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A method for simulating of non-steady thermoplastic processes

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1. Introduction

Metal forming processes are characterized by large plastic strains. Part of the dissipated energy turns into heat. The material properties, especially the yield limit depend on the temperature. The accumulated plastic deformation and the strain-rate lead to hardening of the material, as the temperature increase causes material softening in the same time. The mechanical and the thermal behaviour of the material are highly coupled and it is not admissible to consider them separately.

Determining the destribution of the stress-, strain- and temperature fields during the metal forming process is important for investigating appearance of plastic localization, which leads to structural changes, crack initiation and fracture.

The coupled thermomechanical problem for elastic-plastic and visco-plastic solids was considered in [1, 2, 3, 4 etc.]. Since elastic strains during metal forming processes are neglegible compared to plastic ones, applying the model of a rigid-plastic body is advisable. The solution of that problem, not taking into account temperature influence and heat production, is given in [8, 9, 10]. A method for solving non-steady coupled thermomechanical processes in rigid-plastic bodies will be given here.

2. Constitutive relations for a rigid-plastic body

It is assumed, that a yield condition of the Mises type exists [5]:

$$\bar{\sigma}=\sigma_p(\bar{\varepsilon},\ \dot{\bar{\varepsilon}},\ T), \tag{2.1}$$

where

$$\bar{\sigma}=\sqrt{\frac{3}{2}s_{ij}s_{ij}} \tag{2.2}$$

is the equivalent stress,

$$s_{ij}=\sigma_{ij}-\frac{1}{3}\sigma_{kk}\delta_{ij} \tag{2.3}$$

is the stress deviator, σ_{ij} is the stress tensor, δ_{ij} is Kronecker's delta and σ_p is the yield limit. The material is isotropically hardening, strain-rate sensitive and temperature-softening. The yield limit σ_p depends on the effective strain-rate

(2.4) $$\dot{\bar{\varepsilon}} = \sqrt{\frac{2}{3}\lambda_{ij}\lambda_{ij}},$$

on the effective strain

(2.5) $$\bar{\varepsilon} = \int_0^t \dot{\bar{\varepsilon}}\, dt$$

and on the temperature T. t denotes time. The strain-rate λ_{ij} is linearly connected with the velocity vector v_i:

(2.6) $$\lambda_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}).$$

All variables are time dependent.

The strain-rate is determined by the associated flow rule:

(2.7) $$\lambda_{ij} = \begin{cases} \frac{1}{\Phi} s_{ij} & \text{if } \bar{\sigma} = \sigma_p \\ 0 & \bar{\sigma} < \sigma_p \end{cases},$$

where

(2.8) $$\Phi = \frac{2}{3}\frac{\sigma_p}{\dot{\bar{\varepsilon}}}.$$

The boundary conditions are:

a. kinematical:

(2.9) $$v_i = v_i^0 \quad \text{on } S_V,$$

where v_i^0 is the prescribed velocity of points belonging to the part S_V of the body surface S.

b. static:

(2.10) $$\sigma_{ij} n_j = p_i^0 \quad \text{on } S_p,$$

where p_i^0 is the prescribed traction on the part S_p of S.

c. frictional:

(2.11) $$\sigma_{ij} n_j = \tau_i \quad \text{on } S\tau,$$

where the friction vector τ_i is directed opposite to the tangential velocity of the point, belonging to the part S_τ of S and n_j is the outward unit normal at that point. Siebel's law is usually assumed [6, 8]:

(2.12) $$\tau = m\tau_p,$$

where

(2.13) $$\tau_p = \frac{\sigma_p}{\sqrt{3}}$$

is the shear yield stress and $0 \leqq m \leqq 1$ is the friction factor. Obviously, S_V, S_p, $S_\tau \in S$.

The initial conditions of the problem are:

$$\sigma_{ij}(x_k,\ 0)=0,$$

(2.14) $$\lambda_{ij}(x_k,\ 0)=0 \quad \text{at } t=0,\ x_k\in\Omega,$$

$$\bar{\varepsilon}(x_k,\ 0)=0,$$

where Ω is the region occupied by the body under consideration.

3. Thermal state

The temperature is determined by the equation [1, 5]:

(3.1) $$\frac{dT}{dt}=\frac{\partial T}{\partial t}+T_{,i}v_i=\frac{1}{\rho c_T}(\lambda T_{,i})_{,i}+\frac{\dot{Q}}{\rho c_T},$$

where λ is the conductivity coefficient, c_T is the specific heat supply, ρ is the material density and

(3.2) $$\dot{Q}=k\sigma_p\dot{\bar{\varepsilon}}$$

is the rate of dissipation during the plastic deformation, converted into heat. k is assumed to take values between 0,8 and 0,95 and determines the fracture of the strain-rate energy, which turns into heat.

