

ANALYSIS OF NATURAL FREQUENCY OF POROUS SIGMOID FUNCTIONALLY GRADED SANDWICH PLATES VIA ANOTHER QUADRILATERAL ELEMENT

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ABSTRACT: This paper presents a computational model for vibration behavior of porous sigmoid functionally graded sandwich (PS-FGSW) plate using another four-node quadrilateral element within the concept of considering the influence of neighboring elements on its. The plate includes three layers. The face layers are fabricated of functionally graded material, and the core is ceramic. Imperfect sigmoid functionally graded sandwich plates with porosity are studied. The numerical results are reached as well as compared with known results, and a good agreement is obtained. Frequency analysis of a complex-shaped porous sandwich plate made of functionally graded material has been performed. The effect of various parameters on the dynamic behavior of the plate, such as values of porosity index, power index, and ratio of thicknesses, has been studied.

KEY WORDS: porosity, sigmoid functionally graded sandwich plate, frequency, four-node quadrilateral element.

1 INTRODUCTION

Nowadays, composite materials play an important role in many manufacturing industries such as space shuttles, airplanes, ships, cars, etc. because of their high strength-to-weight, stiffness-to-weight ratios, excellent resistance to corrosive substances and potentially high overall durability. In which functionally graded material has shown superiority because of its continuity in phase change. Some documents related to functionally graded material research can be mentioned here, such as [1–5]. In [1], the authors presented the free vibration analysis of tapered rectangular functionally graded material plates resting on the two-parameter elastic foundation model. Higher-order shear deformation theory was used to satisfy the traction free boundary conditions on the top and bottom surface of the sigmoid functionally graded material plate. The thickness of the plate linearly varied in the y -direction to both sides

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of the middle plane. The variational Hamilton principle established the equation of motions, and Galerkin-Vlasov's method was used to solve the equation of motion. The work [2] was an attempt to develop a simple and accurate finite element formulation based on continuity of transverse displacement for obtaining and comparing nonlinear transient response of porous functionally graded material sandwich plates and shell panels. The volume fraction for the layers made of functionally graded material was computed according to the power-law or sigmoid model. The sandwich panel was subjected to blast loading and thermal environment considering the heat conduction in thickness direction and the material properties were assumed to be temperature-dependent. An eight-noded isoparametric element along with first-order shear deformation theory was used to develop a finite element model. The vibro-acoustic characteristics of the sandwich plate with temperature-dependent parameters in thermal environment was analyzed and discussed in [3], in which the material properties of the sigmoid function-based functionally graded material plate were assumed to vary according to the two-power law distributions of the volume fractions of the constituents. Based on the first-order shear deformation theory, the vibration and vibro-acoustic coupling control equations for these sandwich plates with simply supported edges were established. The thermal stress induced by the temperature distribution along the thickness direction was involved. By Navier method and fluid-solid coupling conditions, the natural frequency and the sound transmission loss were obtained, which were compared with that computed by the finite element simulation to verify the accuracy of the theoretical results. In [4], it was developed a comprehensive finite element model and numerical studies to illustrate the effect of porosity parameter on critical buckling responses of functionally graded power-based and sigmoid plates and spherical caps subjected to various types of edge compression loads. The model, which followed the kinematics of the first-order-shear deformation theory in many aspects, accounts for a quadratic variation of the transverse shear strains and satisfied the zero condition of the transverse shear stresses at the upper and bottom surfaces of the structure. In the paper [5], the free vibration characteristics of sigmoid functionally graded material Levy-type plates were investigated by developing the dynamic stiffness method through the application of the Wittrick-Williams algorithm, as solution technique. Kirchhoff-Love plate theory and Hamilton principle were utilised to derive the governing equation of motion and associated natural boundary conditions. Due to the actual manufacturing process of this material, porosities appeared and they were the cause of significant changes in mechanical properties. The paper [6] reported new analytic free vibration solutions of non-Lévy-type porous functionally graded material rectangular plates, i.e., those without two opposite edges simply supported, by the symplectic superposition method. The primary advantage of the method was that it did not require any assumption of solution forms, but was

conducted via rigorous mathematical treatments, such as the symplectic space-based separation of variables and the symplectic eigen expansion. The effects of various parameters, e.g., porosity distributions, were also investigated by the analytic solutions obtained. A scaled boundary finite element method framework was introduced in [7] for the bending and free vibration analyses of functionally graded porous plates based on the 3D elastic theory. Both the principle of virtual work and Green's theorem were adopted to derive the governing equation. Translating the 2D mesh along the thickness could be obtained the 3D plate geometry by leaving the scaling center at infinity. Consequently, the spatial dimension of the model was reduced, and the computational efficiency was significantly improved. The 2D high-order spectral element was applied to discretize the middle plane of plate due to its superior accuracy, convergence and so on as in [8–12].

