

## ACOUSTIC WAVES MODELING FLOW IN RAREFIED GAS BETWEEN STATIONARY CYLINDERS AT OSCILLATION INHOMOGENEOUS INNER CYLINDER WALL TEMPERATURE

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[Received: 9 December 2024. Accepted: 23 May 2025]

doi: <https://doi.org/10.55787/jtams.25.55.2.133>

**ABSTRACT:** The paper aims at presenting the detailed interplay between acoustic waves, fluid flow, and self-organization phenomena in a rarefied gas environment. The study specifically focuses on the dynamics between two stationary cylinders with final length, where the inner cylinder features a non-uniform temperature distribution along its surface. The emergence of acoustic waves into this rarefied gas system is the primary interest of the current study. Moreover, the article is about to explore the role of acoustic waves in shaping fluid flow patterns.

Acoustic waves are characterized by variations in pressure and density fields. The significance of these waves is particular when it comes to study rarefied gases. As the article proves, the acoustic waves can influence significantly the overall flow patterns and transport phenomena. Simultaneously, the research explores self-organization phenomena, which refers to the spontaneous emergence of ordered structures or patterns in the system. The interplay between acoustic waves, rarefied gas flow, and temperature distribution along the inner cylinder contributes to the self-organization dynamics within this complex environment. Therefore, the proposed model and frequency processing can be implemented used in the modeling and design of Pirani gauge.

**KEY WORDS:** Pirani gauge, Direct Simulation Monte Carlo method, ANSYS Fluent.

### 1 INTRODUCTION

Acoustic wave propagation in rarefied gases confined between stationary coaxial cylinders, where the inner cylinder wall has an inhomogeneous temperature distribution, is a fascinating and complex problem in gas dynamics. This scenario is frequently noticed in environments where the gas is rarefied, which means that the mean free path of the gas molecules is comparable to the characteristic dimensions of the

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system. Acoustic waves in a fluid are pressure disturbances that propagate through the medium due to the compressibility of the gas [1–5]. In a rarefied gas, where the continuum assumption breaks down, the propagation of these waves is influenced by the non-equilibrium effects and the molecular nature of the gas. When an inner cylinder in a coaxial setup has an inhomogeneous temperature distribution, it can generate localized thermal gradients, which, in turn, induces acoustic waves in the gas. These waves can interact with the flow, altering the temperature, pressure, and density fields in complex ways. In rarefied gas conditions, the interactions between gas molecules and the boundary surfaces (like the cylinder walls) become significant [6–8]. The Knudsen number, which compares the mean free path of gas molecules to the characteristic length scale of the system, is a critical parameter. When the Knudsen number is high, gas flows are dominated by molecular collisions rather than continuous fluid motion. Acoustic waves in a rarefied gas behave differently from those in a continuum fluid. In rarefied conditions, the wave speed, attenuation, and interaction with boundaries can be significantly affected by the mean free path of the molecules and the non-equilibrium nature of the flow. The inhomogeneous temperature distribution on the inner cylinder's surface creates spatially varying thermal fields in the gas [9, 10]. These temperature gradients can induce thermal creep flows and generate acoustic waves that propagate through the gas. The resulting acoustic waves are influenced by the temperature distribution, leading to a complex interplay between thermal and acoustic effects [11–14]. The interaction of acoustic waves with the boundaries (the surfaces of the cylinders) is also affected by the rarefied nature of the gas. Boundary conditions need to account for phenomena such as velocity slip and temperature jump, which occur when the mean free path of the molecules is comparable to the characteristic dimension of the system [4, 8, 15]. The possibility of using microchannel with SAWs as a microscale gas pump is studied in [16].

In our papers [17, 18] we studied a rarefied monatomic gas between two coaxial, stationary, unconfined cylinders (one dimensional, axis-symmetrical problem). The temperature of the outer cylinder is constant and equal to the gas equilibrium temperature, while the temperature of the inner cylinder varies in time. The inner cylinder (filament) is subjected to periodically heating-cooling cycle.

