БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ
ТЕОРЕТИЧНА И ПРИЛОЖНА МЕХАНИКА
София, 1984, год. XV, №

BULGARIAN ACADEMY OF SCIENCES
THEORETICAL AND APPLIED MECHANICS
Sofia, 1984, Year XV, No. 1

Synthesis of Time-Optimal Control for Manipulator Dynamics*

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1. Introduction

A basic qualitative characteristic of the industrial robots is the minimum time consumption in handling operations. There are a few papers devoted to this problem. The method of nonlinear programming for optimal control of manipulators is applied in [1]. Using Pontryagin's maximum principle computational algorithm finding the switching times is developed in [2]. Simultaneous starting of joint motions is supposed in this work. A direct method for control synthesis of manipulator dynamics with the same supposition made is proposed in [3]. In another paper [4] a penalty method is used in the optimal control problems with dynamic constraints. The problem of time-optimal control of the gripper of a plannar manipulator along a straight line is solved in [5].

In the present paper a general method for time-optimal control synthesis of manipulator dynamics in handling operations is developed. The determination of the switching values of generalized coordinates is performed so that the conditions at the initial and final positions are satisfied. Sequential starting of joint motions is supposed and the signs of the generalized velocities are not changing during the manipulator motion. The method's verification is illustrated by the synthesis of time-optimal control of a simple manipulator.

2. The optimal control problem

The mathematical model of the manipulator with n degrees of freedom incorporating all relevant kinematic and dynamic mechanical parameters is represented in [6] by a system of complex nonlinear differential eguations

(1) $$M(q)\ddot{q}+N(q, \dot{q})=R(t),$$

where: $q(t)\in R^n$ is a vector of generalized coordinates of the mechanical system, $M(q)$ is an inertial matrix, $N(q, \dot{q})$ is a vector of centrifugal, Coriolis and gravity forces, $R(t)\in R^n$ is the vector of the driving forces and/or torques.

Denote by:

(2) $$q(t^0)=q^0,\ \dot{q}(t^0)=0 \text{ initial phase position,}$$

*The paper has been presented on the IMA/SERC Meeting — Control Theory, University of Warwick, Coventry, July 11—13, 1983.

(3) $q(t^f)=q^f,\ \dot{q}(t^f)=0$ final phase position,

(4) $\operatorname{sign}\dot{q}_i=\operatorname{sign}(q_i^f-q_i^0)=\operatorname{sign}_i,\ (i=1,\dots,n)$ velocities signs condition.

Each control function $R_i(t)$ $(i=1,\dots,n)$ has one switching time t_i^s as it is considered in [2]. A general appearance $\tilde{R}_i(t)$ of these functions and the

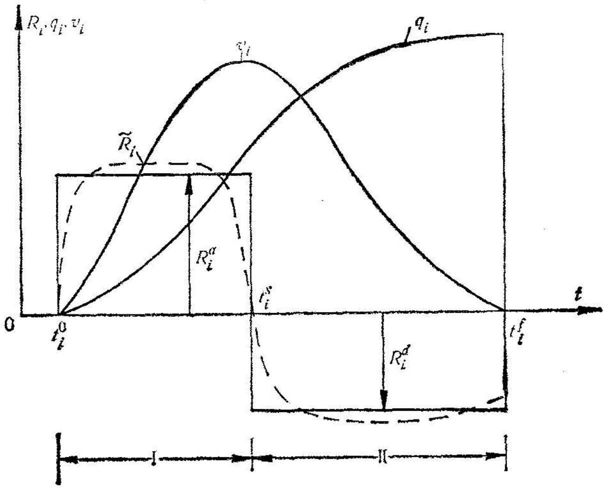

Fig. 1

corresponding coordinates $q_i(t),\ t\in(t_i^0,\ t_i^f)$ are shown on Fig. 1. With the purpose of simply explaining of the method, it will be assumed a relay form of R_i

I regime: $R_i=\operatorname{sign}_i R_i^a,\ t\in(t_i^0,\ t_i^s)$;

II regime: $R_i=\operatorname{sign}_i R_i^d,\ t\in(t_i^s,\ t_i^f)$.

