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Computation of Stabilized Turbulent Fluid Flow and Heat Transfer in Circular Smooth Pipes for Moderate Prandtl Numbers. Part 2. — The Limiting Heat Transfer Coefficients

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Part 1 of this study [1] was concerned with the formulation of a model for the momentum transport and the transport coefficients for hydrodynamically fully developed turbulent flow in smooth circular channels and a computational procedure to realize this model numerically. The model combines elements of previously published models and enables one to find the boundary between the wall region and the core of the flow depending on the value of the Reynolds number Re. Numerical results for the Fanning friction factor F were compared to correlations and experiments previously published to testify the validity of the model.

The purpose of Part 2 is on the basis of the exact solutions [2] for the turbulent Graetz problem [3, 4] to report numerical results for the dimensionless limiting heat transfer coefficient — the Nusselt number — for: (a) — constant wall heat flux boundary condition, denoted by Nu_{∞}^{H} and estimated by means of the Lyon's integral [5]; (b) — constant wall temperature boundary condition, denoted by Nu_{∞}^{T} and obtainable as the first eigenvalue of the corresponding Sturm-Liouville eigenvalue problem utilizing a recently published algorithm for eigenvalue — eigenfunction analysis of Sturm-Liouville problems [6].

It has to be noted that as stated in [11] "experimental data for the wall temperature boundary condition are practically absent, while analytical or semi-analytical results are sparse and based on assumptions that conflict with the practical situations".

The eigenvalue approach has been extensively used to solve Graetz type problems. Several extensions of the classical Graetz problem have been reviewed in [7]. Soviet literature on the subject has been referenced in [8]. Heat transfer for various dilatant and pseudoplastic fluids has been investigated in [9], [10] and for turbulent flow in a plate channel — in [11]. The influence of distorted velocity profiles due to wall suction on the concentration was discussed in [12] while heat generation effects were considered in [13] for Newtonian fluids and in [14] for Nonnewtonian fluids. In all references [9]—[14] Nu_{∞}^{T} for hydrodynamically fully developed pipe flow was calculated by Sturm-

Liouville eigenvalue analysis. Several classical approaches to do this were referenced in [6]. Amongst others one could mention the Stodola-Vianello method [10], finite power-series approximation [12] and WKB-J analysis [14].

To be precise in what follows, we shall be concerned with the heat transfer in a single pipe, subject to the assumptions:

1. The fluid is incompressible with constant physical properties.
2. The fluid flow is hydrodynamically fully stabilized.
3. The heat transfer is time independent.
4. Internal heat generation and axial effects are negligible.

With these assumptions and avoiding the repetition of the derivations in [2, Chap. 8], the steady state dimensionless energy balance equation for the forced convection heat transfer in a circular pipe can be written as

$$\text{(1a)} \qquad U(R)\frac{\partial\theta(X,R)}{\partial X}=\frac{1}{R}\frac{\partial}{\partial R}\left\{RE(R)\frac{\partial\theta(X,R)}{\partial R}\right\},\quad X>0,\ 0<R<1,$$

subject to boundary conditions

$$\text{(1b)} \qquad \frac{\partial\theta(X,0)}{\partial R}=0,\quad X>0,\ R=0,$$

$$\text{(1c)} \qquad \alpha\,\theta(X,1)+\beta E(1)\frac{\partial\theta(X,1)}{\partial R}=\Phi(X),\ X>0,\ R=1,$$

and initial condition

$$\text{(1d)} \qquad \theta(0,R)=\theta_0(R),\quad X=0,\quad 0\leqq R\leqq 1.$$

In Eqs (1a)—(1d) X and R are the dimensionless axial and radial coordinates, $U(R)$ and $E(R)$ are the dimensionless velocity distribution and the diffusivity, $\theta(X, R)$ is the dimensionless temperature, $\Phi(X)$ is the external wall heat flow and α and β are constants, such that $\alpha=0$, $\beta=1$ corresponds to constant wall heat flux boundary condition, and $\alpha=1$, $\beta=0$ — to constant wall temperature boundary condition. The functions $U(R)$ and $E(R)$ are defined as

$$\text{(1e)} \qquad U(R)=U^+(R)/U^+_{avg},$$

$$\text{(1f)} \qquad U^+_{avg}=2\int_0^1 RU^+(R)dR,$$

$$\text{(1g)} \qquad E(R)=1+Pr\,\frac{\varepsilon_H}{\nu},$$

where all notations at the right-hand side of Eqs (1e)—(1g) were explained in [1].