This paper showcases that it is possible for the gas within cylinders at finite length of cylinders containing gas and non-constant filament temperature (two dimensional axis-symmetrical problem) to self-organize under specific circumstances. Once having obtained the information about velocity field, temperature and density, and hence the pressure, we can assess the reliability of the gauge and the frequency of the oscillation through which it is achieved. Rarefied gas dynamics describes gas behavior in regimes where the continuum assumption breaks down, typically characterized by the Knudsen number. DSMC is a probabilistic, particle-based method that simulates

gas molecules individually and tracks their motion and collisions. It is particularly useful for high-Knudsen-number regimes where the continuum assumption is invalid. The statistical model involves dimensionless parameters with scales whose values are determined in accordance with the real values of the continuum. Continuum models, such as the Navier-Stokes-Fourier equations, describe gas dynamics using macroscopic quantities (velocity, pressure, density) and assume a locally equilibrium distribution. The DSMC method is used for Knudsen numbers greater than 0.001, while the Navier-Stokes Fourier is used for Knudsen numbers less than 0.1. The studies were performed in a rarefaction regime suitable for both methods.

When the inner cylinder has a step-wise temperature change, gas molecules gain different amounts of energy at different locations. Molecules near hotter regions move faster than those near colder regions, creating a net gas motion along the temperature gradient. Circulating gas motion forms around the step discontinuity is inhomogeneous temperature effect.

## 2 FORMULATION OF THE PROBLEM

We study a rarefied gas between two coaxial cylinders with finite length - the Taylor-Couette problem. This domain is considered as part of the flow between two infinite cylinders. Thus the recurring periodic structures of the flow (self-organization) can be investigated. For this purpose, periodic boundary conditions are set in the two planar sections bounding the cylinders.

The following notations are used to describe the rarefied gas flow:  $\rho$  is the density,  $T$  is the temperature,  $P$  is the thermodynamic pressure and  $(u, v, w)$  is the velocity components in  $(r, \phi, z)$  directions. The inner cylinder has radius  $R_1$  and temperature  $T_1(z)$  ( $z$  – the coordinate at the cylinder axis) and the outer cylinder –  $R_2$  and  $T_2$ . The two cylinders are stationary.

### 2.1 CONTINUOUS MODEL (NSF) AND NUMERICAL SIMULATION

The continuous model is based on the Navier-Stokes-Fourier (NSF) equations for compressible fluid, completed with the equations of continuity and energy transport. The NSF equations for axisymmetric compressible flow are [19]

$$\begin{aligned}
 (1) \quad & \frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial (r \rho u)}{\partial r} + \frac{\partial (\rho w)}{\partial z} = 0, \\
 (2) \quad & \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial P}{\partial r} - (\nabla \cdot \tau)_r, \\
 (3) \quad & \rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial P}{\partial z} - (\nabla \cdot \tau)_z.
 \end{aligned}$$

The nonzero stress tensor components [19] can be expressed as

$$(4) \quad \tau_{rr} = -\mu \left[ 2 \frac{\partial u}{\partial r} - \frac{2}{3} \left( \frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial w}{\partial z} \right) \right],$$

$$(5) \quad \tau_{\varphi\varphi} = -\mu \left[ 2 \frac{u}{r} - \frac{2}{3} \left( \frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial w}{\partial z} \right) \right],$$

$$(6) \quad \tau_{zz} = -\mu \left[ 2 \frac{\partial w}{\partial z} - \frac{2}{3} \left( \frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial w}{\partial z} \right) \right],$$

$$(7) \quad \tau_{zr} = -\mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right).$$

The equation of energy transport

$$(8) \quad \rho c_P \frac{DT}{Dt} = \text{div} (\lambda \text{grad} T) - p \text{div} \mathbf{V} + \mu \Phi.$$

The dissipation function  $\Phi$  in the equation of energy transport is

$$(9) \quad \Phi = 2 \left[ \left( \frac{\partial u}{\partial r} \right)^2 + \left( \frac{u}{r} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right)^2 - \frac{2}{3} \left[ \frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial w}{\partial z} \right]^2.$$

For a perfect monatomic gas, the viscosity and the heat conductivity coefficients read as [6, 7, 19, 20]:

$$(10) \quad \mu = \mu(T) = C_\mu \rho_0 l_0 V_0 \sqrt{T}, \quad C_\mu = \frac{5}{16} \sqrt{\pi},$$

$$(11) \quad \lambda = \lambda(T) = C_\lambda c_P \rho_0 l_0 V_0 \sqrt{T} C_\lambda = \frac{15}{32} \sqrt{\pi}.$$

