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Multi-Layer Cylinder, Subjected to Internal Loading

A. Baltov, St. Vodenitcharov, I. Nicolova,
G. Kortenski, M. Marinov

Introduction

In great number of practical cases it is nessesary to construct a pot able to get an internal dynamic loading with minimum possible deformation and economic material expences and with guaranteed reliability. For instance, such a case is observed in cameras, where the process of detonation forming is realized. A collective from the Institue of Metal Sciences and Metal Technology, Bulg. Academy of Sciences, has created such a camera on the principle of a multi-layer cylinder [1]. In the work presented some evaluating formulae for the strain of a multi-layer cylinder, dependent on the number of layers, are obtained. In the process of investigation some model simplifications are made with the goal to achieve qualitative conclusions, that make a good basis for quantitative measuring of evaluations also:

a) The cylinder is considered infinitely long, subjected to uniformly distributed impulse internal pressure.

b) The cylinder is considered one-layer, made of a "fictitious", substituting material. The multi-layreness being exhibited as generalized inelasticity of the "fictitious" material, that reflects equivalently the energy dissipation during the wave traverse across the layers.

The investigation is based on:

1) Experimental methods for multi-layer speciments, by the use of the Modified Hopkinson pressure bar.

2) Identification methods for the substitutive material model, based on the data obtained by the following experimental methods [2].

3) Solution of the wave problem for an infinite cylinder made of inelastic material [3].

Brief description of the methods

By help of the Hopkinson pressure bar, the dynamic behaviour of materials is investigated by means of a short cylindric specimen sandwiched between two long cylindric bars with known elastic properties and a high yield point. The one end of the first bar is loaded by a stress pulse, passing across the bar without changing, as far as the bar is deformed elastically. The stress pulse reaches the specimen — part of it is reflected and another part traverses to the second elastic bar. By means of gauges in the elastic bars, the loading, the reflected and the traversed pulses are registered. The stress-strain relation under different strain rates is obtained by comparing the pulses and averaging the dynamic process measures [4]. That is the basis for constructing mechanical mathematical models, describing the dynamic behaviour of the materials studied. In [5] and [6] an expansion of the experimental procedure abilities is made, by proposing to study longer specimens. In this case, the loading and the traversed pulses are compared, the loading pulse being considered as the difference between the falling and the reflected pulses. If there is no change, neither in the form nor in the intensity of the loading pulse, after its passing through the specimen, then the material shows purely elastic properties. If there are changes, they are due to the energy dissipation during the process of wave propagation through the specimen, and these changes indicate to presence of inelastic properties. On this basis the identification of the mechanical mathematical model is made for describing the inelastic behaviour of the material. In the work presented, we propose a new enrichment of the experimental methods by the use of the Hopkinson pressure bar, with a view to the goals set. Cylindric specimens of a given material with different structures and with the same length are tested: an entire specimen or a specimen consisting of two, three or more cylinders placed consecutively. The change of the stress pulse during its propagation through the specimen is registered, dependent on the number of cylinders. These data are the basis for application of the identification methods to the "fictitious" inelastic material, substituting the multilayer medium. In [2] these methods are applied to constructional materials, exhibiting linear elastic properties, for the stress pulses used. When a multi-layer medium is studied, with view to the applications, we propose the description of the "fictitious" substituting material by the simplified Maxwell's model of a visco-elastic body:

(1) $$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^a$$

and

(2) $$\dot{\varepsilon}^e = \frac{1}{B}\dot{\sigma}, \quad \dot{\varepsilon}^a = \frac{|\sigma|}{\eta(n)}$$

where ε denotes the linear strain, σ — the normal stress, ε^e — the elastic part of strain, ε^a — the inelastic part of strain, E — the Young modulus of the material the cylinders are made of, $\eta(n)$ — the viscosity coefficient of the fictitious material, depending on the number n of layers. With point (".") we denote the differentiation with respect to time. Analogically one may apply a more complex inelastic model. After solving the wave problem and comparing its solution with the experimental data for the traversed pulse in cases of different numbers of layers, one obtains the connection $\eta = \eta(n)$ in the form:

$$\eta = 10^8 [a_1 e^{bn} + a_2 n + a_3]$$

where a_1, a_2, a_3 and b are constant coefficients. The relation holds for $n \in [2, 10]$ and $\eta \in [10^8, 79.10^8$ Ncm/s]. An example is given for steel 3 where $a_1 = 12.934$, $a_2 = -3.8194$ 10^4, $a_3 = 123.42$, $b = 2.13$ with a relative standart $b_r = \sqrt{s_r^2} \approx 0.035$ about 3.5 %.

An evaluating formula for the strains of a multilayer cylinder under dinamic loading

We consider a multi-layer cylinder (n layers) with an internal radius $r = a$ and an external $-r = b$. On the internal surface an uniformly distributed dynamic stress pulse $p(t)$ (respectively strain pulse $q(t)$) is applied. A stress wave is created, it passes through the cylinder and causes its deformation leading to displacement of the external surface $u_r|_{r=b} = u_b$. When a, b and k are fixed, we want to define the change of u_b, dependent on the number of layers n, when a model for the "fictitious" material is applied according to [1], [2], [3]. This relation will give us the evaluation searched.

The dynamic process in the cylinder is described by the next system of partia differential equations (in cylindric coordinate system, the axis symmetry taken into account, and all the process measures being functions of r and t only):

$$(4)\quad \begin{cases} \frac{1}{\rho}\cdot\frac{\partial\sigma_r}{\partial r}-\frac{1}{\rho r}(\sigma_r-\sigma_\rho)=\frac{\partial^2 u_r}{\partial r\partial t}, \quad \varepsilon_\rho=\frac{\partial u_r}{\partial r}, \quad \varepsilon_\rho=\frac{u_r}{r} \\ \sigma_r=\lambda[(\varepsilon_r^a-\varepsilon_r^a)+(\varepsilon_\rho-\varepsilon_\rho^a)]+2\mu(\varepsilon_r-\varepsilon_r^a) \\ \sigma_\rho=\lambda[(\varepsilon_r-\varepsilon_r^a)+(\varepsilon_\rho-\varepsilon_\rho)^a]+2\mu(\varepsilon_\rho-\varepsilon_\rho^a) \\ \frac{\partial\varepsilon_\rho^a}{\partial t}=\frac{\sigma_\rho-\sigma_{cp}}{\eta(n)}, \quad \frac{\partial\varepsilon_r^a}{\partial t}=\frac{\sigma_r-\sigma_{cp}}{\eta(n)}, \quad \sigma_{cp}=1/2(\sigma_r+\sigma_\rho) \end{cases}$$

Plain strain is adopted ($\varepsilon_z=0$) and elastic flexibility of the material i. e. $\varepsilon_r^a+\varepsilon_\rho^a=0$, $\varepsilon_r^a=-\varepsilon_\rho^a$. ε_z and σ_r are the main radial components and ε_ρ and σ_ρ are the main peripheral components of strains and stresses, ρ is the material density, $\lambda=\text{const}$, $\mu=\text{const}$ are the elastic Lame constants. The system (4) is to be under the fall lowing boundary and initial conditions:

$$(5)\quad \varepsilon_r(a,t)=-q(t), \quad \sigma_r(r,0)=0$$
$$\sigma_r(b,t)=0, \quad \left.\frac{\partial^2 u_r}{\partial t^2}\right|_{t=0}=0$$

The solution of the system (4) under the constraints (5) is obtained following the correspondence principle between the linear elastic and the linear visco-elastic models [3]. The system of diferential equations describing the behaviour of the linear elastic cylinder is obtained from (4) when $\varepsilon_i^a=0$ and $\varepsilon_\rho^a=0$.