The main objective of this work is to present the flat four-node quadrilateral element for vibration behavior of porous sigmoid functionally graded sandwich (PS-FGSW) plate. The evaluations of the matrices based on the finite element method are built on the concept of considering the influence of neighboring elements on the considered element related to the strategy in literature [13–15] without increasing the total number of degrees of freedom of the system. Numerical examples of PS-FGSW plates are analyzed to verify the effectiveness of this elements. Finally, some concluding remarks are withdrawn.

2 FORMULATIONS

2.1 THE MATERIAL PROPERTIES AND FIRST-ORDER SHEAR DEFORMATION THEORY

Consider a PS-FGSW plate as in Fig. 1. Volume fraction of the layers for sigmoid-law can be expressed as

$$\begin{aligned}
 V^{(1)}(z) &= \frac{1}{2} \left(\frac{z + h/2}{h_p + h/2} \right)^n, & -h/2 \leq z \leq h_p, & \quad h_p = \frac{1}{2} \left(h_1 - \frac{h}{2} \right), \\
 V^{(1)}(z) &= 1 - \frac{1}{2} \left(\frac{z - h_1}{h_p + h_1} \right)^n, & h_p \leq z \leq h_1, \\
 (1) \quad V^{(2)}(z) &= 1, & h_1 \leq z \leq h_2 \\
 V^{(3)}(z) &= 1 - \frac{1}{2} \left(\frac{z - h_2}{h_q + h_2} \right)^n, & h_2 \leq z \leq h_q, & \quad h_q = \frac{1}{2} \left(h_2 + \frac{h}{2} \right) \\
 V^{(3)}(z) &= \frac{1}{2} \left(\frac{z - h/2}{h_q - h/2} \right)^n, & h_q \leq z \leq h/2.
 \end{aligned}$$

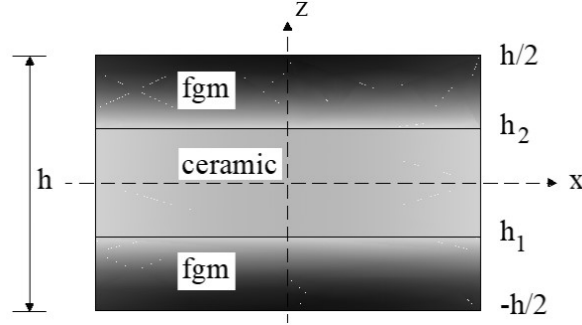


Fig. 1. Functionally graded sandwich plate.

The material properties $M(z)$ for three layers

$$(2) \quad \begin{aligned} M^{(1,3)}(z) &= M_{me} + (M_{ce} - M_{me}) V^{(1,3)}(z) - (M_{ce} + M_{me}) \frac{e}{2}, \\ M^{(2)}(z) &= M_{me} + (M_{ce} - M_{me}) V^{(2)}(z), \end{aligned}$$

in which ce and me denote the ceramic and metal components, respectively; $n \geq 0$ is the volume fraction index; $0 \leq e \leq 1$ is the porosity index; and z is the coordinate along the thickness of the plate.

The displacement field can be expressed by the first-order shear deformation theory (FSDT) as follows:

$$(3) \quad \begin{aligned} u(x, y, z) &= u_0 + z\beta_x \\ v(x, y, z) &= v_0 + z\beta_y \\ w(x, y, z) &= w_0 \end{aligned}$$

u , v , and w are the translational displacements in the x -, y -, and z -directions; u_0 , v_0 , and w_0 correspond to the displacements of the middle plane; β_x and β_y are respectively the rotation of the middle plane about x - and y -axis. The in-plane strain vector can be written

$$(4) \quad \boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_m + z\boldsymbol{\varepsilon}_b,$$

in which

$$(5) \quad \boldsymbol{\varepsilon}_m = \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{Bmatrix}, \quad \boldsymbol{\varepsilon}_b = \begin{Bmatrix} \beta_{x,x} \\ \beta_{y,y} \\ \beta_{x,y} + \beta_{y,x} \end{Bmatrix}$$