The NSF model is completed with the equation of state for ideal gas

$$(12) \quad P = \rho RT.$$

The governing equations are normalized by using the following scales: for density  $\rho_0 = mn_0$  ( $m$  is the molecular mass,  $n_0$  – the average number density), for velocity  $V_0 = \sqrt{2RT_0}$ ,  $R$  is the gas constant, for length – the distance between the cylinders  $L = R_2 - R_1$ , for time  $t_0 = L/V_0$ , for temperature  $T_0$  – the wall temperature of outer cylinder. The Knudsen number is  $\text{Kn} = l_0/L$ , where the mean free path is  $l_0$  and  $\gamma = c_P/c_V = 5/3$  ( $c_P$  and  $c_V$  are the heat capacities at constant pressure

and constant volume respectively). In this way, the characteristic number  $\text{Kn}$  and the constants  $C_\mu$  and  $C_\lambda$  take part in the dimensionless model.

For the problem, formulated first-order slip boundary conditions are imposed at both walls, which can be written directly in dimensionless form as follows [21]:

$$(13) \quad u = 0, \quad w = 0,$$

$$(14) \quad T \pm \zeta_T \text{Kn}_{\text{local}} \frac{\partial T}{\partial r} = \bar{T}_i$$

at  $r = R_i, i = 1, 2$ .

The boundary conditions are periodic at the left ( $z = 0$ ) and right ( $z = L_G$ ) boundaries of both cylinders (15). The profiles of the macrocharacteristics values such as pressure, velocity, temperature are identical in these two cross sections:

$$(15) \quad \begin{aligned} u(r, t) |_{z=0} &= u(r, t) |_{z=L_G} ; \\ w(r, t) |_{z=0} &= w(r, t) |_{z=L_G} ; \\ T(r, t) |_{z=0} &= T(r, t) |_{z=L_G} . \end{aligned}$$

For brevity, the same symbols  $r, t, P, T, u, w$  and  $R_i, i = 1, 2$  are used for the corresponding dimensionless variables. The dimensionless density is denoted with  $\bar{\rho}$ .

For diffuse scattering we have used the temperature jump coefficients  $\zeta_T = 2.1904$  calculated, respectively in [4, 22], from the kinetic BGK equation (a form of the collision integral proposed by Bhatnagar, Gross and Krook). The boundary conditions (14) are modeled by using the local Knudsen number  $\text{Kn}_{\text{local}}$ .

$$(16) \quad \text{Kn}_{\text{local}} = \frac{l}{L} = \left( L \sqrt{2\pi} \sigma^2 \frac{\rho_0}{\rho} n_0 \right) = \frac{\text{Kn}}{\bar{\rho}} .$$

In (16) with  $l$  is denotes the local mean free path,  $\sigma$  – molecular diameter and  $\rho$  – is the dimensional density. For details see [9].

Following [4, 5, 8] we introduce a Stokes number for temperature oscillations  $\beta = \sqrt{\omega L^2 / \nu}$ , where  $\nu$  is kinematic viscosity and  $\omega$  – temperature circular frequency. The relation between dimensionless temperature frequency  $\bar{\omega}$  and Stokes number  $\beta$  can be written, using (10)

$$(17) \quad \bar{\omega} = C_\mu \text{Kn} \beta^2 .$$

The dimensionless value of thermodynamic pressure  $P_0$  corresponding to initial equilibrium state can be calculated as

$$(18) \quad P_0 = \frac{1}{2} \rho_0 T_0 = \frac{1}{2} .$$

In (18)  $\rho_0$  and  $T_0$  are the equilibrium dimensionless density and temperature.

## 2.2 DIRECT SIMULATION MONTE CARLO (DSMC) METHOD

The gas considered is simulated as a stochastic system of  $N$  particles [23, 24]. All quantities used are non-dimensional, so that the mean free path at equilibrium is equal to 1. The basic steps of simulation are as follows:

- A. The time interval  $[0; \hat{t}]$  over which the solution is found, is subdivided into subintervals with step  $\Delta t$ .
- B. The space domain is subdivided into cells with sides  $\Delta z, \Delta r$ .
- C. Gas molecules are simulated in gap  $G$  using a stochastic system of  $N$  points (particles), having position  $x_i(t), r_i(t)$  and velocities  $(\xi_z^i(t), \xi_r^i(t), \xi_\phi^i(t))$ .
- D.  $N_m$  particles are located in the  $m$ -th cell at any given time. This number varies during the computer simulation by the following two stages:

Stage 1. Binary collisions in each cell are calculated, whereas particles do not move. Collision modeling is realized using Bird's scheme "no time counter" [23] which follows next steps:

- Compute Maximum Collision Rate
- Determine the Number of Collisions
- Select Random Particle Pairs and accept collisions to depend on the relative velocity of the two particles
- Compute Post-Collision Velocities. If a collision occurs, update the particle velocities using hard-sphere model collision dynamics.

Stage 2. Particles move with new initial velocities, acquired after collisions and no external forces act on particles. No collisions are accounted for at this stage.

- E. Stage 1 and Stage 2 are repeated until  $t = \hat{t}$ .
- F. Flow macro-characteristics (density, velocity, temperature) are calculated as time-averaged.
- G. Boundary conditions are diffusive at the cylinder walls and periodic at axis  $Oz$ .

### 3 NUMERICAL RESULTS

The ANSYS Fluent density based solution system was used for the numerical solution of the continual model in a dimension form. A 3D model with symmetry about the  $rOz$  plane [25], was used, for pressure-velocity coupling the SIMPLE scheme, for pressure-PRESTO! [26] schemes are applied. DSMC method is based on the 2D flow geometry (axis symmetrical flow) in a dimensionless form.

In our studies grid for DSMC method is  $\Delta z = \Delta r = 0.5$  free molecular path and for continuous model is  $\Delta z = \Delta r = 3.75 \times 10^{-6}$  m  $\approx 0.305$  free molecular path. We study monatomic argon (Ar) gas with pressure  $p_0 = 6.43 \times 10^{-2}$  bar and temperature  $T_0 = 300$  K. The flow parameters are: the most portable speed –  $V_P = \sqrt{2k_B T/M} = 353.38$  m/s, the free molecular path –  $l_0 = \frac{k_B T}{\sqrt{2}\pi\sigma^2 N_A P} = 1.23 \times 10^{-5}$  m, the molar mass –  $M = 40$  amu, cross section –  $\sigma = \pi d^2 = 3.63 \times 10^{-19}$  m, the inner cylinder radius –  $R_1 = 0.01 \times 10^{-3}$  m, the outer cylinder radius –  $R_2 = 0.625 \times 10^{-3}$  m, the Knudsen number  $Kn = l_0/(R_2 - R_1) = 0.02$ , the cylinder length –  $L_G = 8(R_2 - R_1) = 4.92 \times 10^{-3}$  m, the outer cylinder temperature –  $T_2 = 300$  K and the inner cylinder temperature  $T_{W1}$  with profile shown in Fig. 1.

$$(19) \quad T_{W1} = T_{W1,0} [1 + A \sin |(\omega t)|] , \quad A = 0.1 ,$$

where  $T_{W1,0}$  is the temperature profile from Fig. 1. The amplitude value is constant throughout the inner cylinder. The results are presented for two values of the parameter  $\beta = 2$  at relatively low frequencies of oscillation of the profile of Fig. 1 at 4 times higher Stokes number  $\beta = 8$ . (The circular frequency  $\omega$  is 16 times higher).

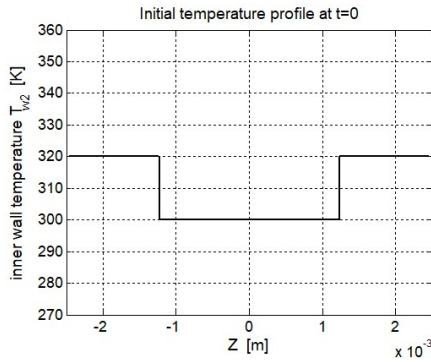


Fig. 1. The inner cylinder temperature profile at the initial moment.

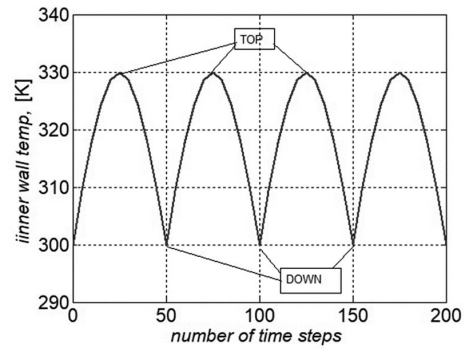


Fig. 2. The time variation of the inner cylinder temperature at  $Z = 0$ .

