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Non-Isothermal Free Convection at High Prandtl Number

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1. Introduction

Ostrach [3] ptesented the first numerical results for free convection past a vertical flat plate for the isothermal ($n=0$) case, the governing equations being solved for the single boundary layer for different values of the Plandtl number. It was revealed in this study that the numerical integration becomes much more difficult when the Prandtl number is much greater than unity. Takhar [6] presented a numerical solution for free convection from a semi-infinite plate which is uniformly heated up to a length l from the base and insulated for the rest of its length. Morgan & Warner [2] considered the thermal layer as a boundary-layer within a boundary layer for high Prandtl number, and used the Prandtl number stretching technique as an aid in determining the asymptotic dependence on Prandtl number of the heat transfer coefficients but they did not proceed to solve the resulting equations. Stewartson & Jones [5] introduced the double boundary layer concept for this problem and they solved the isothermal case numerically using 'matching' to find unknown boundary conditions. Kuiken [1] developed a perturbation solution in the isothermal low Prandtl number problem of free convection from a flat plate. Sparrow & Gregg [4] have given a numerical solution of the boundary layer equations of free convection for a non-isothermal vertical flat plate for two families of wall temperature for various values of the Prandtl number using a similarity transformation involving Grashof number. The non-dimensionalisation used in the present analysis is based on an analogy with forced convection but instead of utilising the free stream velocity a typical free convective velocity is used in forming a free convective Reynolds number. A classical boundary layer stretching technique is employed, the normal variable and the normal velocity being stretched by a stretching factor in the non-dimensional transformations. The temperature of the plate is allowed to vary as a simple power n of the distance x from the heated edge of the plate.

2. Governing equations

Consider a vertical flat plate aligned along the x-axis of a 2-dimensional Cartesian frame of reference and y-axis normal to the plate. The plate is maintained at a temperature directly proportional to various powers of the distance

x from the leading edge, which is taken as the origin of the coordinate system. The density of the fluid is allowed to vary in accordance with the equation of state and the resulting buoyancy forces in an otherwise quiescent Boussinesq fluid cause a boundary layer to develop from the leading edge due to the action of free convective velocities. The equations conserving the mass momentum and energy for a steady laminar flow in a boundary layer over a vertical flat plate are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)+\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u, v are the components of velocity along the x and y directions, ν and a are the viscous and thermal diffusivities, β the coefficient of cubical expansion, T the temperature of the fluid and g the acceleration due to gravity. Boundary conditions on this problem are

$$\left.\begin{array}{lll} u=0=v, & T=T_w & \text{at} \quad y=0 \\ u=0, & T=T_\infty & \text{as} \quad y\to\infty \end{array}\right] \tag{4}$$

where T_w and T_∞ are the values of temperature of the fluid at the wall and at infinity respectively. The continuity equation admits of a stream function defined as

$$\left. u=\frac{\partial \psi}{\partial y}; \quad v=-\frac{\partial \psi}{\partial x}\right] \tag{5}$$

3. Transformations

We shall utilise similarity variables defined as follows:

$$\eta=Cyx^{\frac{n-1}{4}}=\left[\frac{Gr}{4}\right]^{1/4}\frac{y}{x} \tag{6}$$

$$\psi=4\nu Cx^{\frac{n+3}{4}}F^*(\eta)=4\nu\left[\frac{Gr}{4}\right]^{1/4}F^*(\eta) \tag{7}$$

$$\theta^*=\frac{T-T_\infty}{T_w-T_\infty} \tag{8}$$

$$C=\left[\frac{g\beta N}{4\nu^2}\right]^{1/4} \tag{9}$$

$$T_w-T_\infty=Nx^n \tag{10}$$

$$Gr=g\beta(T_w-T_\infty)x^3/\nu^2 \tag{11}$$

$$Pr=\nu/a=\sigma \tag{12}$$

Substitution of these similarity parameters into equations (2) and (3) leads to:

$$F^{*'''}+(n+3)F^{*}F^{*''}-(2n+2)(F^{*'})^{2}+\theta^{*}=0 \tag{13}$$

$$\theta^{*''}+Pr[(n+3)F^{*}\theta^{*'}-4nF^{*'}\theta^{*}]=0 \tag{14}$$

with the boundary conditions

$$\left.\begin{aligned} &F^{*'}(0)=0=F^{*}(0),\ \theta^{*}(0)=1\\ &F^{*'}(\infty)=0=\theta^{*}(\infty)\end{aligned}\right] \tag{15}$$