The classical definition of the Nusselt number is

$$\text{(2a)} \qquad Nu(X)=2\,\frac{\partial\theta(X,1)}{\partial R}\{\theta(X,1)-\theta_{avg}(X)\}^{-1},$$

$$\text{(2b)} \qquad \theta_{avg}(X)=2\int_0^1 RU(R)\theta(X,R)dR.$$

Thus, to find the limiting Nusselt number Nu_∞ one has to solve the initial-boundary value problem, Eqs (1a)—(1d) and to find the limit of the expression, Eq. (2a), as $X \to \infty$. The input data of the problem are the constants α and β and the functions $\theta_0(R)$, $\Phi(X)$, $U(R)$ and $E(R)$. In what follows we shall assume that $\theta_0=0$ and $\Phi=1$, while the computation of $U(R)$ and $E(R)$ was described in Part 1 of this study [1].

In [2, Chap. 8] the solution of Eqs (1a)—(1d) is found using the finite integral transform method. It is in the form of an infinite series of orthonormal eigenfunctions forming a complete family in the corresponding functional space. The temperature distribution $\theta(X, R)$ has the form

$$\text{(3a)}\quad \theta(X, R)=1+\sum_{i=1}^{\infty} \Psi_i(M_i, R)\exp(-M_i^2 X)\frac{1}{M_i^2}\frac{\partial \Psi_i(M_i, 1)}{\partial R}, \quad \alpha=1,\ \beta=0,$$

$$\text{(3b)}\quad \theta(X, R)=\left\{\int_R^1 \frac{\left[2\int_0^R R' U(R')dR'\right]^2}{RE(R)}dR-\int_R^1 \frac{\left[2\int_0^{R'} R'' U(R'')dR''\right]^2}{R'E(R')}dR'\right\}$$

$$-\sum_{i=1}^{\infty}\Psi_i(M_i, R)\exp(-M_i^2 X)\frac{1}{M_i^2}\Psi_i(M_i, 1), \quad \alpha=0,\ \beta=1.$$

In Eqs (3a) and (3b) M_i^2 and $\Psi_i(M_i, R)$, $i=1, 2, \ldots$ are the eigenvalues and the corresponding normalized eigenfunctions of the Sturm-Liouville problem

$$\text{(4a)}\quad \frac{d}{dR}\left\{RE(R)\frac{d\Psi(M, R)}{dR}\right\}+M^2 RU(R)\Psi(M, R)=0, \quad 0<R<1,$$

$$\text{(4b)}\quad \frac{d\Psi(M, 0)}{dR}=0, \quad R=0,$$

$$\text{(4c)}\quad \alpha\Psi(M, 1)+\beta E(1)\frac{d\Psi(M, 1)}{dR}=0, \quad R=1,$$

where α and β take the values mentioned above.

To find the temperature distributions, Eqs (3a)—(3b), one has to solve the eigenvalue problem, Eqs (4a)—(4c). An algorithm for this purpose was recently developed in [6]. Its essence comprises the following major aspects: (a) — the original continuous eigenvalue problem is replaced by a series of piecewise continuous approximations with eigenelements $\{\psi_i^{(j)}(\mu_i, R), \mu_{ij}^2\}$, $i=1, 2, \ldots$, $j=1, 2, \ldots$ which is assumed to converge upon the elements $\{\Psi_i(M_i, R), M_i^2\}$ of the original problem; (b) — for each eigenvalue of each approximate problem, the procedure for the location of the eigenvalue comprises the current generation of sequences of upper and lower bounds thus guaranteeing that eigenvalues will not be missed in the course of the computation; (c) — the resolution of the interval of integration by means of which the piecewise continuous eigenproblems are formulated, is generated automatically, depending on the accuracy requirements to be satisfied by the approximate solution of the original problem.

In view of the iterative procedure for the velocity distribution [1] the automatic stepsize choice just mentioned might become too expensive. For this reason in what follows we have worked with a fixed resolution of the interval, satisfying two requirements: (a) — the largest subinterval has to be