The numerical results are presented, using the dimensionless coordinates  $z$  and  $r$ , calculated as follows:

$$(20) \quad z = \frac{Z}{R_2 - R_1}, \quad r = \frac{R - R_1}{R_2 - R_1},$$

where  $Z$  and  $R$  are the dimensional axial radial coordinates in [m].

When considering a rarefied gas confined between two coaxial cylinders, where the inner cylinder has an inhomogeneous oscillating temperature distribution, the resulting dynamics of the gas can be quite complex. The influence of this oscillating temperature on the gas's temperature, density, and pressure distribution is governed

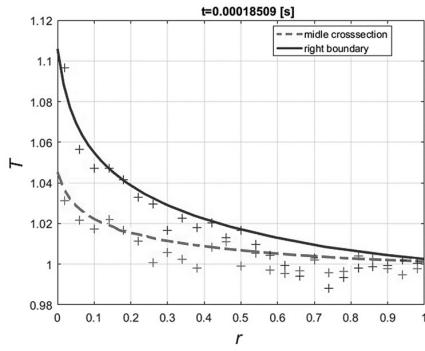


Fig. 3. The temperature profile at the first temperature maximum,  $\beta = 2 + \text{DSMC}$ , - NSF.

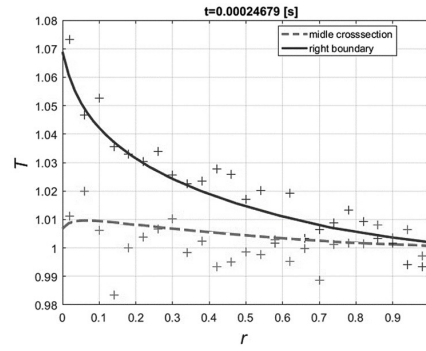


Fig. 4. The temperature profile at the first temperature minimum,  $\beta = 2 + \text{DSMC}$ , - NSF.

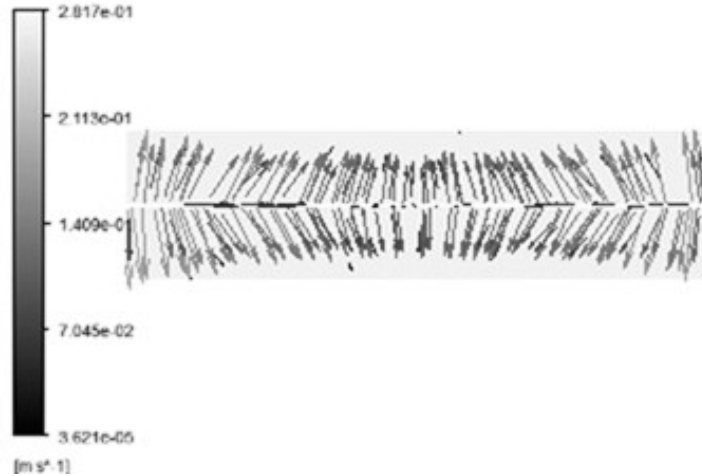


Fig. 5. Velocity vector field at the first temperature maximum  $\beta = 2$ .

by several factors, including the nature of the temperature oscillation, the degree of rarefaction, and the specific geometry of the system.

The inhomogeneous oscillating temperature of the inner cylinder will cause periodic variations in the temperature of the gas near the inner surface. Due to the rarefied nature of the gas, heat transfer will occur primarily through molecular collisions rather than convection, leading to a non-uniform temperature distribution across the gap between the cylinders (Fig. 3 and Fig. 4).

Due to the temperature gradient induced by the oscillating inner cylinder temperature, a thermal creep flow may develop as shown in Fig. 5. This is a flow driven by the temperature gradient, where gas molecules tend to move from colder to hotter regions along the surface. In rarefied gases, this effect can be significant and can alter the density distribution around the inner cylinder (Fig. 6 and Fig. 7).