For large Prandtl number flow we introduce inner and outer variables appropriate for the inner (thermal) layer and the outer (viscous) layer as follows:

$$\left.\begin{aligned} &\eta_{\text{inner}}=\xi=\sigma^{1/4}\eta\\ &f(\xi)\ =\sigma^{3/4}F^{*}(\eta)\\ &\theta(\xi)\ =\theta^{*}(\eta)\end{aligned}\right\} \tag{16}$$

$$\left.\begin{aligned} &\eta_{\text{outer}}=\zeta=\sigma^{-1/4}\eta\\ &F(\zeta)\ =\sigma^{1/4}F^{*}(\eta)\end{aligned}\right\} \tag{17}$$

The equations for the thermal inner layer are given by

$$f'''+\theta+\frac{1}{\sigma}[(n+3)ff''-(2n+2)f'^{2}]=0 \tag{18}$$

$$\theta''+(n+3)f\theta'-4nf'\theta=0 \tag{19}$$

with the boundary conditions: $f=f'=0$ at $\xi=0$.

As the temperature field is contained wholly within this layer we have $\theta(\sigma)=1$ and θ tends to zero as ξ tends to infinity. The only equation in the outer region is given by

$$F'''+(n+3)FF''-(2n+2)F'^{2}=0 \tag{20}$$

with F' tending to zero as φ tends to infinity.

4. Parameter perturbation solutions

The equations (18) to (20) are solved by means of parameter expansions in each layer, the additional boundary conditions required are found by considering the functions in the region or domain where both sets of expansions are valid and can be directly compared by considering the contribution to one solution in its region of validity and the variables of the other.

5. Zero-order solution

The inner functions are expanded as follows:

$$f=f_0+\sigma^{-1/2}f_1+\sigma^{-1}f_2+\cdots \tag{21}$$

$$\theta=\theta_0+\sigma^{-1/2}\theta_1+\sigma^{-1}\theta_2+\cdots \tag{22}$$

Substituting these values of f and θ in the equations (18) and (19) gives for the first term

$$(23)\qquad f_0'''+\theta_0=0$$

$$(24)\qquad \theta_0''+(n+3)f_0\theta_0'-4nf_0\theta_0=0$$

We have four boundary conditions for these equations and thus require one further condition. For large values of ξ, f_0 can be expressed as a simple asymptotic series of the form given below:

$$(25)\qquad f_0=A_0+B_0\xi+C_0\xi^2+O(\xi^3).$$

As f_0''' tends to zero when ξ tends to infinity then all the higher coefficients are all equal to zero. In order to obtain a zero-order equation for $F(\zeta)$ we expand it in terms of the Prandtl number,

$$(26)\qquad F=F_0+\sigma^{-1/2}F_1+\sigma^{-1}F_2+\cdots$$

A series expansion of F_0 for small values of ζ can be written as

$$(27)\qquad F_0=a_0+b_0\zeta+c_0\zeta^2+O(\zeta^3).$$

The matching technique can now be employed to obtain the missing boundary condition. The function F is expressed in term of the inner variable ξ and its contribution in the inner region along with the thermal function's asymptotic representation. As ξ approaches infinity f can be represented by

$$(28)\qquad f=A_0+B_0\xi+C_0\xi^2+\sigma^{-1/2}f_1.$$

The contribution of F in the inner region is

$$(29)\qquad F\sim\sigma^{1/2}(a_0+b_0\sigma^{-1/2}\xi+c_0\sigma^{-1}\xi^2+\cdots+\sigma^{-1/2}f_1).$$

For large σ the parametric functions have to be increasing powers of σ. As there is no term of $O(\sigma^{1/2})$ in the inner region then from the relation (28) it follows that $a_0=0$. It also follows that $B_0=b_0$ and $C_0=0$ therefore $f_0\sim A_0+b_0\xi$ and therefore $f''(\infty)=0$ which is the missing boundary condition. The inner equations can now be integrated from $\xi=x_i$ to $\xi=X$ where X is our 'infinity' and x_i is our starting point. From this integration numerical values of the inner functions at $\xi=x$ are obtained. If K and J denote these numerical values we have $f_0(\xi=X)=K$ and $f_0'(\xi=X)=J$.

Therefore $K=A_0+b_0J$; $J=b_0$.