The transverse shear strain vector is also given as

$$(6) \quad \boldsymbol{\gamma} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \beta_x - \mathbf{w}_{,x} \\ \beta_y - \mathbf{w}_{,y} \end{Bmatrix}.$$

The constitutive relationship can be expressed as

$$(7) \quad \tilde{\boldsymbol{\sigma}} = \tilde{\mathbf{D}} \tilde{\boldsymbol{\varepsilon}},$$

where

$$(8) \quad \tilde{\boldsymbol{\sigma}} = [\bar{\mathbf{N}} \ \bar{\mathbf{M}} \ \bar{\mathbf{T}}]^T, \quad \tilde{\boldsymbol{\varepsilon}} = [\varepsilon_m \ \varepsilon_b \ \boldsymbol{\gamma}]^T$$

$$(9) \quad \tilde{\mathbf{D}} = [\bar{\mathbf{A}} \ \bar{\mathbf{B}} \ 0; \ \bar{\mathbf{B}} \ \bar{\mathbf{D}} \ 0; \ 0 \ 0 \ \bar{\mathbf{C}}]$$

and $\bar{\mathbf{N}} = [N_x \ N_y \ N_{xy}]$ is the in-plane traction resultant, $\bar{\mathbf{T}} = [T_x \ T_y]$ is the out-of-plane traction resultant and $\bar{\mathbf{M}} = [M_x \ M_y \ M_{xy}]$ is the out-of-plane moment resultant. $\bar{\mathbf{A}}, \bar{\mathbf{B}}, \bar{\mathbf{D}}, \bar{\mathbf{C}}$ is the material constant matrices which are given as follows:

$$(10) \quad (\bar{\mathbf{A}}, \bar{\mathbf{B}}, \bar{\mathbf{D}}) = \int_{-h/2}^{h/2} (1, z, z^2) \mathbf{Q}(z) \bar{z}, \quad \bar{\mathbf{C}} = \int_{-h/2}^{h/2} \mathbf{S}(z) \bar{z}$$

with

$$(11) \quad \mathbf{Q}(z) = \frac{E(z)}{1 - \nu(z)^2} \begin{bmatrix} 1 & \nu(z) & 0 \\ \nu(z) & 1 & 0 \\ 0 & 0 & (1 - \nu(z))/2 \end{bmatrix}$$

$$(12) \quad \mathbf{S}(z) = \frac{E(z)}{2(1 + \nu(z))} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

in which $E(z)$ and $\nu(z)$ represent the change in the value of Young's modulus and Poisson's ratio in terms of the variable z , as in formula (2).

2.2 ANOTHER ELEMENT FOR FREQUENCY ANALYSIS

Let $\mathbf{x}_C$ be a point in a four-node quadrilateral element as shown in Fig. 2. Besides, S_i, S_j, S_k and S_m domains are linked with nodes i, j, k and m . The supporting nodes for the point $\mathbf{x}_C$ in this quadrilateral element involve all nodes of domains S_i, S_j, S_k and S_m . By following [13–15], the total support domain of point $\mathbf{x}_C$ is much larger than the standard finite element method support domain, and the trial solution at point $\mathbf{x}_C$ can be written

$$(13) \quad \tilde{u}(\mathbf{x}) = \tilde{N}_1(\mathbf{x})d_1 + \tilde{N}_2(\mathbf{x})d_2 + \cdots = \sum_{l=1}^{n_{sp}} \tilde{N}_l(\mathbf{x})d_l = \tilde{\mathbf{N}}(\mathbf{x}) \mathbf{d}.$$

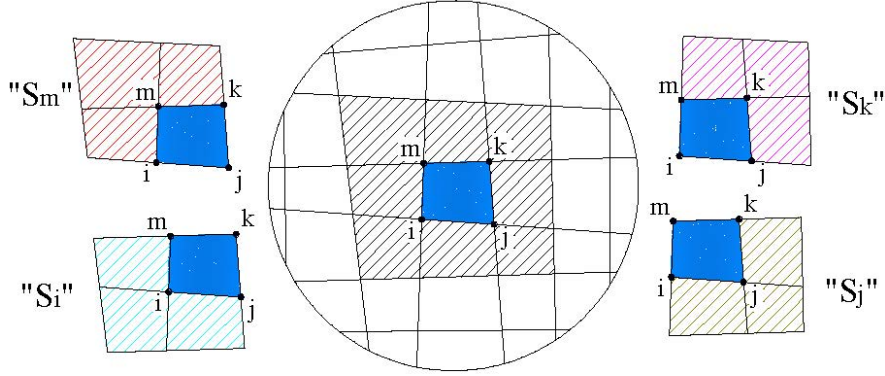


Fig. 2. Schematic sketch of this way for quadrilateral element in 2D.