So, the time-optimal control problem can be formulated as follows: Among all controls $R: R_i^d<R_i<R_i^a$ which drive the system (1) under the condition (4) from the initial state (2) to the required terminal state (3), with sequential starting of the joint motions, that one must be found which minimizes the duration T of the handling operation

(5) $$T=t^f-t^0,\ t^0=\min t_i^0,\ t^f=\max t_i^f,\ i=1,\dots,n.$$

For brevity, the controls R above described are called P-controls. The phase trajectories corresponding to these controls do not leave the parallelipiped defined by the initial (2) and final (3) states.

P-control synthesis

In all these considerations we shall have in mind the strong monotony of the functions q_i (condition (4)) in the corresponding definite intervals $(t_i^0,\ t_i^f)$, $(i=1,\dots,n)$. It affords an opportunity of making the following arguments, purposely realizing the P-control synthesis:

a. The switching times t_i^s $(i=1,\dots,n)$ one-to-one correspond to switching values q_i^s of the general coordinates q_i;

b. Let $(k_1, \ldots, k_n)$ be a permutation of $(1, \ldots, n)$, that assigns the order of the joint motions starts. Then the following relation of the start moments t_k^0 holds:

(6) $$t_{k_1}^0 \leq t_{k_2}^0 \leq \cdots \leq t_{k_n}^0, \quad (t_{k_1}^0 = t^0).$$

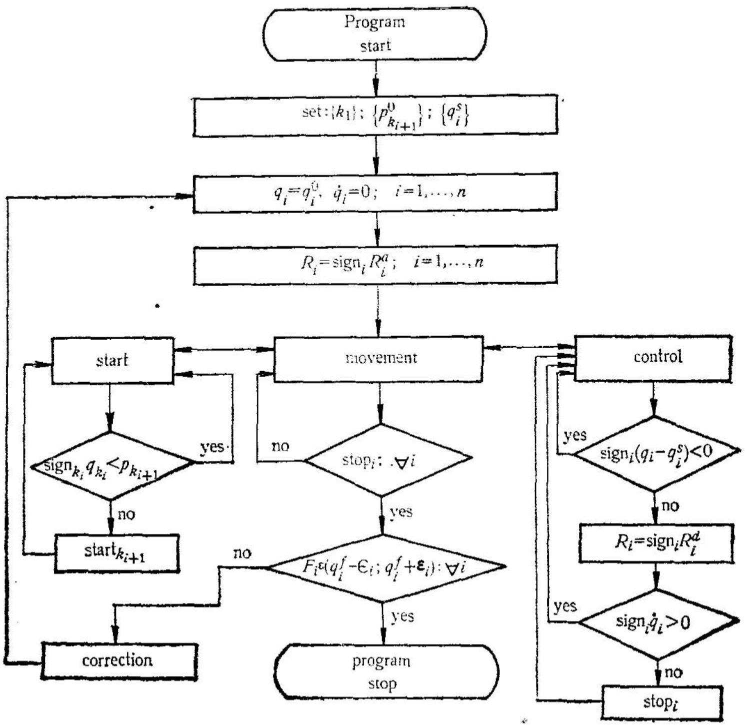

Fig. 2

Moreover, in view of the fact that the manipulator motion is performed continuously, the next conditions are to be fulfilled:

(7) $$t_{k_{i+1}}^0 \in (t_{k_i}^0, t_{k_i}^f), \quad i = 1, \ldots, n-1.$$

Having in mind again the strong monotony of the general coordinates and the conditions (7), we come to the conclusion that the start moments $t_{k_{i+1}}^0$ $(i = 1, \ldots, n-1)$ one-to-one correspond to the values $p_{k_{i+1}}^0$ of the general coordinates q_{k_i}. So, when q_{k_i} gets the values $p_{k_{i+1}}^0$, at the same moment the joint (k_{i+1}) starts.

If the parameters of the system (1) with initial state (2) as well as the permutation $(k_1, \ldots, k_n)$ with the corresponding values $(p_{k_2}^0, \ldots, p_{k_p}^0)$ are fixed, then the final values $q_i(t_i^f) = F_i$, $(i = 1, \ldots, n)$ will depend only on the n-dimensional vector q^s. The moments t_i^f satisfy the conditions $\dot q_i(t_i^f) = 0$. So, the problem of the P-control synthesis reduces to the following shooting equation

(8) $$F(q^s) = q^f.$$

The equation (8), which represents in fact a nonlinear two-point boundary value problem, can be solved using the bisections method.