Hence $A_0=K-JX$ and

$$(30)\qquad f_0=b_0\xi+K-JX.$$

The boundary condition for the outer equations can now be written down. As $a_0=0$ we have $F(0)=0$ and from the solution for the function F_0 we have $F_0'(0)=b_0=J$. The other boundary conditions are already known. The method of integration involves series expansion for smaller values of the independent variable. We have

$$(31)\qquad F_0=b_0\zeta+c_0\zeta^2+\frac{1}{3}(n+1)b_0^2\zeta^3+\frac{1}{12}(3n+1)b_0c_0\zeta^4+O(\zeta^5).$$

6. First order solution

The equations for the first order terms are

(32) $$f_1''' + \theta_1 = 0$$

(33) $$\theta_1'' + (n+3)(f_0\theta_1' + f_1\theta_0') - 4n(f_0'\theta_1 + f_1'\theta_0) = 0.$$

An asymptotic expansion for large ξ could be of the form

$$f_1 \sim A_1 + B_1\xi + C_1\xi^2 + \cdots$$

For small ζ we can expand F_1 as

(34) $$F_1 = a_1 + b_1\zeta + c_1\zeta^2.$$

F is now written in terms of ξ and its contributions to the inner region examined.

(35) $$\begin{aligned} F(\zeta = \sigma^{-1/2}\xi) \sim \sigma^{1/2}[&(b_0\sigma^{-1/2}\xi + c_0\sigma^{-1}\xi^2 + \cdots) \\ &+ \sigma^{-1/2}(a_1 + b_1\sigma^{-1/2}\xi + c_1\sigma^{-1}\xi^2 + \cdots) \\ &+ \cdots\cdots\cdots\cdots\cdots\ ]. \end{aligned}$$

As ξ tends to infinity the inner function f has the form

(36) $$f \sim (K - Tx) + b_0\xi + \sigma^{-1/2}(A_1 + B_1\xi + C_1\xi^2 + \cdots).$$

Matching in the common domain gives

$$A_0 = a_1;\ b_1 = B_1;\ c_0 = C_1.$$

Thus the asymptotic form of f for large ξ is

(37) $$f_1 \sim A + b_1\xi + c_0\xi^2.$$

From the numerical solution we have

$$\begin{aligned} f_1(\xi = X) &= A_1 + b_1X + c_0X^2 = S \\ f_1'(\xi = X) &= b_1 + 2c_0X \qquad = R \\ f_1''(\xi = X) &= 2c_0 \qquad\qquad\ = T \end{aligned}$$

Therefore $b_1 = R - TX$. Also

(38) $$F_1'(0) = b_1 = R - TX.$$

Thus the boundary condition on F_1 comes from the solution of f_1 and we already have $a_1 = K - JX$. The constant term A_1 is given by $S - (R - TX)X - \frac{Tx^2}{2}$. The asymptotic expansion for F_1 is as follows:

(39) $$F_1''' + (n+3)(F_0F_1'' + F_1F_0'') - (4n+4)F_0'F_1' = 0$$

for which all the boundary conditions are known. The series for F_1 for small ζ is

(40) $$F_1 = a_1 + b_1\zeta + c_1\zeta^2 + \frac{1}{6}[(4n+4)b_0b_1 - 2(n+3)c_0a_1]\xi^3 + \cdots$$

7. Second order solution

We have for the second order equations

$$f_2''' + (n+3)f_0 f_0'' - (2n+2)f_0'^2 + \theta_2 = 0 \tag{41}$$

$$\theta_2'' + (n+3)(f_0\theta_2' + f_1\theta_1' + f_2\theta_0') \tag{42}$$

$$-4n(f_0'\theta_2 + f_1'\theta_1 + f_2'\theta_0) = 0.$$

The asymptotic form of f_0 for large ξ is $f_0 = A_0 + b_0\xi$. Substituting this value for f_0 in the inertial terms of equation (41) gives $f_2''' - (2n+2)b_0^2 = 0$. An asympt otic series for f_2 for large ξ can be written as follows:

$$f_2 \sim A_2 + B_2\xi + C_2\xi^2 + D_2\xi^3. \tag{43}$$

Comparing the value of f_2''' in these two relations gives $D_2 = \frac{1}{3}(n+1)b_0^2$. Thus the series for f_2 is given by

$$f_2 \sim A_2 + B_2\xi + C_2\xi^2 + \frac{1}{3}(n+1)b_0^2\xi^3. \tag{44}$$

For small ζ; $F_2 = a_2 + b_2\zeta + c_2\zeta^2 + \cdots$
It now remains to express F in inner variables and to examine its contribution in the inner region alongside the asymptotic form of f for large ξ