The shape function $\tilde{N}_l$ is determined

$$(14) \quad \tilde{N}_l = \Upsilon_i N_l^{[i]} + \Upsilon_j N_l^{[j]} + \Upsilon_k N_l^{[k]} + \Upsilon_m N_l^{[m]} + (\Upsilon_{ix} \bar{N}_{l,x}^{[i]} + \Upsilon_{iy} \bar{N}_{l,y}^{[i]}) \\ + (\Upsilon_{jx} \bar{N}_{l,x}^{[j]} + \Upsilon_{jy} \bar{N}_{l,y}^{[j]}) + (\Upsilon_{kx} \bar{N}_{l,x}^{[k]} + \Upsilon_{ky} \bar{N}_{l,y}^{[k]}) + (\Upsilon_{mx} \bar{N}_{l,x}^{[m]} + \Upsilon_{my} \bar{N}_{l,y}^{[m]})$$

with d_i denotes the nodal displacement vector, while $N_l^{[i]}$ is the shape function with respect to node i , and n_{sp} is the total number of the supporting nodes in regard to the point $\mathbf{x}_C$.

The formulation of the average derivative of the shape functions at node i is given

$$(15) \quad \bar{N}_{l,x}^{[i]} = \sum_{el \in S_i} \left(\omega_{el} N_{l,x}^{[i][el]} \right), \quad \bar{N}_{l,y}^{[i]} = \sum_{el \in S_i} \left(\omega_{el} N_{l,y}^{[i][el]} \right),$$

$N_{l,x}^{[i][el]}$ is the derivative of $N_l^{[i]}$ computed in element el and ω_{el} is the weight function of element $el \in S_i$, which is defined

$$(16) \quad \omega_{el} = \Omega_{el} / \sum_{\hat{el} \in S_i} \Omega_{\hat{el}} \quad \text{with} \quad el \in S_i$$

and Ω_{el} being the area of the element el . Furthermore, the functions Υ_i , Υ_{ix} and Υ_{iy} associated with node i satisfy the following conditions:

$$(17) \quad \begin{array}{lll} \Upsilon_i(\mathbf{x}_l) = \delta_{il} & \Upsilon_{i,x}(\mathbf{x}_l) = 0 & \Upsilon_{i,y}(\mathbf{x}_l) = 0 \\ \Upsilon_{ix}(\mathbf{x}_l) = 0 & \Upsilon_{ix,x}(\mathbf{x}_l) = \delta_{il} & \Upsilon_{ix,y}(\mathbf{x}_l) = 0 \\ \Upsilon_{iy}(\mathbf{x}_l) = 0 & \Upsilon_{iy,x}(\mathbf{x}_l) = 0 & \Upsilon_{iy,y}(\mathbf{x}_l) = \delta_{il} \end{array}$$

with l is any one of the indices i, j, k and m

$$(18) \quad \delta_{il} = \begin{cases} 1 & \text{if } i = l \\ 0 & \text{if } i \neq l \end{cases}$$

Three functions Υ_i , Υ_{ix} and Υ_{iy} are given

$$(19) \quad \begin{aligned} \Upsilon_i &= L_i(1 - L_j^2 - L_k^2 - L_m^2) + L_i^2(L_j + L_k + L_m), \\ \Upsilon_{ix} &= -(x_i - x_j)(L_i + 0.5L_k + 0.5L_m)L_iL_j \\ &\quad - (x_i - x_k)(L_i + 0.5L_j + 0.5L_m)L_iL_k \\ &\quad - (x_i - x_m)(L_i + 0.5L_j + 0.5L_k)L_iL_m, \\ \Upsilon_{iy} &= -(y_i - y_j)(L_i + 0.5L_k + 0.5L_m)L_iL_j \\ &\quad - (y_i - y_k)(L_i + 0.5L_j + 0.5L_m)L_iL_k \\ &\quad - (y_i - y_m)(L_i + 0.5L_j + 0.5L_k)L_iL_m. \end{aligned}$$

