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Adjustment of feedback gains in nonsingular position control systems of manipulators*

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Introduction

In most technological processes, manipulators are mainly used as positioning devices. The path between operation points is not specified and depends on the control algorithm and hardware. In this case, it is important to design an efficient controller which requires less time and energy, provides smooth positioning and desired accuracy.

This problem was the subject of some recent investigations by authors in [1, 2], where suboptimal open-loop control algorithms are proposed with the foregoing goal. With the concept of averaging the manipulator dynamics at each sampling interval, a feedback controller with a weighted minimum time-fuel criterion is developed in [7]. In fast simultaneous coupled joint motions and havier loads, this approach leads to overshoots and settling time prolongation.

Very recently the authors [5, 6] have proposed a method for feedback control synthesis on the base of real manipulator dynamics with bang-bang control in point-to-point motion.

In the present work, a new control law is proposed. The algorithm consists in an interative adjustment of the feedback gains in proportional plus derivative (*PD*) controllers of the joint actuators and the bang-bang control laws are replaced by continuous magnitude control functions. Control laws are in accordance with minimum energy loss-criterion. With given initial and final states, the values of the feedback gains are obtained resolving the corresponding two-point boundary-value problem (*TPBVP*). The exact solution of this problem provides a desired positioning accuracy and improved dynamic performance.

The algorithm with continuous magnitude control functions is illustrated on a dynamic model of a direct current driven manipulator with cylindrical coordinates.

Statement of the Problem

Using the Lagrangian formulation, the mathematical model of manipulators dynamics in coupled joint operations can be written as follows:

$$M(q)\ddot{q}+N(q,\dot{q})=T \tag{1}$$

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where: $M(q)$ is an $n \times n$ inertia matrix, $N(q, \dot{q})$ is a $n \times l$ vector which represents centrifugal, Coriolis, gravitation and friction forces, T is a generalized force/torque vector, and q, $\dot{q}$, $\ddot{q}$ are vectors of n-joint coordinates, velocities and accelerations, respectively. The magnitudes of driving forces or torques are depended on the control inputs:

$$T_i = T_i(q_i, \dot{q}_i, u_i) \quad i = 1, \ldots, n. \tag{2}$$

Boundary conditions:

$$q(t^0) = q^0, \quad \dot{q}(t^0) = 0 \text{ — initial state} \tag{3}$$

$$q(t^f) = q^f, \quad \dot{q}(t^f) = 0 \text{ — final state.} \tag{4}$$

In the case of a relay servosystem with (PD) controllers that has high potential for improving the manipulator performance (time or energy consumption), we have:

$$u_i = U_i \, sign\,(k_i(q_i^f - q_i) - \dot{q}_i). \tag{5}$$

The magnitude function U_i for each joint is composed of two alternatively changing control laws (when crossing the switching line in the phase plane $(q_i, \dot{q}_i)$): U_i^a (acceleration regime) and U_i^d (deceleration regime).

If we assume bang-bang type of control laws (5) in accordance with minimum time-criterion

$$U_i^a = U_{i\,\max}^a, \quad U_i^d = U_{i\,\max}^d, \tag{6}$$

then the problem ~~is to~~ find those values of the feedback gains k_i which solve the *TPBVP* $(1 \div 6)$, i. e. without joint motion oscillation or chattering relay operation.

However, in the case of relatively high levels of the magnitudes U_i^a, U_i^d, bang-bang control laws will not provide a good dynamic behaviour of the manipulator in the switching times. In the presence of backlashes such control strategy may give rise to undesirable vibrations and jerks.

In order to improve the performance capability of manipulators in point-to-point operation, we propose an extension of the relay servo system with (PD) controllers. Instead of using on-off control laws (5), we can apply continuous magnitude control functions (Fig. 1) with

$$r_i = |k_i(q_i^f - q_i) - \dot{q}_i| \cdot (k_i^2 + 1)^{-1/2} \tag{7}$$

and then the problem is to find those values of the feedback gains k_i which solve the *TPBVP* $(1 \div 4)$, (7). The r_i is the distance between the current phase point $(q_i, \dot{q}_i)$ and switching line.

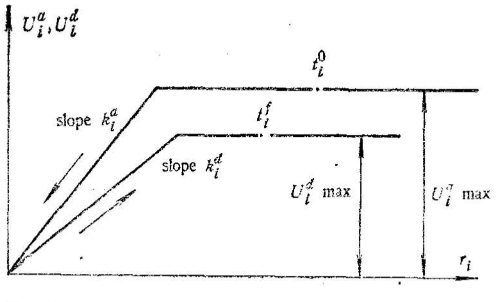

Fig. 1

Besides the improvement of the dynamic behaviour of manipulators in the switching times such control functions provide substantial energy-loss minimization (see [3]) and correspond to Pontryagin Maximum principle for the linear second order plant with energy loss-criterion. Applying appropriate control synthesis algorithm and fixed values of the slopes $k_i^a(k_i^d)$, we can obtain the corresponding feedback gains k_i. Further, making use of the parametric optimization procedure with respect to these slopes and initial times, a satisfactory suboptimal solution can be achieved.