The boundary conditions are:

(3.3) $$T(x_i,\ t)=T_s(x_i,\ t),\quad x_i\in S_{T_1},$$

(3.4) $$\frac{\partial T}{\partial n}=-\frac{H}{\lambda}[T(x_i,\ t)-T_{\text{ref}}(t)],\quad x_i\in S_{T_2},\quad S_{T_1},\ S_{T_2}\in S,$$

where T_s is a prescribed temperature on the part S_{T_1} of the surface of the body S T_{ref} is the temperature of the surrounding medium and H is the surface heat transfer coefficient. If the body under consideration is in contact with another body, then a heat transfer takes place and a contact heat problem should be considered. This is the case on the contact surface between workpiece and tool. A simple expression for determining the value of H is proposed in [7], which enables using the boundary condition (3.4) even in such situations.

The initial conditions of the problem are:

(3.5) $$T(x_i,\ 0)=T_0(x_i)\quad \text{at } t=0,\ x_i\in\Omega.$$

4. Method of solution

The basic equations of the problem, given in § 2 and 3 show that the mechanical and the thermal problems are coupled. A method of solving such coupled problems is their uncoupling in time steps. At each time step $t_{i+1}=t_i+\Delta t$ the mechanical problem is solved, considering the temperature a known parameter, determined at the previous time step t_i. The values of $v_k(x_j,\ t_{i+1})$, $\dot{\bar{\varepsilon}}(x_j,\ t_{i+1})$ and $\bar{\varepsilon}(x_j,\ t_{i+1})$ are found and the new yield limit $\sigma_p^*(x_j,\ t_{i+1})=\sigma_p(\bar{\varepsilon}(x_j,\ t_{i+1}),\ \dot{\bar{\varepsilon}}(x_j,\ t_{i+1}),\ T(x_j,\ t_i))$ is determined. Then the thermal problem is solved, where the heat source $\dot{Q}$ is determined by means of $\sigma_p^*(x_j,\ t_{i+1})$ and $\dot{\bar{\varepsilon}}(x_j,\ t_{i+1})$. The new temperature distribution $T(x_j,\ t_{i+1})$ is found after which the yield limit is corrected: $\sigma_p(x_j,\ t_{i+1})=\sigma_p(\bar{\varepsilon}(x_j,\ t_{i+1}),\ \dot{\bar{\varepsilon}}(x_j,t_{i+1}),T(x_j,t_{i+1}))$. Then the next time step is fulfilled. The flow-chart of the solution is shown in Fig. 1.