The system (4) is reduced to the wave equation for the radial displacements:

$$(6)\quad c^2\frac{\partial^2 u_r}{\partial t^2}-\left[\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r}\right]=0$$

where $c=\sqrt{\frac{\rho}{\lambda+2\mu}}$ is the velocity of the longitudinal wave in elastic medium. The solution is obtained applying the Fourier transform (with parameter ω) and after solving the Bessel equation for the transform of the displacement function $\tilde{u}_r$:

$$(7)\quad \tilde{u}_r(\omega cr)=A(\omega)J_1(\omega cr)+B(\omega)Y_1(\omega cr),$$

where $A(\omega)=-\frac{\tilde{q}(\omega)}{\omega(c)}\cdot\frac{Y_1'(\omega bc)}{J_1'(\omega bc)}\cdot\left[Y_1'(\omega ac)-\frac{Y_1'(\omega bc)}{J_1'(\omega bc)}\cdot J_1'(\omega ac)\right]^{-1}$

$$(8)\quad B(\omega)=\frac{q(\omega)}{\omega c}\left[Y_1'(\omega ac)-\frac{Y_1'(\omega bc)}{J_1'(\omega bc)}\times J_1(\omega ac)\right]^{-1}$$

J_1 and Y_1 are Bessel functions of first order, $\tilde{q}(\omega)$ is the transform of $q(t)$. We denote with "′" the first derivatives of the Bessel functions.

According to the correspondence between the models in the solution for the image (7), (8) we substitute the velocity $c=\sqrt{\frac{\rho}{\lambda+2\mu}}$ by the expression $c_1=[\rho J^*(i\omega\eta)]^{-1/2}$ where $J^*(i\omega\eta)$ is the complex modulus of the Maxwell's model i. e. $J^*(i\omega\eta)=1/E+1/i\omega\eta$.

The displacement $u_r(r, t)$ in the case of Maxwell's model is obtained by inverting the transform $\tilde{u}_r$. It is done numerically with the help of a computational program [7]. Thus $u_r(b, t)$ (for a fixed moment t_1) is expressed as a function of the number of layers n with the help of the viscosity coefficient $\eta=\eta(n)$. On Fig. 1 the function

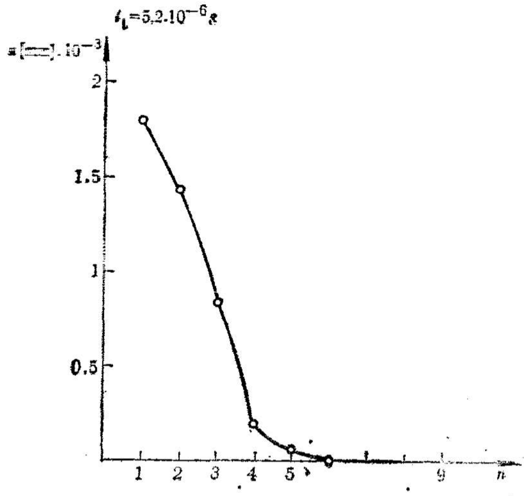

Fig. 1

$u_r(b, t)$ is shown for the next data: 1) geometric $a=5\,cm$, $b=10\,cm$; 2) loading $q(t)=R(t^2-\xi t)$, $R=3.10^7$, $\xi=10^{-5}\,sec$, $t_1=5{,}2.10^{-6}\,s$; 3) for steel 3 $E=2.10^7\,N/cm^2$, $\rho=7.85.10^{-5}\frac{Ns}{cm^4}$ and $\eta(n)$ is obtained following [3] with the data of & 2.

The typical decrease of the displacement of the external radius with the increase of the layer's number n is obvious. This fact grounds the idea for multi-layer cameras. The function obtained is the searched evaluating formula for the deformation of a multi-layer cylinder under dynamically applied internal pressure. It may be used for an important orientation in constructing such cameras.

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