$$F(\zeta = \sigma^{-1/2}\xi) \sim \sigma^{-1/2}\Big[\Big(b_0\xi\sigma^{-1/2} + c_0\xi^2\sigma^{-1} + \frac{1}{3}(n+1)b_0^2\xi^3\sigma^{-3/2} + \cdots\Big) \tag{45}$$

$$+\sigma^{-1/2}(a_1 + b_2\xi\sigma^{-1/2} + c_1\xi^2\sigma^{-1} + \cdots)$$

$$+\sigma^{-1}(a_2 + b_2\xi\sigma^{-1/2} + c_2\xi^2\sigma^{-1} + \cdots)\Big].$$

This is to be matched with

$$f \sim b_0\xi + A_0 + \sigma^{-1/2}(A_1 + b_1\xi + c_0\xi^2) + \sigma^{-1}(A_2 + B_2\xi + c_2\xi^2 + \cdots). \tag{46}$$

Matching gives $c_1 = C_2$, $a_2 = A_1$ and $b_2 = B_2$. The asymptotic series of f_2 for large ξ is of the form

$$f_2 \sim A_2 + B_2\xi + c_1\xi^2 + \frac{1}{3}(n+1)b_0^2\xi^3. \tag{47}$$

Therefore the missing boundary condition for f_2 follows immediately as

$$f_2''(\xi = X) = 2c_1 + 2b_0^2(n+1)X;$$

X being the infinity point in the integration. From the solution we have

$$f_2(\xi = X) = A_2 + B_2X + c_1X^2 + \frac{1}{3}(n+1)b_0^2X^3 = G$$

$$f_2'(\xi = X) = B_2 + 2c_1X + b_0^2(n+1)X^2 = H$$

$$f_2''(\xi = X) = 2c_1 + 2(n+1)b_0^2X = M$$

where G, H, M are the numerical values of these functions.
The equation for F_2 is completely given by

$$F_2''' + (n+3)(F_0F_2'' + F_1F_1'' + F_2F_0'') \tag{48}$$

$$-(2n+2)(2F_0'F_2' + F_1'^2) = 0$$

8. Solution procedure

The integration of the equations requires series solutions in terms of known boundary conditions and unknown parameters that have to be initially given some arbitrary value. The series are for small ξ or ζ. The integrations are started at $\xi=0.1$ and proceed outwards until the infinity (X) boundary conditions are satisfied within a certain limit. A value of ξ between 4 and 10 was found adequate for the values of n considered. Although there was little change in the functions between 4 and 10 this later figure was selected for the solutions. This figure was about the maximum for high values of n but the solution could be carried to higher values of (X) for smaller and negative values of n. The integrations in the outer zone were similarly carried out and again it was found that the smaller values of n seemed to satisfy the boundary conditions at infinity more quickly in terms of the distance than the larger positive values of n. The values of the functions from the first solution have to be fed into the equations for the second and so on. This causes some amplification of errors present initially, even though later sets of equations are linear as opposed to the nonlinear starting equations. The cases of $n=1.2$, 1.5, and 2 only just satisfied the outer conditions in the smaller values of proved easier to solve. The integration method itself is based on a procedure that solves the two-point boundary value problem for a set of ordinary differential equations using Merson's method and Newton iteration.

9. Results

Heat Transfer:

Fourier law gives an expression for local heat transfer

(49) $$q=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

where k is the thermal conductivity.
The local Grashof number is given by

$$Gr=g\beta(T_w-T_\infty)\,x^3/\nu^2,$$

the local heat transfer coefficient is given by

(50) $$h=q/(T_w-T_\infty)$$

and the local Nusselt number is given by

(51) $$Nu=hx/k.$$

This leads to the relationship

(52) $$Nu/\left[\frac{Gr}{4}\right]^{1/4}=-\theta'(0).$$

In terms of the temperature perturbation functions

(53) $$\frac{Nu}{\left[\frac{Gr}{4}\right]^{1/4}}=-\sigma^{1/4}[\theta_0'(0)+\sigma^{-1/2}\theta_1'(0)+\sigma^{-1}\theta_2'(0)]$$

$$(54) \qquad \frac{q}{C_n} = \sigma^{1/4}[\theta_0'(0) + \sigma^{-1/2}\theta_1'(0) + \sigma^{-1}\theta_2'(0)]$$

where C_n is a factor of the transformation constants and variables. To a first approximation $q \sim O(\sigma^{1/4})$ as given by Mordan & Warner (1956). From equations (53) and (54) we get

$$(55) \qquad Nu/\left[\frac{Gr}{4}\right]^{1/4} = -\theta'(0) = q/C_n.$$

Skin Friction:

Skin friction is given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

in terms of the similarity variables used

$$(56) \qquad \frac{\tau_w}{D_n} = \sigma^{-1/4}[f_0''(0) + \sigma^{-1/2}f_1''(0) + \sigma^{-1}f_2''(0)],$$

where D_n are constants for a particular x depending on n.

