

SAFETY ZONE WITHIN THE LOAD SPACE OF DIFFUSIVELY SURFACE-REINFORCED STRUCTURAL COMPONENTS

ANA YANAKIEVA*, ANGUEL BALTOV, GERGANA NIKOLOVA

*Institute of Mechanics, Bulgarian Academy of Sciences, Acad. G. Bontchev
St., bl. 4, 1113 Sofia, Bulgaria*

[Received: 13 January 2022. Accepted: 05 April 2022]

doi: <https://doi.org/10.55787/jtams.22.52.4.351>

ABSTRACT: The paper proposes a new Methodology for determining the safety zone of a structural element in the load space. To calculate the load-bearing capacity of the selected composite structural element, the Approximate Computational Model (ACM) created by the authors is applied, adapted, and extended for strength load-bearing capacity. The structural element is reinforced on both sides with a reinforcing substance and subjected to overloads (loads above the permissible norms). Through appropriately selected dimensionless parameters, a load space is built, and boundary surfaces of load-bearing capacity are built in it. These surfaces enclose the required volume safety zone.

KEY WORDS: reinforced structural elements, overloads, safety zone, load space, load capacity.

1 INTRODUCTION

Nowadays structure overloading is a common phenomenon mostly affecting industrial structures and resulting in accidents involving damage/fracture and casualties. The most common unfavourable factors yielding eventual overloading are: (i) technological - due to the insufficient reconstruction funding, and/or delayed manufacture and supply of structural components; (ii) climatic - due to wind or heavy snowfall, earthquake, etc.; (iii) accidents- explosions, fire, etc. Hence, one needs to assess the resource of construction load-bearing capacity and to find the limit loads of its safety performance.

The load-bearing capacity of a structure/component can be lost under various impacts: (i) overloading due to an externally applied force, which exceeds the strength of the material or that of some of its components (basic material or reinforcement)

*Corresponding author e-mail: aniyanakieva@imbm.bas.bg

when considering a composite material; (ii) material fatigue due to persistent vibrations; (iii) stability loss or subjection to inadmissible deformations due to other factors. We treat in our study the violation of material strength under mechanical overloading due to an external force, only.

Consider diffusively reinforced structural components consisting of a concrete core and a high modulus reinforcing material deposited on the surface. Note that the initial material can undergo loading to a specific limit called safe loading. Assume now reinforcement-induced material strengthening. Then, the material can undergo loading higher than the above limit without damage/fracture. We call that loading safe overloading, and we propose a new method of identifying a safe overloading zone within a specially introduced load space.

Consider now that a structural component loses its load-bearing capacity under violation of the strength (tensile or compressive) of the weaker material (concrete in our case). We select in our case study a specific structural component – a cantilever beam, undergoing a longitudinal distributed loading yielding bending. It also undergoes axial compression due to an axially applied force, Fig. 1(a).

There are various methods of application of reinforcement known in the literature. We consider herein a diffusive deposition of reinforcement on the material surface, whereas data on material load-bearing capacity is scarce. However, the technology of surface reinforcement is much more reliable than that of reinforcement using a folio [1, 2]. It is proved that folio debonds comparatively easier under overloading, while the diffusive reinforcement yields a smooth and solid bond between the reinforcing layer and the substrate, where a transition bonding area emerges in-between, [2–4]. The result is that the composite material thus obtained turns stronger. We use an approximate computational method (ACM) designed in [1] to calculate the load-bearing capacity of the composite structural component. It is adapted such as to estimate the material load-bearing capacity. We consider static loading, reducing some dynamic impacts to static ones in conformity with the normative calculation documents Eurocode – (1-8) [5]. Loading is assumed to be simple from a viewpoint of structural mechanics [6]. We design a load space via suitably selected non-dimensional parameters, thus drawing limit surfaces of load-bearing capacity loss. They bound a zone of so-called safe overloading.

2 CALCULATION MODEL

2.1 ASSUMPTIONS

1. A basic concrete core and thin transition areas between concrete and the reinforcing material, emerge because of a diffusive reinforcement of the structural component, Fig. 1(b).