Other functions can be calculated in the same way by a circulatory permutation of indices i, j, k and m . L_i, L_j, L_k and L_m are the area coordinates of the point $\mathbf{x}_C$ in the quadrilateral element with four nodes i, j, k and m . Three shape functions are complete polynomials, satisfy properties of the partition of unity, and possess Kronecker's delta function property. From the FSDT, the membrane strain at an arbitrary point $\mathbf{x}_C$ can be obtained as follows:

$$(20) \quad \boldsymbol{\varepsilon}_m(\mathbf{x}_C) = \mathbf{B}_m(\mathbf{x}_C)\mathbf{d},$$

where $\mathbf{B}_m$ is the gradient matrix, $\mathbf{d}$ is the vector of degrees of freedom associated with the supporting nodes

$$(21) \quad \mathbf{d}_i = [u_{0i} \ v_{0i} \ w_{0i} \ \beta_{xi} \ \beta_{yi}]^T$$

$$(22) \quad \mathbf{B}_m(\mathbf{x}_C) = \begin{bmatrix} \tilde{N}_{1,x} & 0 & 0 & 0 & 0 & \cdots & \tilde{N}_{n_{sp},x} & 0 & 0 & 0 & 0 \\ 0 & \tilde{N}_{1,y} & 0 & 0 & 0 & \cdots & 0 & \tilde{N}_{n_{sp},y} & 0 & 0 & 0 \\ \tilde{N}_{1,y} & \tilde{N}_{1,x} & 0 & 0 & 0 & \cdots & \tilde{N}_{n_{sp},y} & \tilde{N}_{n_{sp},x} & 0 & 0 & 0 \end{bmatrix}_{3 \times 5n_{sp}},$$

$\mathbf{d}_i$ is the vector of nodal degrees of freedom, n_{sp} is the total number of the supporting nodes in regard to the point $\mathbf{x}_C$. In a similar way, the bending strain over the supporting domain is expressed as

$$(23) \quad \boldsymbol{\varepsilon}_b(\mathbf{x}_C) = \mathbf{B}_b(\mathbf{x}_C)\mathbf{d},$$

where $\mathbf{B}_b$ is the bending gradient matrix given as

$$(24) \quad \mathbf{B}_b(x_C) = \begin{bmatrix} 0 & 0 & 0 & \tilde{N}_{1,x} & 0 & \cdots & 0 & 0 & 0 & \tilde{N}_{n_{sp},x} & 0 \\ 0 & 0 & 0 & 0 & \tilde{N}_{1,y} & \cdots & 0 & 0 & 0 & 0 & \tilde{N}_{n_{sp},x} \\ 0 & 0 & 0 & \tilde{N}_{1,y} & \tilde{N}_{1,x} & \cdots & 0 & 0 & 0 & \tilde{N}_{n_{sp},x} & \tilde{N}_{n_{sp},x} \end{bmatrix}_{3 \times 5n_{sp}}$$

The shear strain is also expressed by

$$(25) \quad \boldsymbol{\varepsilon}_s(\mathbf{x}_C) = \mathbf{B}_s(\mathbf{x}_C) \mathbf{d},$$

where $\mathbf{B}_s$ is the transverse shear gradient matrix given as

$$(26) \quad \mathbf{B}_s(x_C) = \begin{bmatrix} 0 & 0 & \tilde{N}_{1,x} & \tilde{N}_1 & 0 & \cdots & 0 & 0 & \tilde{N}_{n_{sp},x} & \tilde{N}_{n_{sp}} & 0 \\ 0 & 0 & \tilde{N}_{1,y} & 0 & \tilde{N}_1 & \cdots & 0 & 0 & \tilde{N}_{n_{sp},y} & 0 & \tilde{N}_{n_{sp}} \end{bmatrix}_{2 \times 5n_{sp}}.$$