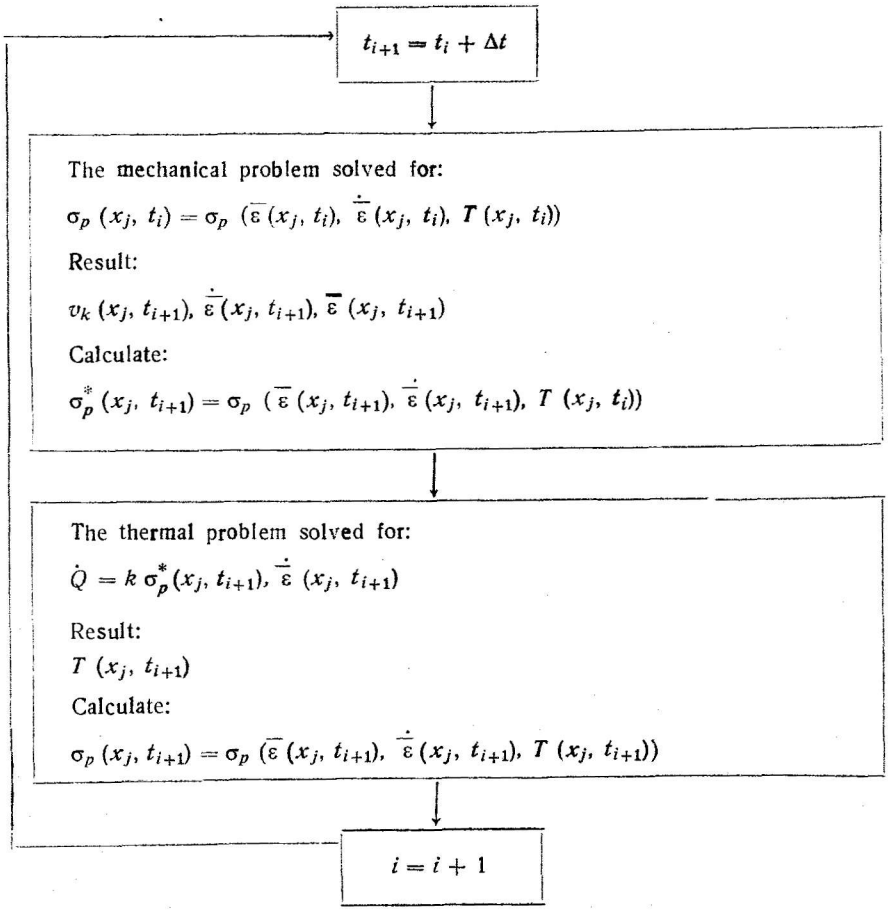

Fig. 1

4.1. Method for solving the mechanical problem

The finite-element method will be used for solving the mechanical problem. A variant of the method for rigid-plastic solids is given in [8, 9, 10]. The method is based on the modified Markov's variational principle:

$$\pi = \int_\Omega \sigma_p \dot{\bar{\varepsilon}} \, dV + \int_\Omega \sigma_m \dot{\varepsilon}_{ii} \, dV + \int_{S_\tau} \tau v_i \, dS - \int_{S_p} p_i^0 v_i \, dS \Rightarrow \text{stat}, \tag{4.1}$$

where the hydrostatic stress $\sigma_m = \frac{1}{3}\sigma_{kk}$ is a Lagrangian multiplier. The vector of nodal parameters:

$$\{z\}^T = \{\{q\}^T, \{\sigma_m\}^T\} \tag{4.2}$$

is built by the vector of nodal velocities $\{q\}$ and the vector of the nodal hydrostatic stress $\{\sigma_m\}$. The element stiffness matrix is:

$$[K] = \begin{bmatrix} [K^*] & [C]^T \\ [C] & [0] \end{bmatrix}, \tag{4.3}$$

where

(4.4) $$[K^*]=\int_{V^e}\Phi[B]^T[B]\mathrm{d}V$$

(4.5) $$[C]=\int_{V^e}[N_\sigma]^T\{C\}[B]\,\mathrm{d}V$$

(4.6) $$[B]=[D][N]$$

(4.7) $$\{C\}^T=\{1,\ 1,\ 1,\ 0,\ 0,\ 0\}$$

$D]$ is the differential matrix and $[N]$ and $[N_\sigma]$ are matrices of the shape functions for the velocity and the hydrostatic stress. The strain-rate vector $\{\lambda\}$ and the velocity vector are:

(4.8) $$\{\lambda\}=[D]\{v\}$$

(4.9) $$\{v\}=[N]\{q\}.$$

The basic equations system of the finite-element method is:

[4.10] $$[K]\{z\}=\{R\},$$

where

(4.11) $$\{R\}^T=\{\{F\}^T,\ \{0\}\}$$

and $\{F\}$ is the nodal force vector.

As the stiffness matrix depends on the yield stress, and hence on the strain and strain-rate, the problem is nonlinear and the Newton-Raphson iteration procedure should be applied. In rigid elements $\dot{\bar{\varepsilon}}=0$ and $\Phi\to\infty$. In order to avoid numerical difficulties, it is assumed that $\dot{\bar{\varepsilon}}=\dot{\bar{\varepsilon}}_{\min}$ if $\dot{\bar{\varepsilon}}<\dot{\bar{\varepsilon}}_{\min}$.

A computer program, named FARM is built on the basis of the above relations, for solving nonstational metal forming processes, neglecting thermal effects [8, 9, 10]. Linear shape functions are used in FARM and $\bar{\varepsilon}$, $\dot{\bar{\varepsilon}}$ as well as σ_m are constant within a finite element. The problem is solved stepwise in time. The nodal points are considered as material points and at each time step undergo displacements $u_i=v_i\Delta t$. If an element degeneration takes place, a rezoning procedure for mapping the values of $\bar{\varepsilon}$, $\dot{\bar{\varepsilon}}$ and Φ from the degenerated to the new regular mesh is realized.

4.2. Method for solving the thermal problem

As $\bar{\varepsilon}$, $\dot{\varepsilon}$ and σ_p are assumed in FARM to be constant within a finite element, the heat source $\dot{Q}$ will be there constant too. A constant value of σ_p requires, a constant value of the temperature as the yield limit is temperature dependent. The application of the finite element method for solving the thermal problem leads to a temperature distribution inside the element, which is not constant, even if linear shape functions are assumed [11]. That is why we shall apply the "free point method" [12, 13, 14] for solving the thermal problem, connecting it with the finite-element mesh of the mechanical solution.