2. The plane cross-section hypothesis holds within the concrete core [6].
3. A quadratic approximation of the distribution of the material mechanical properties is assumed within the transition areas.
4. A quadratic distribution of the linear normal strain is assumed within the transition area.
5. The stress and strain transition are smooth through the transition area.
6. Strains and displacements are small conforming to the linear geometrical theory of deformation.
7. The structural component deforms linear-elastically.
8. Concrete tensile/compressive strength is the measure of the load-bearing capacity of the structural component [2].
9. Loading is simple – static [6].

2.2 GEOMETRY MODEL AND LOADING

Model geometry is shown in Fig. 1, introducing a coordinate system $Oxyz$ while axis x runs along the beam axis.

The cantilever beam is bilaterally reinforced, its length is l , and its cross-section is rectangular – $b \times h$. The thickness of the concrete core is $2z_L$, and $z_L = h/2 - h_R$,

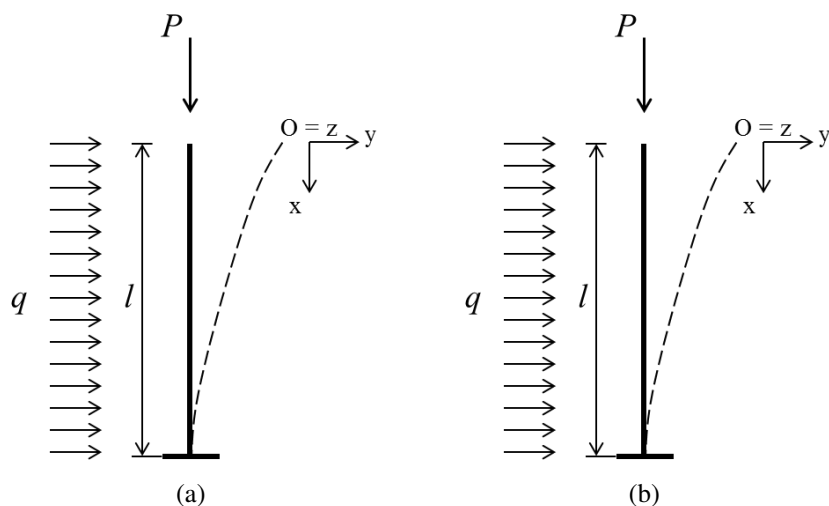


Fig. 1: Geometry of the model: (a) cantilever beam; (b) Cantilever beam cross section.

while h_R is the thickness of the transition area. The beam undergoes an axial load P , and a uniformly distributed bending loading with intensity q yielding moment $M(x)$, $x \in [0, l]$. The axial load P yields normal force N^- .

2.3 FINDING THE NORMAL STRESSES IN THE CONCRETE CORE IN THE PRESENCE OF REINFORCEMENT AND A TRANSITION AREA

The load-bearing capacity of the concrete core is found accounting for the distribution of the normal stresses applied to the cross-section of the structural component. A linear-elastic behavior of the structural component is assumed (Assumption 7). Hence the superposition principle holds, i.e., normal stresses within the concrete core for $z \in [-z_L, z_L]$ read

$$(1) \quad \sigma_x^\pm = \sigma_x^\pm(M) + \sigma_x^-(N^-),$$

where $\sigma_x^\pm(M)$ is the normal stress due to the distributed load with intensity q , and $\sigma_x^-(N^-)$ is the normal stress due to the axial load P .

2.3.1 NORMAL STRESS DUE TO THE DISTRIBUTED LOADING

$$(2) \quad \sigma_x^\pm(M) = E_B \varepsilon_x^\pm(M),$$

where $\varepsilon_x^\pm(M) = \chi_x z$, for $\chi_x = M(x)/K_{II}^*$ – curvature of the elastic line of the reinforced cantilever beam, K_{II}^* is the cantilever bending stiffness accounting for the reinforcement and the presence of a transition area.

The bending stiffness has the form [1]:

$$K_{II}^* = 2b \int_0^{h/2} E(z) z^2 dz = \frac{2}{3} b z_L^3 E_B + 2 \sum_{i=1}^3 K_i h_R^i$$

for

$$K_1 = \frac{bh^2}{12}(2E_B + E_R), \quad K_2 = -\frac{bh^2}{12}(5E_B + E_R), \quad K_3 = \frac{b}{12}\left(3E_B + \frac{1}{3}E_R\right),$$

where E_B and E_R are the elasticity moduli of concrete and reinforcement, and h_R is a model parameter found employing the method specified in [1].