In studied cases, taking into account the final length of the cylinders, the self-organization of the flow for the first period of the pulsations shows a relatively intense

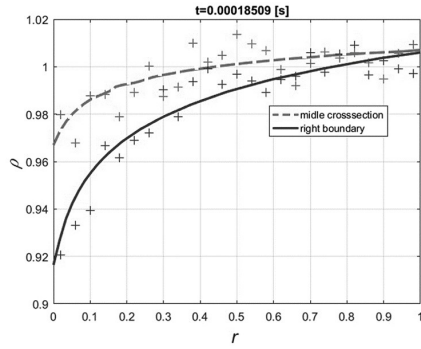


Fig. 6. The density profile at the first temperature maximum,  $\beta = 2 + \text{DSMC} - \text{NSF}$ .

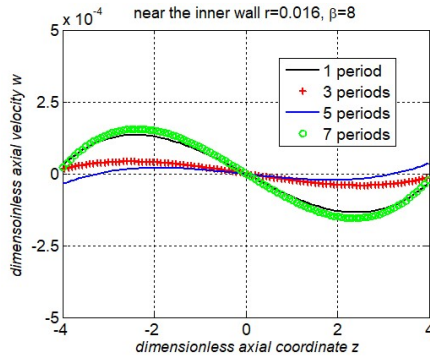


Fig. 8. The axial velocity for  $\beta = 8$  at 4 temperature maximums near the inner cylinder.

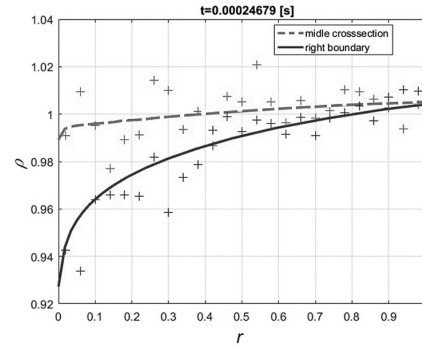


Fig. 7. The density profile at the first temperature minimum,  $\beta = 2 + \text{DSMC} - \text{NSF}$ .

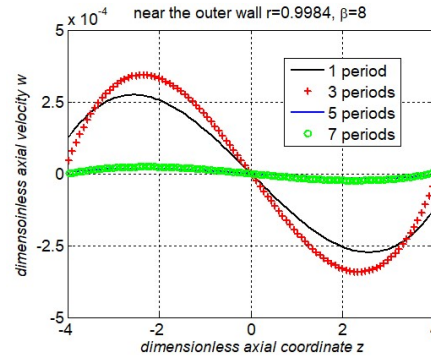


Fig. 9. The axial velocity for  $\beta = 8$  at 4 temperature maximums near to outer cylinder.

movement of the gas phase in the direction of the axis of symmetry, which, however, quickly changes and after the second cycle, draws near sustained oscillations. The presented results were obtained after a sufficient number of oscillation cycles (from 10 to 20) to reach sustained oscillations. This is also confirmed by the low value of the axial velocity (Fig. 8 – Fig. 13). The velocity and pressure figures do not show data from the statistical method because the values variety are of order  $10^{-3}$ , which is within the accuracy of the DSMC method which is of order  $10^{-2}$ .

The pressure in the gas will vary in response to the temperature oscillations, following the ideal gas law (12). Oscillations in temperature lead to oscillations in pressure, but these are also influenced by the density variations and the gas's ability to transmit these pressure changes across the domain. In a rarefied gas, the pressure distribution may not instantaneously adjust to temperature changes due to the finite

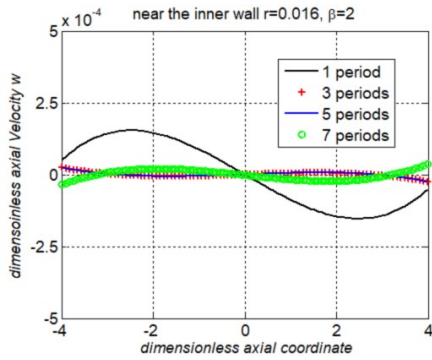


Fig. 10. The axial velocity for  $\beta = 2$  at 4 temperature maximums near the inner cylinder.