Table 1

Re 10^{-4}		1.00	2.00	5.00	10.00	20.00
Ref.				Pr=0.7		
[15]	Nu_∞^H	33.82	54.68	106.20	178.64	304.36
[8]	Nu_∞^H	30.69	51.82	99.21	164.83	274.15
[17]	Nu_∞^H	31.61	55.03	114.54	199.42	347.21
[4]	Nu_∞^H	31.17	51.49	104.37	181.52	318.56
[4]	Nu_∞^T	32.19	54.50	109.50	187.95	325.60
this	Nu_∞^H	31.19	50.96	100.07	180.37	302.42
work	Nu_∞^T	29.73	48.98	96.84	175.04	294.24
Ref.				Pr=1.0		
[15]	Nu_∞^H	38.96	64.46	128.22	218.86	377.53
[8]	Nu_∞^H	36.92	62.09	123.44	207.59	349.13
[17]	Nu_∞^H	36.45	63.47	132.10	230.00	400.45
[4]	Nu_∞^H	39.05	65.61	134.91	236.27	416.69
[4]	Nu_∞^T	40.73	69.95	142.60	246.55	429.70
this	Nu_∞^H	37.28	61.84	123.62	223.63	379.67
work	Nu_∞^T	35·82	59.77	120.08	217.68	370.10
Ref.				Pr=8.0		
[15]	Nu_∞^H	92.97	164.95	355.74	640.77	1160.38
[8]	Nu_∞^H	91.21	162.73	348.93	620.10	1100.09
[17]	Nu_∞^H	83.75	145.81	303.49	528.40	920.00
[4]	Nu_∞^H	93.28	165.23	357.34	644.52	1165.74
[4]	Nu_∞^T	88.30	156.75	342.80	616.00	1135.50
this	Nu_∞^H	91.28	159.78	341.68	640.31	1136.25
work	Nu_∞^T	88.67	155.04	330.44	616.13	1091.16

small enough; (b) — the increment of the coefficient functions over one subinterval should not exceed some prescribed value. This explains the restrictions on the stepsize in the integration of the initial value problem, Eqs (8a) and (8b) in [1].

For the special case $X \to \infty$ the limiting Nusselt numbers, corresponding to Eqs (3a) and (3b) are shown to be [2, p. 370]

(5a) $$Nu_\infty^T = M_1^2, \quad \alpha=1, \ \beta=0,$$

(5b) $$Nu_\infty^H = \left\{ 2 \int_0^1 \frac{\left[\int_0^{R'} R' U(R') dR'\right]^2}{R E(R)} dR \right\}^{-1}, \quad \alpha=0, \ \beta=1,$$

the latter relation being known as the Lyon's integral.

Table 1 contains data for Nu_∞^H and Nu_∞^T obtained using various correlations previously published and also — our results. For the latter Nu_∞^H was calculated through Eq. (5b) utilizing the trapezoidal rule and Nu^T_∞ — by means of the modified algorithm [6] described above.

The following correlations were used:

[4]
$$Nu^H_\infty = 5.00 + 0.016\, Re^a Pr^b,$$
$$a = 0.88 - 0.24/(4.00 + Pr),$$
$$b = 0.33 + 0.50 \exp(-0.60\ Pr),$$
$$10^4 \leq Re \leq 10^6,\ 0.1 \leq Pr \leq 10^4;$$

[15]
$$Nu^H_\infty = Re\, Pr\{\sqrt{F/2}\,[12.48\, Pr^{2/3} + 3.613 \ln(Pr)$$
$$-7.853\, Pr^{1/3} + 5.8 + 2.78\ \ln(Re\sqrt{F/8}/45)]^{-1}\},$$
$$Re \leq 10^6,\ 0.7 \leq Pr \leq 10^5;$$

[8]
$$Nu^H_\infty = Re\ Pr\ F\{2[1.07 + 12.7\ (Pr^{2/3} - 1.00)\ \sqrt{F/2}]^{-1}\},$$
$$0.5 \leq Pr \leq 120;$$

[17]
$$Nu^H_\infty = 0.023\ Re^{0.8} Pr^{0.4},$$
$$Pr \leq 120.$$

The missing ranges for Re are given in [1].

We could not find numerical data for Nu^T_∞ with the exception of [4, Part 3]. It has to be noted that the model used in the reference just mentioned is slightly different from ours and the method of solution of the eigenvalue problem is not explicictly indicated, so that a straightforward comparison of the results is not available. But in general in heat transfer literature [17], [4], [22] it is clearly stated that $Nu^H_\infty > Nu^T_\infty$ and the (relative) difference between these two parameters decreases and becomes negligible as $Pr \to \infty$. It is usually assumed that for $Pr > 10$ the heat transfer coefficients are practically equal. Until now we have not found a satisfactory explanation for such a statement. Accepting this to be true, it can be seen that our results do not violate it, while the same is not true for [4] and [16, Chap. 7, Tables 9 and 10 for the case $Pr = 0.7$ and $Re = 100\,000$].

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