The maximal tensile stress within the concrete core for $z = -z_L$ is

$$(3) \quad \sigma_{xL}^+(M) = E_B \chi_x z_L.$$

The maximal compressive stress within the core for $z = z_L$ is

$$(4) \quad \sigma_{xL}^-(M) = E_B \chi_x z_L.$$

2.3.2 NORMAL STRESS DUE TO THE AXIAL LOAD

$$(5) \quad \sigma_x^-(N^-) = -E_B \varepsilon_x^-,$$

where $\varepsilon_x^- = P/K_I^*$, K_I^* is the axial stiffness of the cantilever beam which accounts for the reinforcement and the availability of transition areas.

The axial stiffness has the form [1]

$$K_I^* = 2b \int_0^{h/2} E(z) dz = \frac{2}{3} b z_L E_B + 2b h_R (E_B + 0.215 \Delta E_R),$$

$$\Delta E_R = E_R - E_B.$$

Thus, we find the normal stresses within the concrete core accounting for the reinforcement and the transition areas via stiffness K_I^* and stiffness K_{II}^* .

3 STRENGTH CRITERIA FOR CONCRETE

In this work, to quickly assess the load-bearing capacity of the cantilever column due to external impact, it is assumed that the cantilever beam loses its load-bearing capacity only when reaching the maximum transverse permitted tensile and/or compressive stresses. The beam-cross section at the beam fixing point $x = l$ is a vulnerable cross-section where the bending moment is maximal, i.e., $M(l) = \frac{1}{2} Ql$, $Q = ql$. The cross-section of concrete at which the normal tensile stress is maximal is at $z = -z_L$, while the maximal normal compressive stress occurs at $z = z_L$, Fig. 1(b).

To attain the admissible limit tensile strength of the concrete core, one needs to apply an external loading Q that would yield tension close to the concrete limit tensile strength. This requires a specific ratio between the external loads P and Q .

Based on these considerations we introduce the First criterion for the load-bearing capacity of the cantilever beam.

$$(6) \quad \sigma_{xL}^+ = \sigma_{B\text{per}}^+,$$

where σ_{xL}^+ is the maximal normal tensile stress of concrete, $\sigma_{xL}^+ = E_B \varepsilon_{xL}^+$, $\varepsilon_{xL}^+ = \frac{M(l)}{K_{II}^*} z_L - \frac{P}{K_I^*}$; $\sigma_{B\text{per}}^+$ is the admissible limit tensile stress of concrete, $\sigma_{B\text{per}}^+ = \sigma_{BC}^+ / C^+$, σ_{BC}^+ is the limit strength of concrete under tension, and C^+ is a generalized safety factor under tension found following the actual normative requirements Eurocode [5].

Redoing Eq. (6), we find the following form of the criterion:

$$(7) \quad E_B \left(\frac{Ql}{2K_{II}^*} z_L - \frac{P}{K_I^*} \right) = \sigma_{B\text{per}}^+.$$

We can apply the newly proposed method to a wide range of strengthened structural components fabricated from various materials. The non-dimensional form of Eq. (7) reads finally

$$(8) \quad a_q \bar{q} - a_p \bar{p} = a_I,$$

where $a_q = \frac{Q_0 l^2 \bar{z}_L}{2K_{II}^*}$, $\bar{q} = \frac{Q}{Q_0}$, $a_p = \frac{P_0}{K_I^*}$, $\bar{p} = \frac{P}{P_0}$, $a_I = \frac{\sigma_{B\text{per}}^+}{E_B}$, while Q_0 and P_0 are normative loads, and $\bar{q}$ and $\bar{p}$ are relative overloads whereas $Q \geq Q_0$ and $P \geq P_0$, i.e. $\bar{q} \geq 1$ and $\bar{p} \geq 1$, while $\bar{z}_L = z_L/l$.

It follows from Eq. (8) that Q and P ($\bar{q}$ and $\bar{p}$ respectively) should obey the condition

$$(9) \quad \frac{\bar{q}}{\bar{p}} > \frac{a_p}{a_q}$$

to realize tension.