Completely similar to the traditional finite element method, the global stiffness matrix $\mathbf{K}$ can be obtained. Besides, the global mass matrix $\mathbf{M}$ is calculated based on the mass matrix $\mathbf{m}$ as

$$(27) \quad \mathbf{m} = \begin{bmatrix} I_0 & 0 & 0 & I_1 & 0 \\ & I_0 & 0 & 0 & I_1 \\ & & I_0 & 0 & 0 \\ & & & I_2 & 0 \\ sym & & & & I_2 \end{bmatrix}, \quad (I_0, I_1, I_2) = \int_{-h/2}^{h/2} (1, z, z^2) \rho(z) z.$$

Finally, for frequency analysis

$$(28) \quad |\mathbf{K} - \omega^2 \mathbf{M}| = 0.$$

3 NUMERICAL RESULTS AND DISCUSSIONS

In this section, the proposed element is verified through numerical examples. The shear correction factors are equal to $\pi^2/12$. Material properties of this structure are provided in Table 1.

Table 1. Material properties

	Al	Al ₂ O ₃	Si ₃ N ₄	SUS304	ZrO ₂	Ti-6Al-4V
E (GPa)	70	380	322.27	207.78	162	105.69
ρ (kg/m ³)	2707	3800	2370	8166	3000	4427
ν	0.3	0.26	0.24	0.33	0.28	0.33

Table 2. Comparison of normalized frequencies with $n = 2$

e	Method	1/0/1	1/1/1	1/2/1	2/1/2	2/2/1	2/1/1
0	[16]	1.1617	1.3119	1.4155	1.2427	1.3594	1.2797
	Article	1.1598	1.3087	1.4104	1.2401	1.3566	1.2741
0.1	[16]	1.1039	1.2595	1.3718	1.1862	1.3113	1.2262
	Article	1.1085	1.2616	1.3755	1.1921	1.3172	1.2317
0.2	[16]	1.0315	1.2011	1.3256	1.1208	1.2580	1.1632
	Article	1.0386	1.2158	1.3382	1.1301	1.2699	1.1764

The first test problem for a square, simply supported, three-layer plate made of Al/Al_2O_3 with $a/h = 10$ is considered. Note that a is the length of the side of the square plate and h is the plate thickness. The simply supported boundary conditions are expressed as $u_0 = w_0 = 0, \beta_x = 0$ on edges in x -direction and $v_0 = w_0 = 0, \beta_y = 0$ on edges in y -direction, respectively. Table 2 demonstrates a comparison of the calculated dimensionless frequency $\bar{\omega} = \omega a^2 \sqrt{\rho_0/E_0}/h$ with results reported by [16] in relation with the ratios of thicknesses. This table indicates a good agreement between the results obtained and those known in [16]. In article [16], by using a higher-order shear deformation theory, the porous sigmoid functionally graded sandwich plate is considered. But here the author only needs to use FSDT on the expanding influence of neighboring nodes to achieve results approximately with [16].

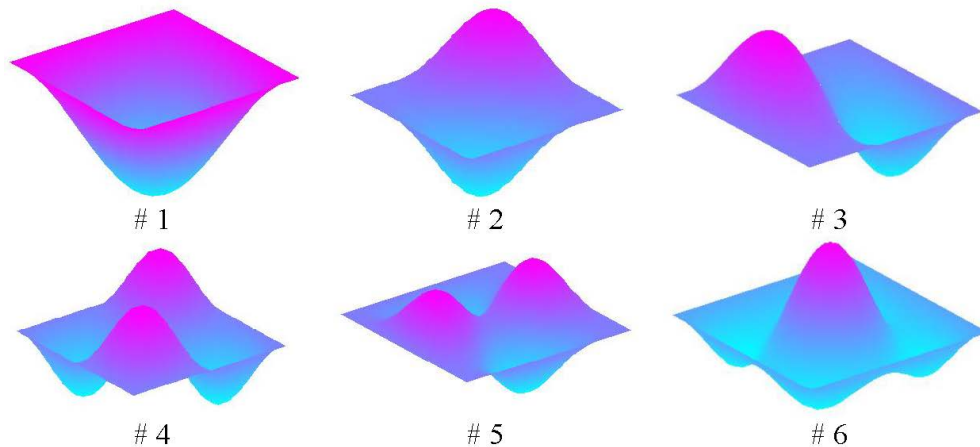


Fig. 3. The first six mode shapes of simply supported plate with ratio of thicknesses 1/1/1, $e = 0.1$ and $n = 2$.

The computational rate of the proposed element will be considered in another study. Figure 3 also depicts the first six mode shapes of this plate with ratio of thicknesses 1/1/1.

