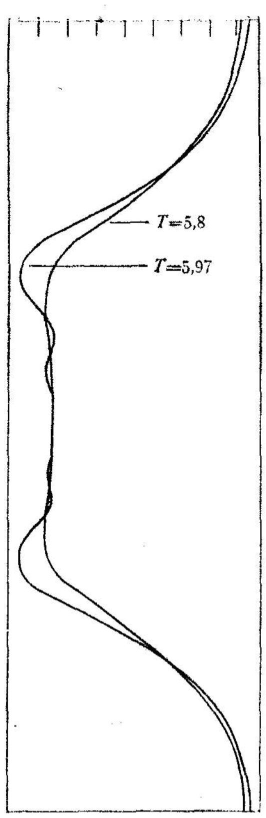


Fig. 5. $\lambda=9.016\,R_N$, $\varepsilon=0.02$, $\sigma=72.5$ erg/cm², $\mu=0.01$ g/cm . s

In the present paper the numerical results of the infinite-jet problem are shown. The investigation of the process of the desintegration is made under the assumptions: 1) The jet is initially an infinite cylinder of viscous incompressible fluid with radius R_N. 2) The jet has a circular cross-section and therefore is axially symmetric. 3) The gravitational effect and that of the ambient medium are neglected. 4) The jet is rotation free. 5) The jet is assumed to be infinite and therefore the results are valid at a considerable distance from the nozzle.

2. Formulation of the problem. The axis of the jet is taken as direction of the z-coordinate. The governing equations are the Navier-Stokes's equations in formulation $\psi-\omega$ (ψ — stream-function, ω — vorticity) in cylindrical coordinates:

$$\frac{1}{r}\left(\frac{\partial^2\psi}{\partial r^2}-\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{\partial^2\psi}{\partial z^2}\right)=\omega,$$

$$\frac{\partial\omega}{\partial t}+u\frac{\partial\omega}{\partial r}+v\frac{\partial\omega}{\partial z}=\frac{u\omega}{r}+\nu\left(\frac{\partial^2\omega}{\partial r^2}+\frac{1}{r}\frac{\partial\omega}{\partial r}+\frac{\partial^2\omega}{\partial z^2}-\frac{\omega}{r^2}\right),$$

$$u=\frac{1}{r}\frac{\partial\psi}{\partial z},$$

$$v=-\frac{1}{r}\frac{\partial\psi}{\partial r}.$$

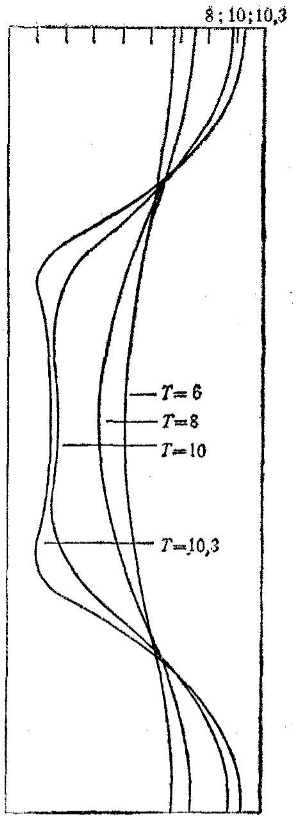


Fig. 6. $\lambda=9.016\,R_N$, $\varepsilon=0.02$, $\sigma=72.5$ erg/cm^2, $\mu=0.01$ g/cm . s

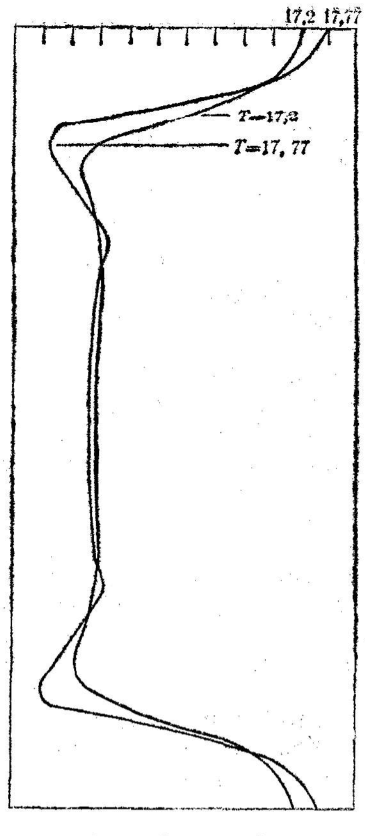


Fig. 7. $\lambda=20\,R_N$, $\varepsilon=0.02$, $\sigma=72.5$ erg/cm^2, $\mu=0.01$ g/cm . s

Here ν is the kinematic viscosity of the fluid, u and v are the radial and axial components of velocity, respectively.

The boundary conditions are:

At the free surface

1) The shear stress vanishes (the effect of the ambient medium is neglected):

$$\left(\frac{\partial u}{\partial r}-\frac{\partial v}{\partial z}\right)\sin 2\alpha+\left(\frac{\partial u}{\partial z}+\frac{\partial v}{\partial r}\right)\cos 2\alpha=0$$

2) The difference in the normal stress between the inside and the outside is due to the interfacial tension:

$$p_1-p_2=2\mu\left(\frac{\partial u}{\partial r}\cos^2\alpha+\frac{\partial v}{\partial z}\sin^2\alpha\right)-\mu\left(\frac{\partial u}{\partial z}+\frac{\partial v}{\partial r}\right)\sin 2\alpha+\sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right),$$

where $\operatorname{tg}\alpha=\frac{\partial h}{\partial z}$, p_1 is the pressure of the liquid, p_2 is the pressure of the ambient medium, R_1 and R_2 are the mutually orthogonal principal radii of curvature of the surface.

3) The condition for the free surface:

$$\frac{\partial h}{\partial t}=u-v\frac{\partial h}{\partial z}.$$

At the centerline:

$$\psi=0, \quad \frac{\partial\psi}{\partial r}=0.$$

At the ends of one-wavelength jet segment periodic conditions are imposed.

The problem has been nondimensionalized with reference to the radius R_N, as a characteristic length and to $T=\sqrt{\rho R_N^3/\sigma}$ as a characteristic time (σ — surface tension, ρ — density of the liquid). The equations are solved after a transformation of coordinates: $x=r/h$, $z=z$ ($h(z,t)$ — the free surface of the jet), that makes the domain of calculation rectangular, but complicates the equations and the boundary conditions.

3. Algorithm of solution. For given initial values of ω, ψ, u, v, and h one uses the algorithm:

1. Numerically differentiate h — cubic splines are used.
2. Solve the equation for ω — alternating direction implicit method is used.
3. Obtain the finite differences scheme and solve the equation for ψ — nine points differences scheme is used to overcome the difficulty of the two boundary conditions for ψ at the free surface.
4. Calculate ω and ψ at the free boundary.
5. Obtain u, v and new h.

4. Numerical results. One obtains the configuration of the jet for various wavelengths, initial disturbance amplitudes, surface tensions, and viscosities.

The initial velocity profile is uniform. The initial geometry of the jet is determined by using the value of h from the linearized solution of Weber for various time arguments.

The numerical results which show the dependence of the time of desintegration on the wavelengh, the initial disturbance amplitude, the surface tension and the viscosity are plotted on Fig. Fig. 1-7.

References

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Received 10.08. 1986