The second criterion for loss of the load-bearing capacity of the cantilever beam states that one should exceed the admissible limit compressive strength of the concrete core. The latter is found using the elasticity limit of concrete and a respective safety factor:

$$(10) \quad |\sigma_{xL}^-| = |\sigma_{B\text{per}}^-|,$$

where σ_{xL}^- is the maximal normal compressive stress of concrete, $|\sigma_{xL}^-| = E_B |\varepsilon_{xL}^-|$, $|\varepsilon_{xL}^-| = \frac{M(l)}{K_{II}^*} z_L + \frac{P}{K_I^*}$, $\sigma_{B\text{per}}^-$ is the admissible limit compressive strength, $\sigma_{B\text{per}}^- = \frac{\sigma_{B\text{per}}^-}{C^-}$, $\sigma_{B\text{per}}^-$ is the elastic limit of concrete under compression, and C^- is a generalized safety factor under compression found according to the actual normative requirements Eurocode [5].

Expanding Eq. (10), we find the following form of the criterion:

$$(11) \quad E_B \left(\frac{Ql}{2K_{II}^*} z_L + \frac{P}{K_I^*} \right) = |\sigma_{B\text{per}}^-|.$$

It reads as follows in a non-dimensional form:

$$(12) \quad a_q \bar{q} + a_p \bar{p} = a_{II},$$

where $a_{II} = |\sigma_{B\text{per}}^-|/E_B$.

4 LOAD SPACE. VOLUME SAFETY ZONE

In terms of simple loading (Assumption 9), overloads (P, Q) exceeding the normative loads (P_0, Q_0) should depend on one parameter. In our case both loads are due to one and the same impact. We introduce a dimensionless load parameter ξ expressing the cause for the overload. The values of parameters P and Q , and which of them is the higher one, are determined for each case. The probability for P to overload the structure is $v_p \in [0, 1]$ while that for $Q - v_q \in [0, 1]$. We assume v_p and v_q to be independent probabilities. Based on this assumption, we can present loads P and Q as

$$(13) \quad P = v_p P_0 \xi, \quad Q = v_q Q_0 \xi.$$

Design a load space $\Pi = \{v_p, v_q, \xi\}$ determined by the three non-dimensional positive parameters v_p, v_q and ξ . Introduce a Cartesian coordinate system $Ov_p v_q \xi$, and consider the first octant where the real parameter values are positive, Fig. 2. Define an admissible set $D_C = \{v_p \in [0, 1]; v_q \in [0, 1]\}$ in the plane $Ov_p v_q$. Map limit surfaces $\xi_I = \xi_I(v_p, v_q)$ and $\xi_{II} = \xi_{II}(v_p, v_q)$ over it. They cross each other along a curve denoted by ξ^* . In this way, a discontinuous surface is defined as follows:

$$(14) \quad \xi_f = \min(\xi_I, \xi_{II}).$$

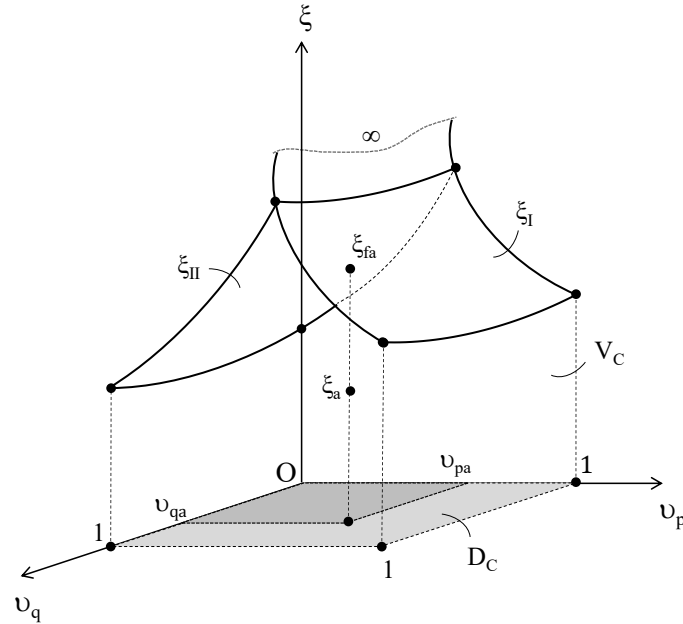


Fig. 2: Loading space. Volume safety zone.