By considering the PS-FGSW plate with complex geometry as depicted in Fig. 4, the plate is clamped along the whole boundary, including the hole. The clamped boundary conditions are expressed as $u_0 = w_0 = v_0 = 0$, $\beta_x = \beta_y = 0$ on boundaries, respectively. The geometric parameters of the given plate are: $a/h = 10$, $a/b = 1$, $a_1/a = 0.45$, $b_1/b = 0.30$, $r_1/a = 0.1$ and $r_2/a = 0.2828$ with h is the plate thickness. The effect of volume fraction index n on normalized fundamental frequency for PS-FGSW (Al/Al₂O₃) plates with sigmoid porosity distribution is shown in Table 3. This table shows that the natural frequencies decrease while the volume fraction index n increases for all the considered ratios of thicknesses. The normalized frequencies of the plates with ratio 1/2/1 are greater than corresponding frequencies of the plates with another ratios. For ratio 1/0/1, the values of the normalized frequencies less than those obtained from other ratios. This is explained by the lower rigidity of the plate with ratio 1/0/1. Figure 5 further clarifies this statement.

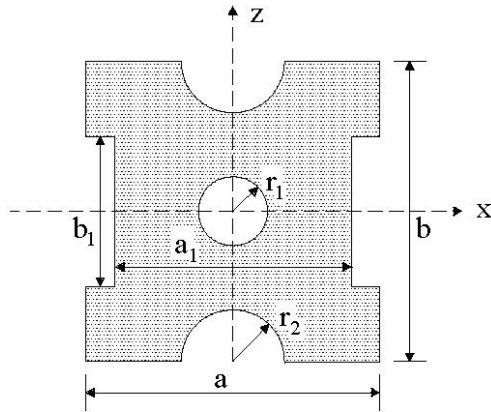


Fig. 4. PS-FGSW plate with complex geometry.

Table 3. Normalized natural frequency of clamped plates with complex geometry

n	1/0/1	1/1/1	Al/Al ₂ O ₃ , $e = 0.2$			
			1/2/1	2/1/2	2/2/1	2/1/1
0	9.5311	9.6664	9.8678	9.5970	9.8191	9.7188
1	8.1234	8.8903	9.4369	8.5357	9.1563	8.7437
2	7.5922	8.6557	9.3172	8.1862	8.9571	8.4243
5	7.1309	8.4749	9.2284	7.8987	8.8009	8.1604
10	6.9921	8.4238	9.2027	7.8241	8.7559	8.0891

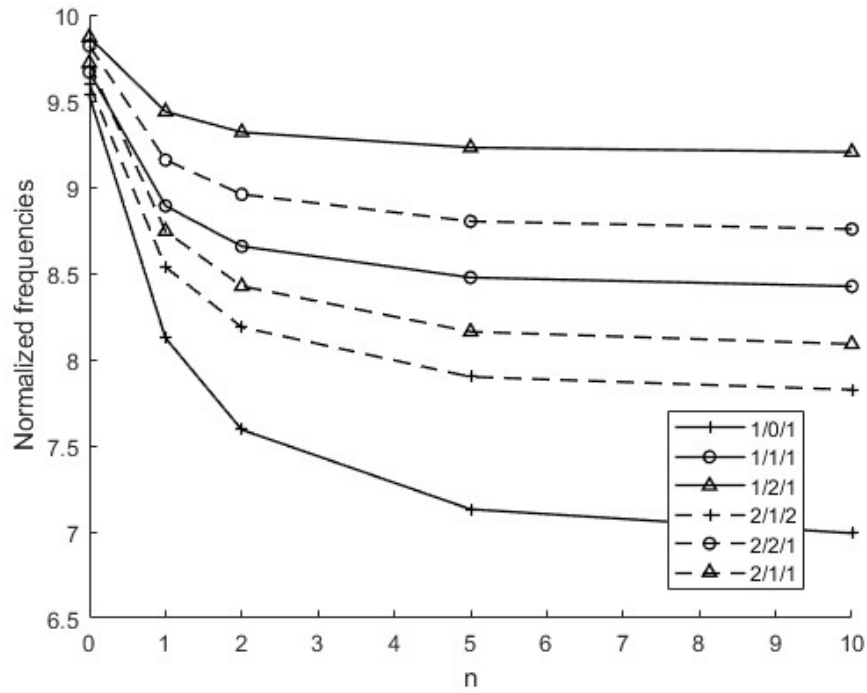


Fig. 5. Normalized natural frequencies of clamped PS-FGSW plates with complex geometry, several ratios of thicknesses and $e = 0.2$.