Control Synthesis Algorithm

Control synthesis algorithm which exactly solves the *TPBVP* ($1\div4$, 7) consists in performing several test movements from the given initial state to some terminal states, converging to the required one. In the off-line sence, each such movement means that the system of differential equations (1) and (2), with initial values (3) and control laws (7) with some approximative feedback gains k_i and slopes $k_i^a(k_i^d)$, is integrated until satisfying the final conditions for the velocities $(4)_2$: $\dot{q}_i(t_i^f)=0$ ($t^f=\max t_i^f$, $i=1,\dots n$) — it means that each joint motion stops after the moment t_i^f with some terminal value of the corresponding coordinate: $F_i=q_i(t_i^f)$. The n-dimensional vector $\boldsymbol{F}$ depends on the vector of feedback gains k_i (if there is no chattering relay operation). So, in order to satisfy another final condition $(4)_1$, one has to solve the following shooting equation:

$$F(k)=q^f. \tag{8}$$

This equation is solvable using the well known gradient's, bisection's or other search methods (see [4], where convergence conditions are given, too).

Having obtained the exact values in the off-line case, the proposed iterative procedure can be performed on the manipulator itself for final adjustment of these values. Such a self-learning procedure is needed since it is practically impossible the dynamic model ($1\div2$) to present the real manipulator dynamics.

Numerical Example

To be more illustrative, the proposed method is applied to a dynamic model of manipulator with cylindrical coordinates. Including direct current (*DC*) actuators and taking some specified numerical values of the model parameters, the equations of the two coupled joint motion are written as [5]:

$$\begin{aligned}
\ddot{q}_1 &= (u_1-6.8\dot{q}_1-0.34(10.0q_2+5.0(q_2+1.0))\dot{q}_1\dot{q}_2)/ \\
&\quad (0.4+0.17(20.0+10.0q_2^2+5.0(q_2+1.0)^2), \\
\ddot{q}_2 &= (u_2-6.8\dot{q}_2+0.17(10.0q_2+5.0(q_2+1.0))\dot{q}_1^2)/2.95.
\end{aligned} \tag{9}$$

The accepted slopes k_i^a and $k_1^d: k_1^a=k_1^d=k_2^a=k_2^d=100.0$.

The control and state parameters are accepted the same values as in the case of bang-bang control [5].

The following values of the feedback gains are obtained: $k_1=-2.3$ and $k_2=-4.0$, with the corresponding final times: $t_1^f=2.3$ *s* and $t_2^f=1.7$ *s*. The corresponding joint motions are depicted on Figs 2 and 3.

We have an energy loss reduction about 42.7 % in comparision with the bang-bang control [5].

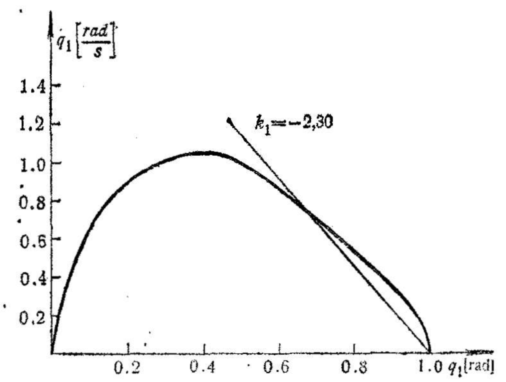

Fig. 2

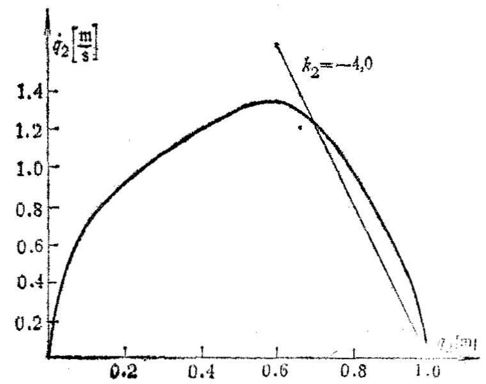

Fig. 3

Conclusions

A method for adjustment of feedbeak gains in nonsingular position control systems of manipulators is proposed. On the basis of real manipulator dynamics and continuous magnitude control functions, a control synthesis algorithm for the system with (*PD*) controllers of the joint actuators is developed. Control laws are in accordance with minimum energy loss-criterion. The values of the feedback gains are obtained from the exact solution of the corresponding *TPBVP*. An illustrative example of a dynamic model of (*DC*) manipulator with cylindrical coordinates is presented. The method is simple for on-line implementation as an adaptive self-learning procedure. Such a procedure is very useful for final adjustment of the feedback gains since the exact identification of manipulator parameters is in general impossible and computation errors are unavoidable.

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