Surface ξ_f and the admissible set D_C outline the volume safety zone V_C of the operation of the structural element undergoing overloads. It can be found from the following:

$$(15) \quad V_C = \int_0^1 \int_0^1 \xi_f(v_p, v_q) dv_p dv_q .$$

To determine V_C it is necessary to find functions ξ_I , ξ_{II} and ξ^* .

4.1 DETERMINATION OF ξ_I AND ξ_{II}

We introduce in (13) the relative loads $\bar{p}$ and $\bar{q}$ as follows:

$$(16) \quad \bar{p} = v_p \xi, \quad \bar{q} = v_q \xi .$$

Substitute consecutively those new forms of the relative loads $\bar{p}$ and $\bar{q} = v_q \xi$ in (8) – the criterion for load-bearing capacity loss under tension, and in (12) – the criterion for load-bearing capacity loss under compression. Then, we find:

- For tension

$$(17) \quad \xi_I = \frac{a_I}{a_q v_q - a_p v_p} .$$

The condition of concrete core tension – inequality (9), takes the form

$$(18) \quad \frac{v_p}{v_q} > \frac{a_q}{a_p} .$$

Note that if $v_q = \frac{a_p}{a_q} v_p$, then $\xi_I \rightarrow \infty$ in (18), Fig. 2.

bullet For compression

$$(19) \quad \xi_{II} = \frac{a_{II}}{a_q v_q + a_p v_p} .$$

4.2 DETERMINATION OF ξ^*

As mentioned above, the curve ξ^* is defined by the intersection of the two limit surfaces and determined for $\xi_I = \xi_{II}$. Its equation reads

$$(20) \quad \frac{a_I}{a_q v_q - a_p v_p} = \frac{a_{II}}{a_q v_q + a_p v_p} .$$

Equation (20) can also be represented as a ratio between probabilities

$$(21) \quad \frac{v_q}{v_p} = \frac{a_p (a_I + a_{II})}{a_q (a_{II} - a_I)} .$$

Expression (21) is positive since $a_{II} > a_I$, and concrete tensile strength is lower than concrete compressive strength, $\sigma_{B\text{per}}^+ < |\sigma_{B\text{per}}^-|$.

Thus, the volume safety zone V_C is determined.

We can calculate ξ using Eq. (12), and knowing beam geometry $b \times h, l$, thickness of the transition area h_R , the limit admissible tensile/compressive stress for concrete $(\sigma_{B\text{per}}^+, \sigma_{B\text{per}}^-)$, normative loads (P_0, Q_0) and probabilities (v_p, v_q) . We denote the value of ξ in this specific case by ξ_a , Fig. 2. We find in space II the limit point $\xi_{fa} = \min(\xi_{Ia}, \xi_{IIa})$ of surface $\xi_f = \min(\xi_I, \xi_{II})$. When $\xi_a > \xi_{fa}$ the cantilever beam has lost its load-bearing capacity, while the overloading within the safety zone is admissible when $\xi_a < \xi_{fa}$, i.e., the beam has not lost its load-bearing capacity. One can assess the reserve of the latter via parameter R_C specified as

$$(22) \quad R_C = \frac{\xi_f - \xi}{\xi_f} \%.$$

When analyzing a particular structural component and expected overloading, we can recommend a particular reinforcement of the component assessing the zone of overloading and accounting for the strength requirements to the structure.

5 NUMERICAL EXAMPLE AND RESULTS

The presented methodology for assessing the bearing capacity of surface-reinforced concrete structural elements will be illustrated on a concrete cantilever beam. The cantilever beam is diffusively and bilaterally reinforced with a polymer composite containing nanoparticles [3]. The geometric and mechanical characteristics are as follows:

- Geometric: $b = 20$ cm, $h = 30$ cm, $l = 400$ cm, $z_L = 14$ cm, $h_R = 1.0$ cm;
- Stiffness of the beam: $K_I^* = 2.062 \times 10^9$ N, $K_{II}^* = 16.4 \times 10^{10}$ N/cm³
- Mechanical characteristics of concrete:

$$E_B = 3.5 \times 10^6 \text{ N/cm}^2, \quad \sigma_{B\text{per}}^+ = 300 \text{ N/cm}^2 \\ \sigma_{PB}^- = -2500 \text{ N/cm}^2, \quad \sigma_{B\text{per}}^- = -1000 \text{ N/cm}^2$$

- Mechanical characteristics of the reinforcement [3]: $E_R = 30 \times 10^6$ N/cm².
- Normative initial loading [7]: $Q_0 = 80$ N, $P_0 = 2 \times 10^3$ N.

