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On the decoupled drive system design of industrial robots

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Introduction

In designing industrial robots it is necessary to have efficient methods for modelling and control of their dynamic behaviour. The continuously increasing demands for enhanced productivity and improved precision have imposed strong requirement on the integrated design of the mechanical configuration, drive and control system. Industrial robots are highly nonlinear coupled dynamic systems and their dynamics become even more complicated when the load/speed increases.

An inherent approach to overcome the dynamic complexity is to modify the design of arm linkages and joint actuators so that the links are to be driven more independently. The central role in designing a robot with appropriate dynamic characteristics plays the inertial matrix.

Some conceptual frameworks have been developed in order to specify the inertial matrix, leading to a significant reduction of the dynamics complexity, see Tourassis and Neuman [1], Asada [2], Youcef-Toumi and Asada [3]. Mass distribution techniques and corresponding arm configurations are presented in these works to achieve diagonal dominant inertial matrices.

In this work we start from such a motivated condition of diagonal dominance of the inertial matrix to meet the next problem concerning the design of a decoupled drive system. The decoupling concept is in the sense that the sign of the maximum force/torque applied at any joint is equal to the sign of the corresponding acceleration. The availability of such independent dynamics controllability (IDC) condition will provide several advantages in the control of robot dynamics:

— reliability and stability of terminal feedback control systems, see, e. g., Young [4] Arimoto and Miyazaki [5] etc.,

— robustness of continuous path control schemes, see, e. g., Dubowsky and DesForges [6], G. De Maria and all. [8], Slotine and Sastry [7] etc.,

— on-line dynamics avoidance of obstracles,

— efficient loading of joint actuators (descrepancy between signs of a joint force/torque, and corresponding acceleration may result in violating current limitations, energy loss, wear, etc.), and

— the IDC condition has been presumed in several works for control synthesis of point-to-point motions based on natural robot dynamics and actuator's maximum capabilities, see Kiriazov and Marinov [9], Marinov and Kiriazov [10, 11].

2. Robot Dynamics and Drive Decoupling

The next considerations will concern only those "n" joints of a robot which are in dynamic coupling. The corresponding joint motion equations, via Lagrangian formulation, may be written in the form:

$$(1)\qquad \sum_{j=1}^{n} d_{ij}(q)\ddot{q}_j + c_i(q, \dot{q}) + s_i\dot{q}_i + g_i(q) = T_i(t), \quad i=1, \ldots, n,$$

where: q_i are joint coordinates; $d_{ij}(q)$ are inertial coupling coefficients; c_i represent centrifugal and Coriolis forces; $c_i = \dot{q}^T C_i(q)\dot{q}$ — quadratic forms; $s_i > 0$ — viscous friction; $g_i(q)$ — stand for gravitation and Coulumb friction forces; T_i — joint forces/torques.

The work space for the joint coordinates represents a compact set in the joint space:

$$(2)\qquad Q = \{q \mid q_i^{\min} \leq q_i \leq q_i^{\max}\}.$$

All functions of q in (1) are continuous and therefore bounded.

In the following decoupling drive design we start from a diagonal dominance condition for the inertial matrix $D = (d_{ij})$:

$$(3)\qquad \sum_{j=1,\ j\neq i}^{n} |d_{ij}| < d_{ii}/(n-1), \quad \forall q \in Q, \quad i=1, \ldots, n.$$

Such a condition holds for several well-known three DOF positioning system configurations:

— cartesian,

— cylindrical with sufficiently small exentricity, see Troch, Kopacek and Desoyer [12],

— spherical with sufficiently small exentricity, and

— articulated with special geometrical configuration and mass distribution as well — Tourassis and Neuman [1], Asada [2], Youcef-Toumi and Asada [3].

It follows from inequality (3) that the inverse inertial matrix $D^{-1} = (d_{ij}^{-1})$ will be diagonally dominant in the usual sense:

$$(4)\qquad \sum_{j=1,\ j\neq i}^{n} |d_{ij}^{-1}| < d_{ii}^{-1}, \quad \forall q \in Q, \quad i=1, \ldots, n.$$

A detailed proof of this inequality is rather mathematical and therefore given in the Appendix.