Dimensionless Heat Flux Integral:

Integrating the energy equation (3) across a section of the boundary layer yields:

$$(57) \qquad \frac{\partial}{\partial x}\int_0^\infty uT dy = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

This in terms of the dimensionless similarity variables used, becomes

$$(58) \qquad H^* = 16\,\sigma\left(\frac{Gr}{4}\right)^{5/4}\int_0^\infty T\frac{\partial F^*}{\partial y}dy.$$

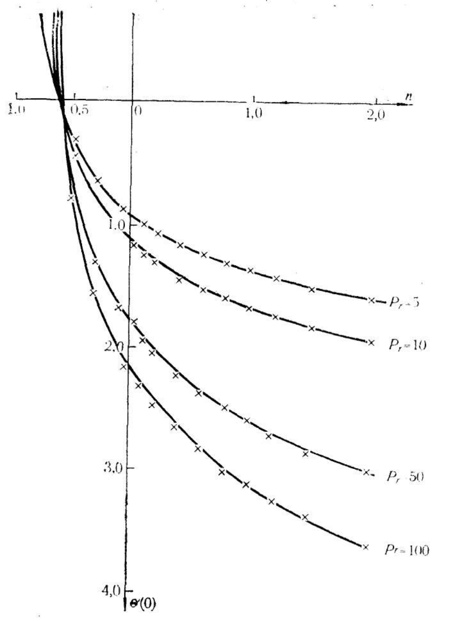
As all temperature differences are confined only to the inner region, we have

$$(59) \qquad \frac{H^*}{\left[\frac{Gr}{4}\right]^{5/4}} = 16\sigma\int_0^\infty \theta\frac{\partial f}{\partial \xi}\,d\xi.$$

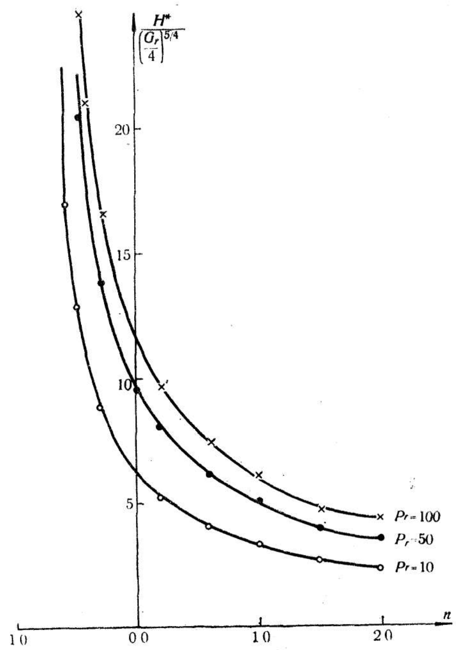
The integral (59) can be found using a modified Simpson's rule involving products of f and θ. If $\frac{\partial T}{\partial y} = 0$ then H^* is independent of x, otherwise it is similar to the local Grashof or Nusselt number. The quantity $H^*/(Gr/4)^{5/4}$ is solely dependent upon the Prandtl number and the exponent, n.

10. Conclusions

Heat transfer is in general increasing with increasing values of the Prandtl number and n. For $n = -0.6$ we have the so-called insulated case when there is no heat transfer at the surface. When n is less than -0.6 the



Фиг. 1. Heat transfer for various values of Prandtl number and n



Фиг. 2. Dumensionless heat flux integral number for various values of the Prandtl number and n

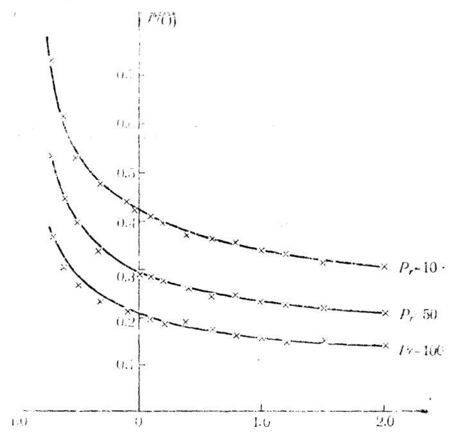
sign of heat transfer reverses which implies that for a certain range of distance ξ from the wall the fluid would be hotter than the wall itself, as seen from Figure 1. The only results available for large Pr are due to Ostrach (1953) for the isothermal flat plate, $n=0$. A comparison of the present values of θ' is made with those of Ostrach in Table 1, which also compares the value of skin friction in these two analyses.