To determine the volume safety zone, we follow the sequence of operations described in the previous Section 4:

1. Introduce load space Π and consider the first octant of the Cartesian coordinate system $Ov_p v_q \xi$.
2. Map the limit surfaces $\xi_I(v_p, v_q)$ and $\xi_{II}(v_p, v_q)$ over the admissible set D_C , (15) and (17).
3. Determine the cross points of the two curves ξ_I and ξ_{II} , i.e. $\xi_I = \xi_{II} = \xi^*$ for v_{qi}^* , and $v_{pi} = i\Delta v_p$. Their set maps a curve ξ^* .
4. Using (15) define the volume safety zone V_C .

For a better understanding of the presented methodology, we will find the volume of the safety zone using another technique. It will clearly show how the structural element operates at each point of the loading space subjected to loads exceeding the normative ones, i.e., overloads. Being like medical scanning, this technique will also help to find areas with specific properties.

For this purpose, we cut the load space into plane sections Ov_p by a step $\Delta v_p = 0.1(v_{pi} = i\Delta v_p, i = 1, 2, \dots, 10)$. Each of these sections is bounded by $\xi_I(i\Delta v_p, v_q)$, and there is an intersecting curve between them $\xi^* = \xi_I = \xi_{II}$.

The results for ten sections are shown in Fig. 3.

It is important to note that the choice of step $\Delta v_p = 0.1(v_{pi} = i\Delta v_p, i = 1, 2, \dots, 10)$ is computational, and it depends on each specific selected structural element. Summation of sections Ov_p yields an approximate V_C . To refine its value we look for that probability interval, which is located between the starting and the end point of the intersection between ξ_I and ξ_{II} , and which one belongs to the originally introduced interval – $\{v_p \in [0, 1]; v_q \in [0, 1]\}$. At these points, the structural element is subjected to both tension and compression. Thus, we find the new real interval probabilities – $\{v_p \in [0.05, 0.76]; v_q \in [0.07, 1]\}$, Fig. 4.

The volume of the safety zone is defined as follows:

$$(23) \quad V_C \approx \Delta v_p \left(\sum_{i=1}^n S_{Ii} + \sum_{i=1}^n S_{IIi} \right), \quad n = 1, 2, 3, \dots,$$

where the choice of n may be different for each structural element, while S_{Ii} and S_{IIi} are areas under the limit curves $\xi_{fi} = \min(\xi_{Ii}, \xi_{IIi})$, Fig. 5. They have the following form:

$$S_{Ii} = \int_0^{\xi_i^*} \xi_{Ii}(v_p, v_q) dv_q, \quad S_{IIi} = \int_{\xi_i^*}^1 \xi_{IIi}(v_p, v_q) dv_q.$$