We assume that the mass centre $P^{(k)}$ of the finite element bears the information about the element temperature. We represent the continuum in the region Ω, occupied by the body, as a mani-fold of descrete points $P^{(k)}$, $k=1,\ldots,n$, where n is the num-

ber of finite elements in the mesh, built during the previous solution. We represent the continuous temperature field $T(x_i, t)$ by the manifold of descrete values $T_k^*(t)$, $k=1, \ldots, n$ of the temperature at the points $P^{(k)}$. The temperature field at time $t+\Delta t$ may be determined by means of a linear operator L, applied to the temperature field at time t:

$$T(x_i, t+\Delta t)=LT(x_i, t). \tag{4.12}$$

The value of the function $T(x_i, t)$ at a point $P^{(k)}$ may be calculated by means of an approximation operator A, applied to m values $T_j^*(t)$, $j=1, \ldots, m$, at m points, belonging to the neighbourhood of the point under consideration $P^{(k)}$:

$$T(x_i^{(k)}, t)=AT^*(t). \tag{4.13}$$

This is true also at time $t+\Delta t$:

$$T(x_i^{(k)}, +\Delta t)=AT^*(t+\Delta t). \tag{4.14}$$

Substituting (4.12) and (4.13) into (4.14) we obtain:

$$AT^*(t+\Delta t)=LAT^*(t). \tag{4.15}$$

As only the values of the temperature at the descrete points are of interest, we may write:

$$T_k^*(t+\Delta t)=LAT^*(t). \tag{4.16}$$

Convergence of the solution means that:

$$T_k^*(t)-T(x_i^{(k)}, t)\underset{\Delta t\to 0}{\to}0 \qquad \begin{matrix} t\in(0, t_f) \\ k=1, \ldots, n, \end{matrix} \tag{4.17}$$

where $(0, t_f)$ is the considered time interval.

At time $t+\Delta t$ the following is valid:

$$T_k^*(t+\Delta t)-T(x_i^{(k)}, t)=LA(T^*(t)-T(x_i, t))+L(AT^*(t)-T(x_i^{(k)}, t)). \tag{4.18}$$

After introducing the norm:

$$\|\varphi\|=\max_i \varphi_i \tag{4.19}$$

it follows from (4.17) and (4.18) [12]:

$$\begin{aligned} \|T_k^*(t+\Delta t)-T(x_i^{(k)}, t+\Delta t)\| \leq \|LA\| . \|T^*(t)-T(x_i^{(k)}, t)\| \\ + \|L(AT^*(t)-T(x_i^{(k)}, t))\|. \end{aligned} \tag{4.20}$$

The condition (4.20) will be fulfilled if

$$\begin{aligned} &\|LA\|\leq 0, \\ &\|L(AT^*(t)-T(x_i^{(k)}, t))\|=0(\Delta t). \end{aligned} \tag{4.21}$$

The first condition in (4.21) is a stability condition. The second one is a condition for the approximation. It ensures that the approximation error does not increase in time, e. g. the approximation A is sufficiently good.

In order to prove the conditions (4.21) we shall consider the one-dimensional case. The temperature equation (3.1) then takes the form:

$$\frac{dT}{dt}=\frac{1}{\rho c_T}\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\dot{Q}}{\rho c_T}. \tag{4.22}$$

As in the solution of the mechanical problem the nodal points move during the deformation process like material points, the constitutive terms do not appear in (4.22).

Assuming a second-order polynomial approximation A, we obtain:

$$T(x,\ t)=Z_1(t)+Z_2(t)\xi+Z_3(t)\xi^2, \tag{4.23}$$

where $\xi=x-x^{(k)}$.

The approximation accuracy in (4.23) is of order Δx^3. The second derivatives accuracy is of order Δx. The condition $(4.21)_2$ has then the form:

$$\| L(AT^*(t)-T(x^{(k)},\ t))\| =\frac{\lambda}{\rho c_T}\Delta x\Delta t. \tag{4.24}$$

Taking the temperature equation (3.1) in the form:

$$\frac{dT}{dt}=\frac{1}{\rho c_T}q_{i,i}+\frac{\dot{Q}}{\rho c_T},\quad q_i=\lambda T_{,i} \tag{4.25}$$

or in the one-dimensional case:

$$\frac{dT}{dt}=\frac{1}{\rho c_T}\frac{\partial q}{\partial x}+\frac{\dot{Q}}{\rho c_T},\quad q=\lambda\frac{\partial T}{\partial x} \tag{4.26}$$

and applying independent approximations for the temperature and for the heat flux q:

$$\begin{aligned} T(x,\ t)&=Z_1(t)+Z_2(t)\xi+Z_3(t)\xi^2\\ q(x,\ t)&=P_1(t)+P_2(t)\xi+P_3(t)\xi^2 \end{aligned} \tag{4.27}$$

we have accuracy for the first derivatives of order Δx^2 and $(4.21)_2$ takes the form

$$\| L(AT^*(t)-T(x^{(k)},\ t))=\frac{\lambda}{\rho c_T}\Delta x^2\Delta t \tag{4.28}$$

which is better than (4.24).