The existence of IDC condition is shown in the following

Theorem: Under condition (3) which specifies robot model (1), (2), there exist bounds $T_i^{\text{extr}} = \pm T_i^{\max}$ of the joint forces/torques and a margin Q' for the joint velocities such that:

$$(5)\qquad \operatorname{sign} T_i^{\text{extr}} = \operatorname{sign} \ddot{q}_i, \quad i=1, \ldots, n$$

for any $q \in Q$, $\dot{q} \in Q'$ and $-T_j^{\max} \leq T_j \leq T_j^{\max}$, $j \neq i$. Proof: Making use of the inverse inertial matrix and the system of differential eqs (1), IDC condition (5) can be written in the form:

$$(6)\qquad \operatorname{sign} T_i^{\text{extr}} = \operatorname{sign} \sum_{j=1}^{n} d_{ij}^{-1}(q)\,(T_j - g_j(q) - c_j(q, \dot{q}) - s_j\dot{q}_j),$$

where: $T_j = T_i^{\text{extr}}$ for $j = i$.

Denoting the right parts of eq. (6) by $\operatorname{sign} R_i = \operatorname{sign} R_i(q, \dot{q})$, we present each of them as a sum of two functions:

(7) $$R_i(q, \dot{q}) = M_i(q) + N_i(q, \dot{q}),$$

where:

(8) $$M_i(q) = d_{ii}^{-1}(q)\, T_i^{\text{extr}} + \sum_{j=1,\, j\neq i}^{n} d_{ij}^{-1}(q) T_j - \sum_{j=1}^{n} d_{ij}^{-1}(q)\, g_j(q)$$

and

(9) $$N_i(q, \dot{q}) = \sum_{j=1}^{n} d_{ij}^{-1}(q)\left(-c_j(q, \dot{q}) - s_j \dot{q}_j\right).$$

Since all of the functions in (8), as well as (9), are constrained and inequality (4) holds, then there exists a sufficiently large $\widehat{T}$ such that for any $q \in Q$:

(10) $$d_{ii}^{-1}(q)\widehat{T} - \sum_{j=1,\, j\neq i}^{n} |d_{ij}^{-1}(q)|\, \widehat{T} - \left| \sum_{j=1}^{n} d_{ij}^{-1}(q) g_j(q) \right| > 0.$$

Therefore if we take $T_i^{\max} = \widehat{T}$, $i = 1, \ldots, n$, then the following inequality holds true:

(11) $$T_i^{\text{extr}} . M_i(q) > \Delta_i > 0, \quad \forall q \in Q \text{ and}$$
$$\forall T_j \in [-T_j^{\max};\ T_j^{\max}],\ j \neq i,\ i = 1, \ldots, n.$$

The boundness of functions $d_{ij}^{-1}(q)$ and components of matrices $C_i(q)$ together with the fact $N_i(q, 0) = 0$ leads to a conclusion that there exists a neighbouring set Q' of the origin in the space of joint velocities with the property that:

(12) $$|T_i^{\text{extr}} . N_i(q, \dot{q})| < \Delta_i, \quad q \in Q,\ \dot{q} \in Q'.$$

From eqs (7), (11), and (12) it follows that: $T_i^{\text{extr}} . R_i(q, \dot{q}) > 0$ which had to be obtained.

3. Discussion

Some concepts on the design of decoupled drive systems have been presented. With given work space Q and given mechanical structure of a robot which satisfies diagonal dominance condition (3), one can find torque levels T_i^{extr} (systems motorgear) satisfying IDC condition (5) with a margin Q' of the joint velocities.