The algorithm of this method for the P-control synthesis is described with the help of Fig. 2, where MOVEMENT symbolically expresses the manipulator motion from the initial state (2) to a terminal state $(F_1, \ldots, F_n)$, and CONTROL and START are control subroutines. The subroutine CORRECTION performs iterative corrections of the switching values q_i^s until satisfying the accuracy condition

$$F_i(q^s) \in (q_i^f - \varepsilon_i;\ q_i^f + \varepsilon_i),\ i = 1, \ldots, n. \tag{9}$$

The optimal P_0-control with minimal T can be found by a consecutive setting of permutations $(k_1, \ldots, k_n)$ and the corresponding values $(p_{k_2}, \ldots, p_{k_n})$.

3. Illustrative numerical example

With the help of the proposed method for time-optimal control a simple manipulator with three degrees of freedom was synthesized, Fig. 3. The following system of differential equations describes the motion (the third degree of freedom is independent) [7]:

$$\{[J_1 + J_2 + m_2 q_2^2 + M(q_2 + a)^2]\dot{q}_1\}^{\cdot} = R_1(t); \tag{10}$$

$$(m_2 + M)\ddot{q}_2 - [m_2 q_2 + M(q_2 + a)]\dot{q}_1^2 = R_2(t),$$

where: (q_1, q_2) — polar coordinates of the mass centre of the third link 3, M — total mass (mass of the gripper and the tip mass), m_1 — mass of the second link 2, m_2 — mass of the third link 3, J_1 — total momentum of inertia of the links 1 and 2 with respect to the X_3-axis, J_2 — momentum of inertia

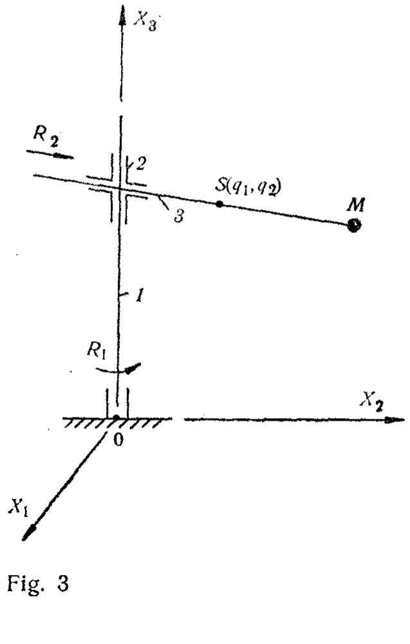

Fig. 3

Table 1

No	(k_1, k_2)	$p^0_{k_2}$	q^s_1	q^s_2	$F_1(q^s_1, q^s_2)$	$F_2(q^s_1, q^s_2)$	$T[s]$
1	(1,2)	0.000	1.150	0.325	1.998	1.009	2.796
2	(2,1)	1.000	1.000	0.500	2.000	1.000	4.295
3	(1,2)	2.000	1.000	0.500	2.000	1.000	3.232
4	(1,2)	0.655	1.237	0.213	1.991	1.002	2.228
5	(1,2)	1.114	1.120	0.365	2.001	1.000	2.180
6	(1,2)	0.912	1.182	0.294	2.003	1.006	2.155
7	(1,2)	1.011	1.151	0.326	2.002	0.998	2.121