The coefficients Z_i and P_i (i=1, 2, 3) have the values:

$$\begin{aligned} Z_1&=T_k^* & P_1&=q_k^*\\ Z_2&=\frac{1}{2\Delta x}(T_{k+1}^*-T_{k-1}^*) & P_2&=\frac{1}{2\Delta x}(q_{k+1}^*-q_{k-1}^*)\\ Z_3&=\frac{1}{2\Delta x^2}(T_{k+1}^*-2T_k^*+T_{k-1}^*) & P_3&=\frac{1}{2\Delta x^2}(q_{k+1}^*-2q_k^*+q_{k-1}^*). \end{aligned} \tag{4.29}$$

The application of the L operator leads to:

$$T_k^*(t+\Delta t)=T_k^*(t)+\frac{\Delta t}{2\rho c_T\Delta x}(q_{k+1}^*-q_{k-1}^*)+\frac{\dot{Q}}{\rho c_T}\Delta t \tag{4.30}$$

and the substitution of (4.27) in (4.29) yields:

$$T_k^*(t+\Delta t)=T_k^*(t)+\frac{\lambda\Delta t}{4\rho c_T\Delta x^2}[(T_{k+2}^*-T_k^*)+(T_{k-2}^*-T_k^*)]+\frac{\dot{Q}}{\rho c_T}\Delta t. \tag{4.31}$$

The physical reality of heat transfer requires the fulfillment of the following condition