Table 4. Comparison of the normalized natural frequency for various alloys

e	2/1/2		
	Al/Al ₂ O ₃	Si ₃ N ₄ /SUS304	ZrO ₂ /Ti-6Al-4V
0.0	8.6220	8.3902	6.9018
0.1	8.4512	8.3781	6.8282
0.2	8.1463	8.3415	6.7669
0.3	7.8171	8.2927	6.7055
0.4	7.3049	8.2073	6.5951
0.5	6.4878	8.0976	6.4356
0.6	4.9634	7.8659	6.2270

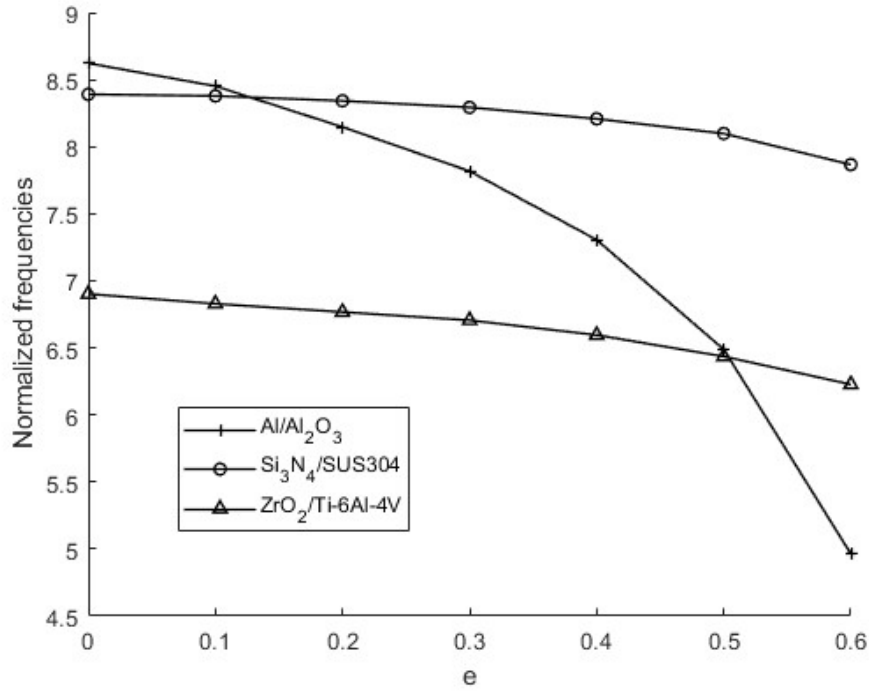


Fig. 6. Comparison of normalized natural frequencies with $n = 2$, ratio of thicknesses 2/1/2, complex geometry, several alloys and porosity index e .

In Fig. 6, the effect of the porosity index e on natural frequencies of the clamped sandwich plate (2/1/2) for various alloys ($\text{Al}/\text{Al}_2\text{O}_3$), ($\text{Si}_3\text{N}_4/\text{SUS304}$) and ($\text{ZrO}_2/\text{Ti-6Al-4V}$) are presented. It should be noted that for all considered functionally graded materials, the porosity index e totally affects the frequencies, especially rapidly reducing the frequency value for the ($\text{Al}/\text{Al}_2\text{O}_3$) alloy as shown in Table 4, respectively.

4 CONCLUSION

In this paper, the presented element is applied to free vibration analysis of a porous sigmoid functionally graded sandwich plate in the framework of the FSDT. The evaluations of the matrices based on the finite element method are built on the concept of considering the influence of neighboring elements on the considered element without increasing the total number of degrees of freedom of the system. Numerical examples have been carried out, and the presented element is found to yield satisfactory results in comparison with other available numerical results using different methods. The success of it provides further demonstration of efficient flat quadrilateral elements

for vibration analysis. Last but not least, extensive analysis regarding the advantages and disadvantages of the proposed element will be presented in the next study. In addition, the intention to extend the method to nanostructure analysis will also be introduced.

CONFLICT OF INTEREST

The author declares no conflict of interest.

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