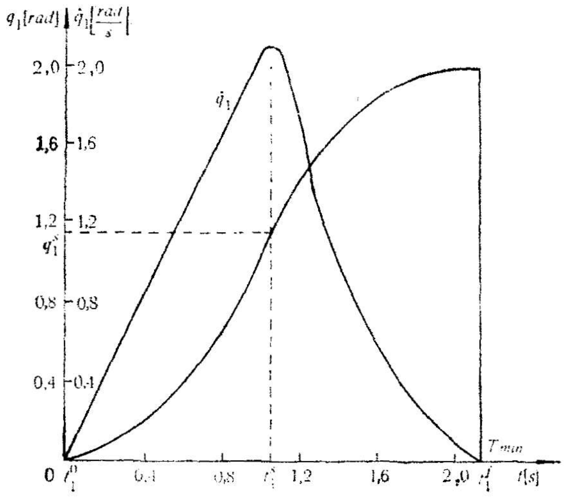

Fig. 4

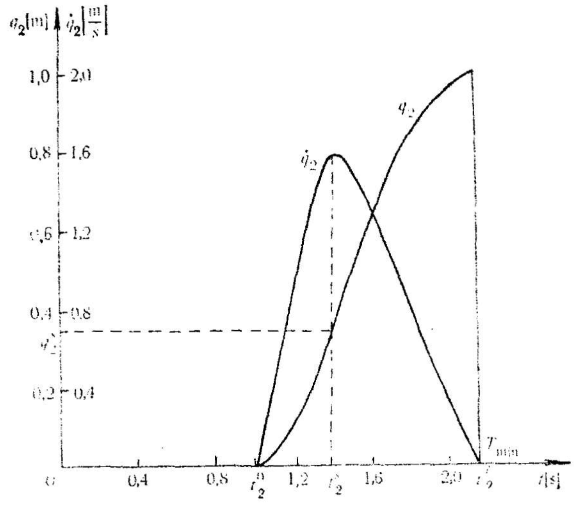

Fig. 5

of the link 3 about the axis which passes through the point S and is parallel to the X_3 — axis, a — distance between the mass centre M and S.

The following parameters are accepted:

$$m_1=210\,[kg],\ m_2=97\,[kg],\ M=100\,[kg],\ a=1\,[m],\ J_1+J_2=193\,[kgm^2];$$

$$R_1^a=-R_1^d=600\,[Nm],\ R_2^a=-R_2^d=500\,[N],\ t^0=0\,[s],\ q_1^0=0\,[rad],$$

$$q_1^f=2\,[rad],\ q_2^0=0\,[m],\ q_2^f=1\,[m];\ \varepsilon_1=0{,}01\,[rad],\ \varepsilon_2=0{,}01\,[m].$$

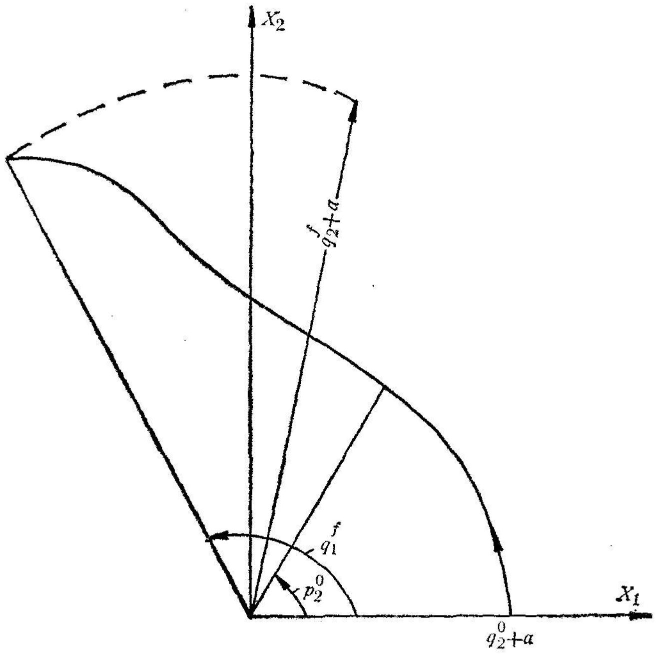

Fig. 6

The time-optimal control synthesis is carried out by means of some values of the variable $p_i^0\,(i=1,2)$ assumed at each iteration. The results obtained are given in Table 1 and the execution of the optimal motion is shown in Figs. 4, 5, 6.

4. Conclusions

A general method for synthesis of time-optimal control for manipulator dynamics in handling operations has been suggested. Satisfaction of the joint constraints imposed on the generalized coordinates is presumed. The method is applicable also in the more general case of smooth nonlinear control functions ($\tilde{R}$ rather than R, Fig. 1). Finally, the presented algorithm can be applied for precise adjustment of $\{p_i^0\}$, $\{q_i^s\}$ using the manipulator controller.

Received 20. X. 1982

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