$$\frac{2\lambda\Delta t}{4\rho c_T\Delta x^2}<1 \tag{4.32}$$

$$\Delta t<\frac{2\rho c_T\Delta x^2}{\lambda} \tag{4.33}$$

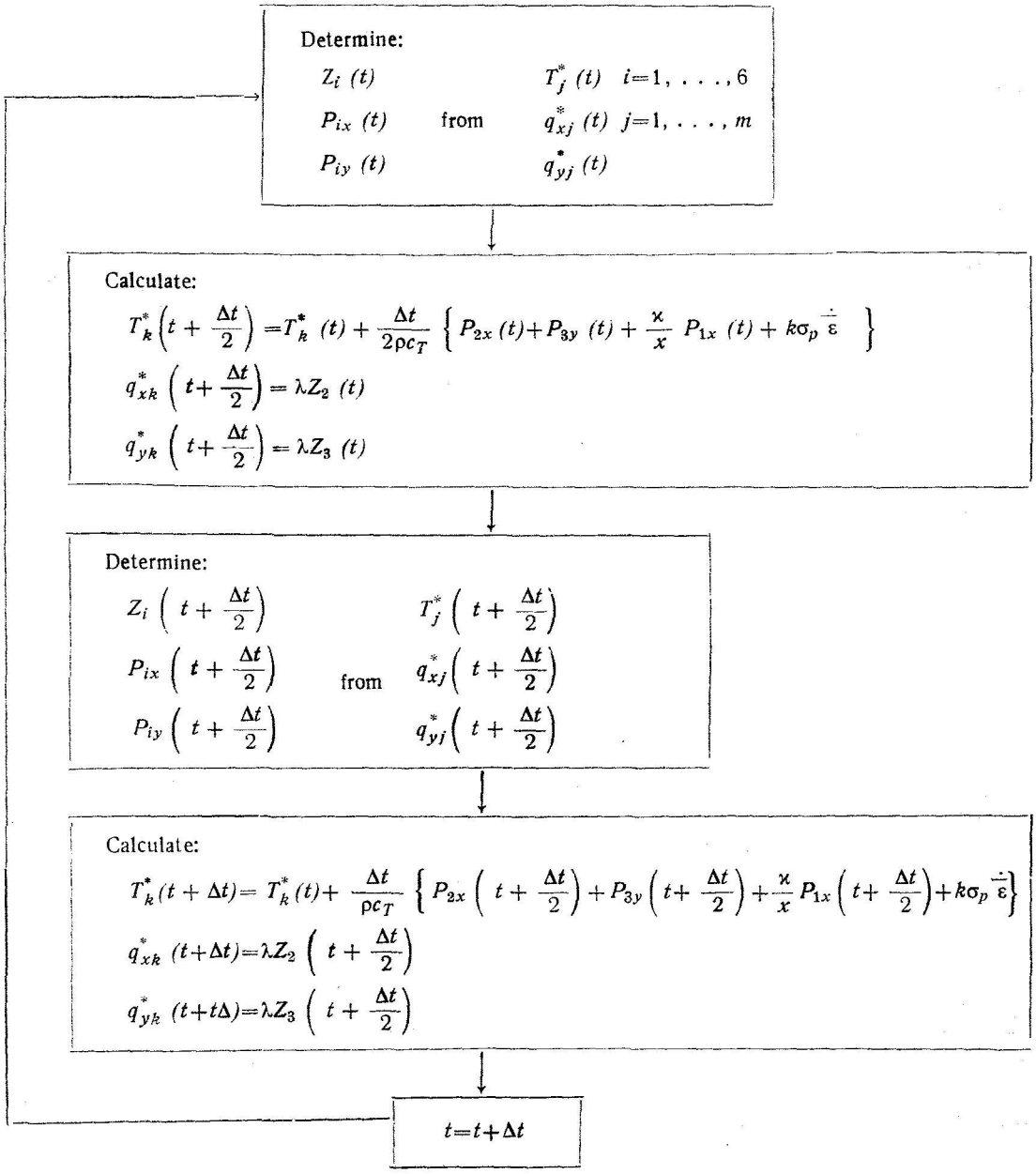

Fig. 2

This is the stability condition $(4.21)_1$. Obviously, decreasing the finite element size leads to a rather strong decreasing of the time-step in the temperature solution.

It is known in literature, that a good stability in the numerical solution is obtained applying either the Lax-Wendroff method [14], or an implicit scheme for time integration.

The temperature equation (4.25) takes in the two-dimensional case the following form:

$$\frac{dT}{dt}=\frac{1}{\rho c_T}\left[\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\varkappa}{x}q_x+k\sigma_p\dot{\varepsilon}\right]$$

(4.34)
$$q_x=\lambda\frac{\partial T}{\partial x}$$

$$q_y=\lambda\frac{\partial T}{\partial y}$$

where $\varkappa=0$ at plane flow and $\varkappa=1$ at axisymmetric flow, at which Oy is the symmetry axis [7].

We assume the following independent approximations for temperature and for both components of the heat flux vector:

$$T(x,\ y,\ t)=Z_1(t)+Z_2(t)\xi+Z_3(t)\eta+Z_4(t)\xi\eta+Z_5(t)\xi^2+Z_6(t)\eta^2$$

(4.35)
$$q_x(x,\ y,\ t)=P_{1x}(t)+P_{2x}(t)\xi+P_{3x}(t)\eta+P_{4x}(t)\xi\eta+P_{5x}(t)\xi^2+P_{6x}(t)\eta^2$$

$$q_y(x,\ y,\ t)=P_{1y}(t)+P_{2y}(t)\xi+P_{3y}(t)\eta+P_{4y}(t)\xi\eta+P_{5y}(t)\xi^2+P_{6y}(t)\eta^2,$$

$$\xi=x-x^{(k)},\ \eta=y-y^{(k)}.$$

Figure 2 shows the flow-chart of the solution according to the Lax-Wendroff method.

The implicit scheme requires solution of the following equations:

$$T_k^*(t+\Delta t)=T_k^*(t)+\frac{\Delta t}{\rho c_T}\left[P_{2x}(t+\Delta t)+P_{3y}(t+\Delta t)+\frac{\varkappa}{x}P_{1x}(t+\Delta t)+k\sigma_p\dot{\varepsilon}\right]$$

(4.36)
$$q_{xk}^*(t+\Delta t)=\lambda Z_2(t+\Delta t)$$

$$q_{yk}^*(t+\Delta t)=\lambda Z_3(t+\Delta t)$$

which lead to an iterative procedure. The flow-chart of the solution is shown in Fig. 3.

According to the methods mentioned above, the temperature $T_k^*(t+\Delta t)$ in all "internal elements" is calculated. "Internal elements" are those elements, whose number of nodes, lying on the surrounding surface of the body is less than two (Fig. 4). The approximation coefficients Z_i, P_{ix} and P_{iy} $(i=1,\ldots,6)$, are obtained by means of the least square method from the temperature and heat-flux values at the "neighbouring elements". Those are all elements, having at least one common node with the considered element. After determining the temperature at the "internal elements" we turn to the "boundary elements". These are elements, having two nodes at the surrounding surface. (Fig.5a). The normal direction through the point $P^{(k)}$ of the considered element intersects the line l, connecting the points $P^{(r)}$ and $P^{(q)}$ of two neighbouring elements at the point $P'(x',\ y')$. We find the temperature T'^* at that point interpolating the temperatures $T_q^*(t+\Delta t)$ and $T_r^*(t+\Delta t)$. We represent the boundary condition (3.4) in the form:

(4.37)
$$\frac{T_k^*(t+\Delta t)-T'^*(t+\Delta t)}{\Delta n}=-\frac{H}{\lambda}\left[T_k^*(t+\Delta t)-T_{\text{ref}}(t+\Delta t)\right]$$

which yields:

(4.38)
$$T_k^*(t+\Delta t)=\frac{1}{1+\frac{H\Delta n}{\lambda}}\left[T'^*(t+\Delta t)+\frac{H\Delta n}{\lambda}T_{\text{ref}}(t+\Delta t)\right],$$

where

$$\Delta n=\sqrt{(x^{(k)}-x')^2+(y^{(k)}-y')^2}.$$

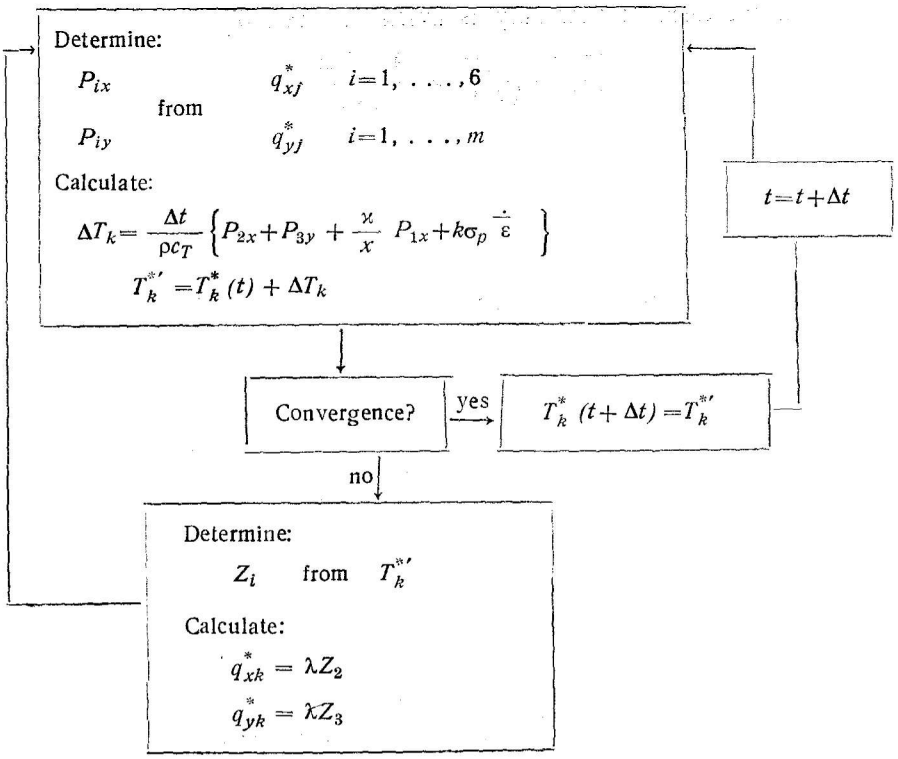
Determine:
P_{ix} q^*_{xj} $i=1, \ldots, 6$
from
P_{iy} q^*_{yj} $i=1, \ldots, m$
Calculate:
$\Delta T_k = \frac{\Delta t}{\rho c_T}\left\{P_{2x}+P_{3y}+\frac{\varkappa}{x}P_{1x}+k\sigma_p\frac{\dot{-}}{\varepsilon}\right\}$
$T^{*'}_k = T^*_k(t)+\Delta T_k$
$t=t+\Delta t$
Convergence?
yes
$T^*_k(t+\Delta t)=T^{*'}_k$
no
Determine:
Z_i from $T^{*'}_k$
Calculate:
$q^*_{xk}=\lambda Z_2$
$q^*_{yk}=\lambda Z_3$