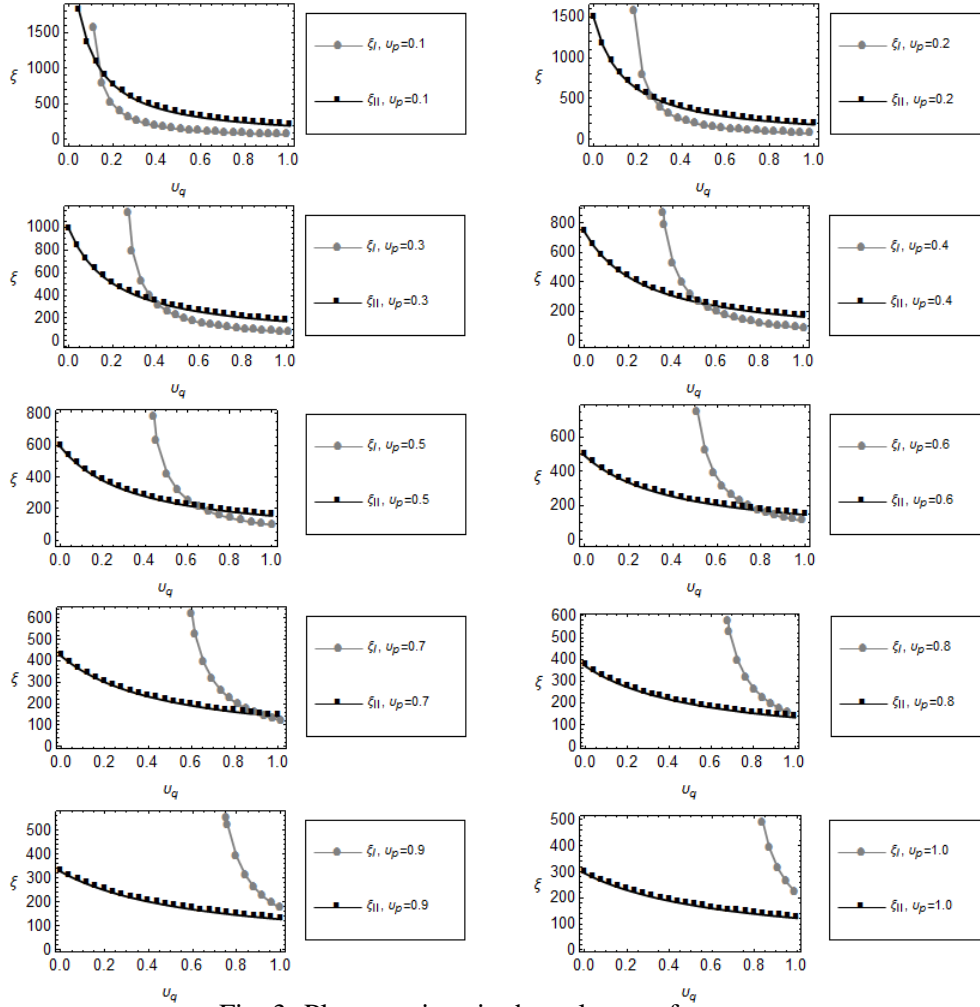


Fig. 3: Plane sections in the volume safety zone.

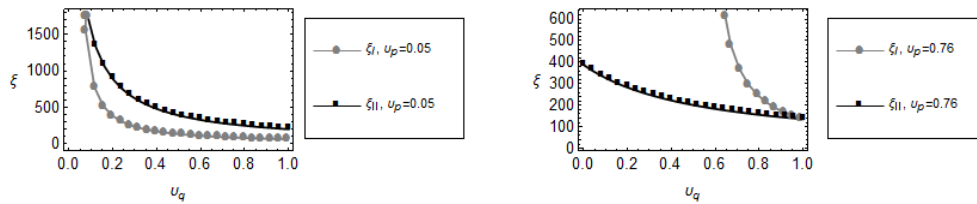


Fig. 4: A plane section containing the starting and end point of the intersection between ξ_I and ξ_{II} .

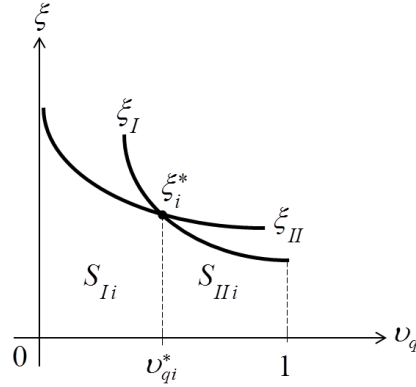


Fig. 5: General view of the section of the volume safety zone.

Figure 6 shows curve protection in (ξ^*, v_q^*) obtained from the intersection of the two limit surfaces. In its physical sense, it can be called “The curve of equalled strengths” because it is the curve along which we reach the maximum allowable strength values, both in tension and compression.

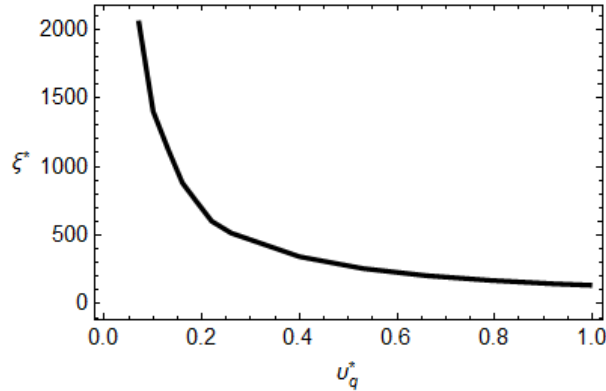
Fig. 6: The protection of the curve of equalled strengths in plane (ξ^*, v_q^*) .