Table 1

Comparison of the skin friction and the heat transfer functions for $n=0$

Prandtl No.	Ostrach		Present Theory	
	$F''(0)$	$H'(0)$	$f''(0)$	$\theta'(0)$
2	0.5713	—0.7165	0.6002	—0.7240
10	0,4192	—1.1694	0.4210	—1.1700
100	0.2517	—2.1910	0.2517	—2.1913
1000	0.1450	—3.9660	0.1449	—3.9654

The agreement is very good for large values of Prandtl number, even the case of $Pr=2$ shows a large measure of agreement. Thus is would seem that as far as the temperature is concerned the theory can be extended to lower values of Prandtl number. The Nusselt-Grashof number relationship is also given by $-\theta'_{(0)}$ so the same agreement would be expected here. An interesting feature of the present analysis is the relative importance of the three pertur-



Фиг. 3. Skin friction for various of Prandtl number and n

bations in determining the characteristics of flow and heat transfer. This was examined by considering the perturbation functions as a percentage of the relevant zero order function. The results of this analysis are given in Table 2. It is seen that at $Pr=2$ the higher order perturbations make their greatest contribution. For large values of Pr the second perturbation is negligible indicating that there is little need in proceeding to higher perturbations. The first and second order perturbations contribute significantly more to the skin friction than the heat transfer functions. The case $n=-0.7$ shows the greatest departure from these trends, the contribution of the higher order terms being somewhat greater. The quantity $H^*/\left[\frac{Gr}{4}\right]^{5/4}$ is a measure of the heat flux and can be treated as a characteristic number of the problem. Figure 2 shows its variation with n for various values of the Prandtl number. The trend of this quantity is similar to that of heat transfer except that there is no change in sign at $n=-0.6$. $H^*/\left[\frac{Gr}{4}\right]^{5/4}$ increases with σ and decreases with n. Takhar (1968) calculated this quantity $H^*/\left[\frac{Gr}{4}\right]^{5/4}$ for different values of σ in a single boundary layer problem for $n=0$ and $n=-0.6$. The results presented here agree very well with Takhar's for $\sigma=10$. For smaller values of σ, there is increasing divergence of values but even for $\sigma=2$ the agreement is still very good.

Table 2

Percentage perturbation contribution for skin friction and heat transfer for $n=1$

Prandtl No.	1st Perturbation		2nd Perturbation	
	θ'_1	f''_1	θ'_2	f''_2
2	15.35	23.25	3.51	9.94
10	6.86	10.39	0.70	1.99
100	2.17	3.29	0,07	0.20
1000	0.68	1.04	0.007	0.02

The skin friction is proportional to the normal derivative of the tangential velocity, $f''(0)$. The values of this quantity obtained in this work are compared with those of Ostrach (1953) for the isothermal case in Table 1. Again agreement is very good for high Prandtl numbers and moderate for low Prandtl numbers. However, it must be remarked that the results for skin friction are significantly poorer for $\sigma=2$ than was the case with heat transfer. The skin friction exhibits the opposite trend to that of heat transfer as σ increases and also decreases as n increases. Figure 3 shows that the cases $n=-0.6$ and $n=-0.7$ do not exhibit abnormal behaviour and just continue the trend for increasingly negative values of n. The contributions of highe order perturbations as a percentage of the zero-order skin friction are give in Table 2. The contributions to skin friction of the perturbations f_1'' and f_2'' are significantly greater than was the case for the heat transfer functions θ_1' and θ_2'. Again if we take σ large enough the need to consider these perturbations is reduced considerably.

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References

1. Kuiken, H. K. J. Fluid Mech. 37, 775, 1969.
2. Morgan, G. W., W. H. Warner, J. Aero. Sci. 23, 937, 1956.
3. Ostrach, S. NACA Report. 1111, 1953.
4. Sparrow, F. M., J. L. Gregg, Trans. ASME, 80, 379.
5. Stewartson, K. J. Aero. Sci. 24, 379, 1958.
6. Takhar, H. K. J. Fluid Mech., 34, 81, 1968.

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