If we don't observe the constraint Q' on the joint velocities, then in case the work space is too large, the velocity dependent forces could violate the IDC condition, no matter the inertial matrix is diagonally dominant. But for a regularly designed robot (whose torques' levels dominate gravitation torques), we always can slow down the robot motion so that the IDC condition to be fulfilled. This reasonable statement can be easily verified using Hamilton's canonical form of robot dynamics:

(13) $$\dot{q} = \frac{\partial K}{\partial p} = D^{-1}(q)\, p$$
$$\dot{p} = -\frac{\partial K}{\partial q} - \frac{\partial U}{\partial q} + T,$$

where: K and U are kinetic and potential energies, respectively.

As a consequence of eqs (13), the time-derivative of the kinetic energy may be presented as follows:

(14) $$\frac{d}{dt}K=\frac{\partial K}{\partial p}\dot{p}+\frac{\partial K}{\partial q}\dot{q}=\sum_{i=1}^{n}\left(T_i-\frac{\partial U}{\partial q_i}\right)\dot{q}_i.$$

One can see from eq. (14) that the kinetic energy will be strongly decreasing when all the actuators are in a breaking regime. Therefore, after some time $\dot{q}$ will belong to Q', and the IDC condition will be achieved.

In our opinion, such transient violation of the IDC condition may be apparent in point-to-point motions, but near the target point it will take place again to provide the necessary controllability.

4. Conclusions

On the base of diagonal dominance character of the inertial matrix, we have considered the existence of a condition for independent dynamic controllability of joint motions. In case of higher speeds or heavier manipulation loads this reasonable condition is indispensable to improve the dynamic performance and to guarantee reliability and robustness of robot control systems.

Appendix

The objective of this appendix is to prove that inequality (3) is sufficient condition for the inverse inertial matrix to be diagonal dominant one. The basic relations from the matrix theory are used:

$$\sum_{k=1}^{n} d_{ik}^{-1}\, d_{kj}=0, \quad i\neq j$$

(A-1) $$\sum_{k=1}^{n} d_{ik}^{-1}\, d_{ki}=1.$$

First, we will show that for any fixed i, $i=1,\dots,n$, the following estimation exists:

(A-2) $$|d_{ij_0}|=\max|d_{ij}^{-1}|<|d_{ii}^{-1}|, \quad j=1,\dots,n;\ j\neq i.$$

Actually, if we assume $|d_{ij_0}^{-1}|\geq|d_{ii}^{-1}|$ then from (3) and (A-1) (**when** $j=j_0$) we obtain the following consequences:

(A-3) $$|d_{ij_0}^{-1}|\,|d_{j_0j_0}|\leq \sum_{k=1,\,k\neq j}^{n} |d_{ik}^{-1}|\,|d_{kj_0}|\leq |d_{ij_0}^{-1}| \sum_{k=1,\,k\neq j}^{n} |d_{kj_0}|<|d_{ij_0}^{-1}|\,|d_{j_0j_0}|/(n-1)$$

which are contradictory.

Further more, (A-1), (A-2), and (3) lead to:

(A-4) $$|d_{ij}^{-1}\, d_{jj}|=\left|\sum_{k=1,\,k\neq j}^{n} d_{ik}\, d_{kj}\right|<|d_{ii}^{-1}|\,|d_{jj}|/(n-1)$$

and

(A-5)
$$\sum_{j=1,\, j\neq i}^{n} |d_{ij}^{-1}| < |d_{ii}^{-1}|$$

Finally, the positiveness of diagonal elements can be seen by the relations following from (A-1), (A-2), and (3) as well:

(A-6)
$$d_{ii}^{-1} d_{ii} = 1 - \sum_{k=1,\, k\neq i}^{n} d_{ik}^{-1} d_{ki} \geq 1 - \sum_{k=1,\, k\neq i}^{n} |d_{ik}^{-1}|\, |d_{ki}|$$
$$> 1 - |d_{ii}^{-1}| \sum_{k=1,\, k\neq i}^{n} |d_{ki}| > 1 - |d_{ii}^{-1}|\, |d_{ii}|/(n-1)$$

Thus the proof is completed.

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