Fig. 3

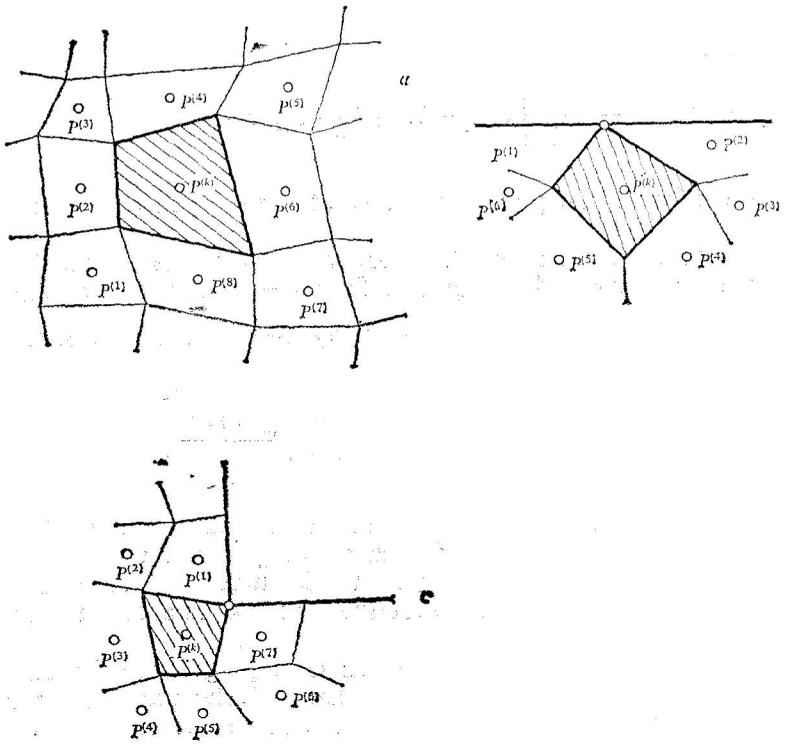
a
b
c

Fig. 4

If friction at the element boundary is available, then [7]:

$$\left(\frac{\partial T}{\partial n}\right)^{-}-\left(\frac{\partial T}{\partial n}\right)^{+}=\frac{mk_s\sigma_p|\Delta v_t|}{\sqrt{3}\lambda}. \tag{4.39}$$

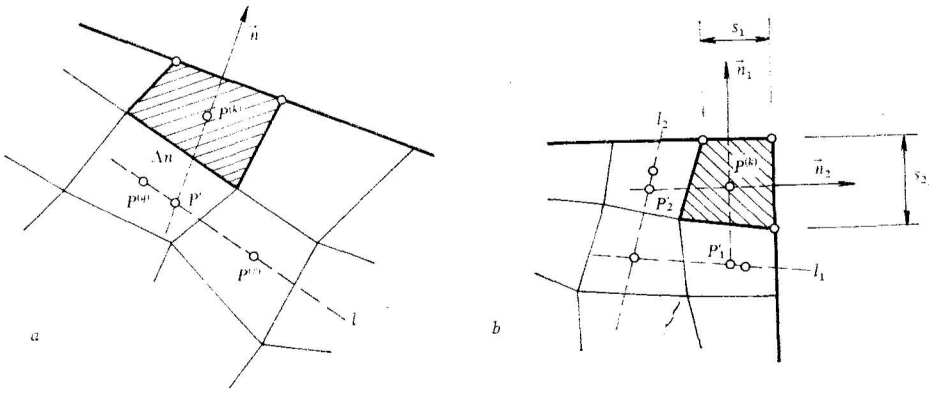

Fig. 5

We assume:

$$\left(\frac{\partial T}{\partial n}\right)^{+}=-\frac{H}{\lambda}[T_k^*(t+\Delta t)-T_{\mathrm{ref}}(t+\Delta t)]$$
$$\left(\frac{\partial T}{\partial n}\right)^{-}\approx\frac{T_k^*(t+\Delta t)-T'^*(t+\Delta t)}{\Delta n} \tag{4.40}$$

and it follows that:

$$T_k^*(t+\Delta t)=\frac{1}{1+\frac{Hsn}{\lambda}}\left[T'^*(t+\Delta t)+\frac{H\Delta n}{\lambda}T_{\mathrm{ref}}(t+\Delta t)+\frac{mk_s\sigma_p|\Delta v_t|\Delta n}{\sqrt{3}\lambda}\right], \tag{4.41}$$

where Δv_t is the relative tangential velocity of both surfaces, where friction takes place.

At the end the temperature of the "corner elements" is obtained. These are elements with three boundary nodes (Fig. 5b). Such elements have two outward normals. The temperatures $T^*_{k(1)}(t+\Delta t)$ and $T^*_{k(2)}(t+\Delta t)$ by means of the normals $\vec{n}_1$ and $\vec{n}_2$ are calculated and then:

$$T_k^*(t+\Delta t)=\frac{T^*_{k(1)}(t+\Delta t)s_1+T^*_{k(2)}(t+\Delta t)s_2}{s_1+s_2} \tag{4.42}$$

is found.

Now the temperature in all finite elements is obtained at time $t+\Delta t$ and the next time step may ce undertaken. It is worth mentioning, that the time step for solving the temperature problem must not be equal to the time step used in the mechanical solution. Moreover, the latter should be derived into a number of substeps, in order to satisfy the condition (4.33).

According to the described method the computer programme TFARM was built for solving two-dimensional nonstationar coupled thermoplastic problems of metal forming. The programme uses the programme FARM, in which some changes were made.

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