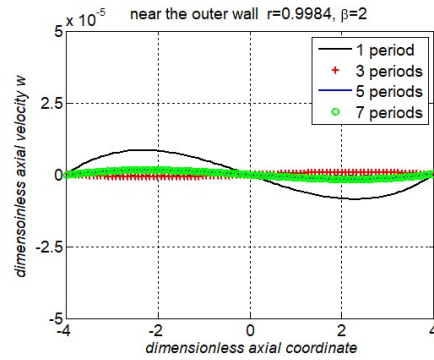


Fig. 11. The axial velocity for  $\beta = 2$  at 4 temperature maximums near the outer cylinder.

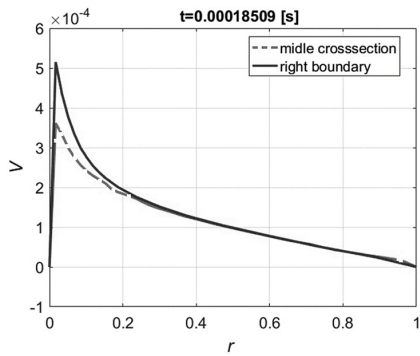


Fig. 12. The radial velocity at the first temperature maximum,  $\beta = 2$  – NSF.

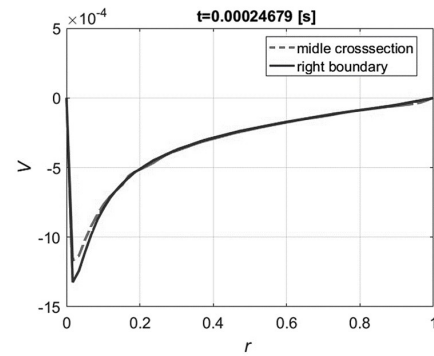


Fig. 13. The radial velocity profile at the first temperature minimum,  $\beta = 2$  – NSF.

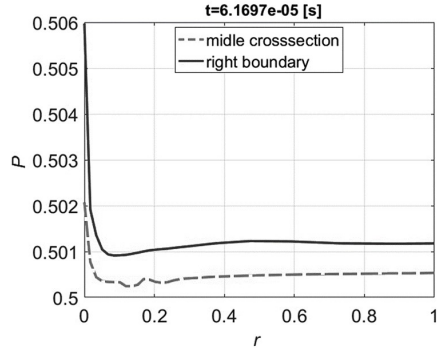


Fig. 14. The pressure profile at the first temperature maximum  $\beta = 2 - \text{NSF}$ .

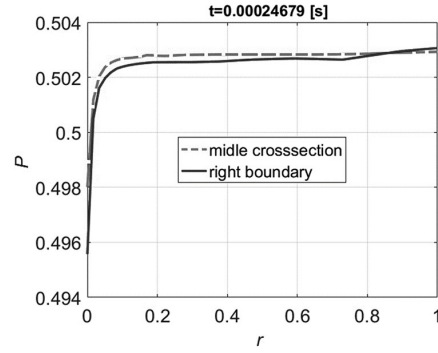


Fig. 15. The pressure profile at the second temperature maximum  $\beta = 2 - \text{NSF}$ .

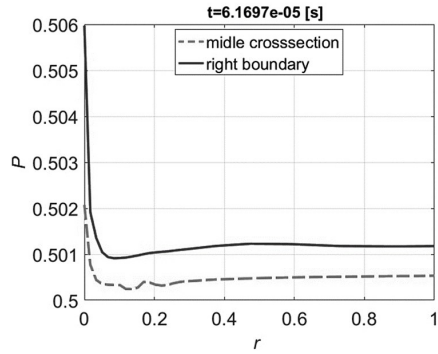


Fig. 16. The pressure profile at the first temperature minimum  $\beta = 2 - \text{NSF}$ .

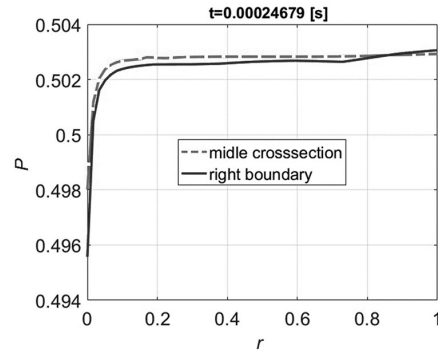


Fig. 17. The pressure profile at the second temperature minimum  $\beta = 2 - \text{NSF}$ .