Table 1 shows the calculation results.

To calculate the coefficient of the bearing capacity reserve R_C we consider for instance section $v_p = 2\Delta v_p = 0.2$, Fig. 7. We also consider a specific overloading $\xi_a < \xi_{fa}$ in the volume safety zone and calculate R_C according to (22).

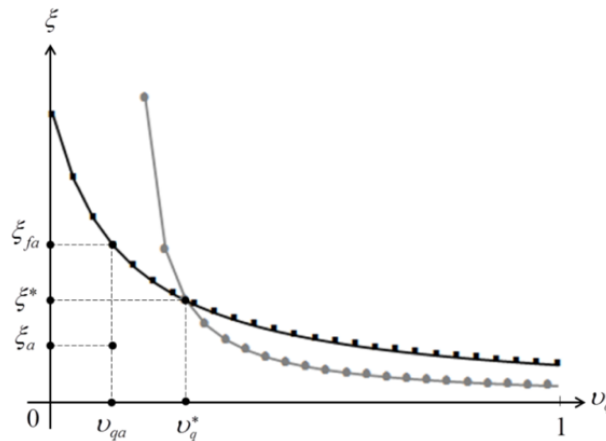
We find for $v_p = 2\Delta v_p = 0.2$ that:

$$R_C = \frac{\xi_f - \xi}{\xi_f} \% = \frac{810 - 258}{810} \% = \frac{552}{810} \% = 68\%,$$

where $\xi_f = 810$, $\xi = 258$, $\xi^* = 516$, $v_q = 0.101$, $v_q^* = 0.264$.

Table 1

Step	v_p	v_q^*	ξ^*	S_I	S_{II}	$S_I + S_{II}$
1	0.05	0.07	2062	216.76	0	216.76
2	0.1	0.13	1031	170.91	219.73	390.64
3	0.2	0.26	516	122.42	219.73	342.15
4	0.3	0.40	344	91.55	219.73	311.28
5	0.4	0.53	258	67.56	219.73	287.29
6	0.5	0.66	206	47.22	219.52	266.74
7	0.6	0.79	172	28.42	219.56	247.98
8	0.7	0.92	147	10.43	219.58	230.01
9	0.76	1.00	136	0	219.47	219.47
Total:					$V_C = 231.62$	

Fig. 7: An arbitrary plane section for $v_p = 2\Delta v_p$.

6 CONCLUSIONS

The study presents an efficient method of determining the reserve and limits of the bearing capacity of diffusion-reinforced concrete structural elements subjected to loads exceeding the normative ones, i.e., overloads, whereas tensile areas develop in the concrete core. A specific loading space is designed, where a Volume safety zone is outlined. Regarding a particular structural component, one can perform an expert assessment of the overloading probability within the loading space. The method is illustrated on a cantilever beam, but it can also be applied to other components with different mechanical characteristics of the concrete core and the reinforcing material.

REFERENCES

- [1] . BALTOV, A. YANAKIEVA (2018) Energy saving and surface-reinforced structural elements with transition areas. *Series in Applied Mathematics and Mechanics* **10** p. 90, Inst. of Mech.- BAS, Sofia, ISSN: 1314-3034.
- [2] . BALTOV, A. YANAKIEVA, G. NIKOLOVA (2019) Residual Resource of Emergency-loaded Concrete Beams under Pressure and Micro-damage. *Comptes rendus de l'Académie bulgare des Sciences* **72**(8) 1086-1094.
- [3] I-ÈRE ECOLE D'AUTOMNE (1986) "Endommagement des matriaux et des structures des composites, dégradation - Durabilité". Paris.
- [4] I. IVANOVA, J. ASSIH, D. DONTCHEV (2020) Influence of anchorage length of composite fabrics and bonded surface on the strengthened short reinforced concrete corbel by bonding CFRF. *European Journal of Environmental and Civil Engineering* **24**(12) 1993-2009.
- [5] EUROPEAN COMISSION (2000) [Eurocode: vols. 1-8](#).
- [6] D.C. DRUCKER (1967) "Introduction to Mechanics of Deformable Solids". Mc. Craw-Hill Book Company, N.Y.
- [7] . VELINOV, N. ANGELOV, A. GEORGIEV (2021) BDS N 1992-1-1: User manual, Second expanded edition, Sofia (in Bulgarian).