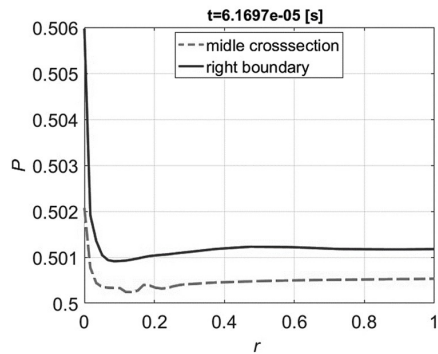


Fig. 18. The pressure profile at the first temperature maximum  $\beta = 8 - \text{NSF}$ .

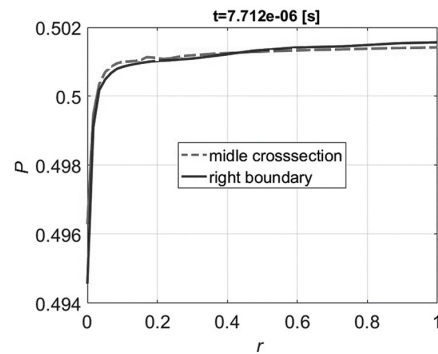


Fig. 19. The pressure profile at the first temperature minimum  $\beta = 8 - \text{NSF}$ .

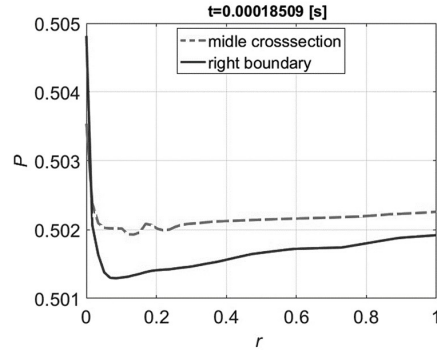


Fig. 20. The pressure profile at the second temperature maximum  $\beta = 8$  – NSF.

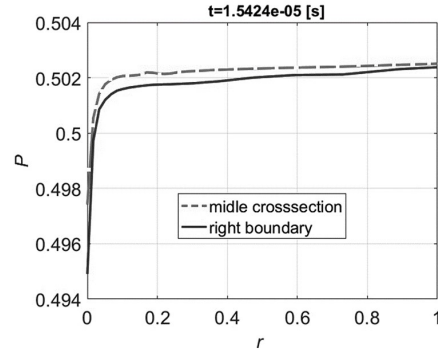


Fig. 21. The pressure profile at the second temperature minimum  $\beta = 8$  – NSF.

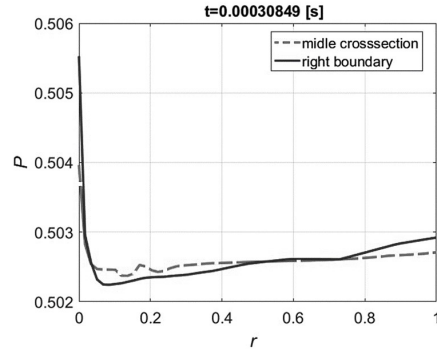


Fig. 22. The pressure profile at the third temperature maximum  $\beta = 8$  – NSF

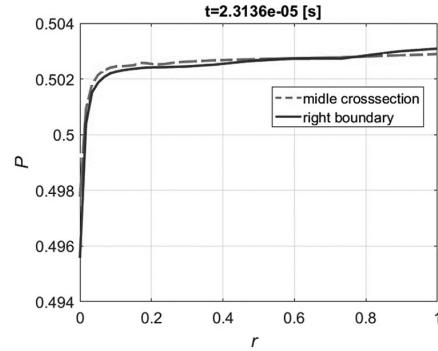


Fig. 23. The pressure profile at the third temperature minimum  $\beta = 8$  – NSF.

time required for molecular interactions to equilibrate the system. This can result in phase shifts between temperature and pressure oscillations, and potentially complex, non-linear pressure fields (Fig. 14 – Fig. 23).

#### 4 CONCLUSION

The inhomogeneous oscillating temperature of the inner cylinder in a rarefied gas system creates complex and dynamic variations in the temperature, density, and pressure of the gas. These effects are strongly influenced by the degree of rarefaction, the frequency and amplitude of the temperature oscillation, and the specific geometry of the coaxial cylinder setup. Understanding and predicting these influences typically requires sophisticated numerical simulations, such as those provided by the Direct Simulation Monte Carlo (DSMC) method and the ANSYS Fluent packet.

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