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A Study of the Nonlinear Behaviour of Reinforced Concrete Frame to Strong Ground Motions

Introduction

Earthquakes have caused considerable loss of life and property damage in the Balkan Region and are considered as the most important natural hazard in this region. Improved building practices leading to better structures to resist the seismic forces are effective means for reducing these losses. Buildings designed as earthquake-resistant structures should be able to resist frequent, minor earthquakes without any significant damage in the nonstructural components. Such structures should resist the moderate earthquakes without significant structural damage. In case of severe seismic action, the structure should be able to resist the earthquake loading without major failure of the structural system to maintain life and to minimize major economical and cultural losses. For the satisfactory behaviour of the structure in the inelastic range, the members and connections should have adequate ductility and energy dissipation capacity. It should be noted that the designer has different means to compute the system ductility of the structure. One of the ways is the study of experimental behaviour of members and structures. The other way used is "time history linear and nonlinear dynamic analysis" as a research tool.

The main aim of this paper is to present basic principles and methods in the above mentioned analysis of earthquake resistant reinforced concrete frames and to describe, in a comparative way, the behaviour of R/C frame subjected to two earthquakes.

Outline of numerical model

Some simplifying assumptions are adopted in the structural idealization of the frame: masses are lumped at floor levels, and represent the total mass of the structure, soil-structure interaction is not included and the dynamic excitation is applied at the supports. It is also assumed that viscous damping is a linear combination of the constant mass matrix and the variable stiffness matrix; the resulting viscous forces are assumed to act at floors levels. The inelastic models for seismic analysis involve parameters whose definition is complicated; simple structural models are useful for knowing the behaviour of the structures only in a general context. A simplified model for inelastic systems must include relevant parameters such as the hysteretic behaviour. Accurate models have been proposed by Otani [1] correlating the response of a multi-degree-of-freedom system (MDOF) to that of equivalent shear beam-model. The latter

models use a trilinear moment-curvature relationship for the structural members (cracking, first yield in the tensile steel, and crushing of the most stressed fibre of concrete), and Takeda's hysteresis rules.

In this paper for the dynamic analyses a version of the programme SAKE (2)

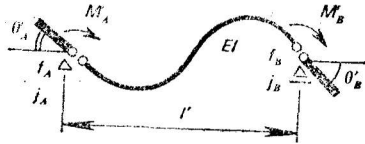


Fig. 1. Moment-rotation relation of flexible portion of member

implemented in a IBM 370 computer is used. The programme allows modelling structural members with four different elements (Fig. 1): a flexible elastic line element, an inelastic flexural spring, a spring to simulate the perfect bond between concrete and steel reinforcement, and a rigid element at each end. The inelastic moment-rotation relationship of a flexural spring under load reversals is assumed to follow Takeda hysteresis rules (2). The hysteresis rules are developed on the basis of observed behaviour of many reinforced concrete members under static lateral load reversals with light to medium amount of axial loads. The major characteristics of the hysteresis rules are: (1) the stiffness changes with a stress (strain) history, and (2) some energy can be dissipated even from a small amplitude stress reversal.

The method of analyzing a nonlinear structure can be summarized as follows:

1. Linearization of a nonlinear member end force-displacement relation by an instantaneous stiffness matrix for a small increment of force and displacement. The member stiffness matrix is formed as follows: The rotation at support A' of a member between two rigid zones is the sum of the rotations of two rotational springs (instantaneous flexibility constants of f_A and j_A) and a rotation at the end of the flexible elastic line element (flexural rigidity of EI and length l'), (Fig. 1). Therefore, in an incremental form:

$$(1) \quad \begin{Bmatrix} \Delta \theta'_A \\ \Delta \theta'_B \end{Bmatrix} = \begin{bmatrix} (l'/3EI + f_A + j_A) & -(l'/6EI) \\ -(l'/6EI) & (l'/3EI + f_B + j_B) \end{bmatrix} \cdot \begin{Bmatrix} \Delta M'_A \\ \Delta M'_B \end{Bmatrix}.$$

This relation can be easily rewritten by inverting the flexibility matrix into a form

$$(2) \quad \begin{Bmatrix} \Delta M'_A \\ \Delta M'_B \end{Bmatrix} = \begin{bmatrix} k'_{11} & k'_{12} \\ k'_{21} & k'_{22} \end{bmatrix} \cdot \begin{Bmatrix} \Delta \theta'_A \\ \Delta \theta'_B \end{Bmatrix}.$$

Incremental moments ΔM_A and ΔM_B at the ends of a member are related to incremental moments $\Delta M'_A$ and $\Delta M'_B$ at the rotational spring by the following matrix relation:

$$(3) \quad \begin{Bmatrix} \Delta M_A \\ \Delta M_B \end{Bmatrix} = \begin{bmatrix} 1 + \beta_A & \beta_B \\ \beta_A & 1 + \beta_B \end{bmatrix} \begin{Bmatrix} \Delta M'_A \\ \Delta M'_B \end{Bmatrix}$$

in which β_A, β_B are shown in Fig. 2

Incremental rotation $\Delta \theta_A$ and $\Delta \theta_B$ at the ends of a member and those $\Delta \theta'_A, \Delta \theta'_B$ inside rigid zones are related in the following form:

$$(4) \quad \begin{Bmatrix} \Delta \theta'_A \\ \Delta \theta'_B \end{Bmatrix} = \begin{bmatrix} 1 + \beta_A & \beta_B \\ \beta_A & 1 + \beta_B \end{bmatrix} \begin{Bmatrix} \Delta \theta_A \\ \Delta \theta_B \end{Bmatrix}.$$

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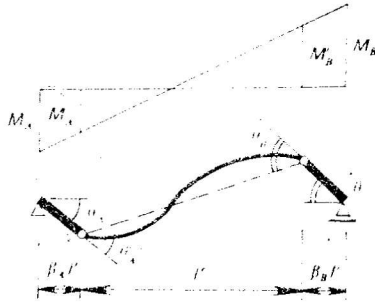


Fig. 2. Moment-rotation relation of member

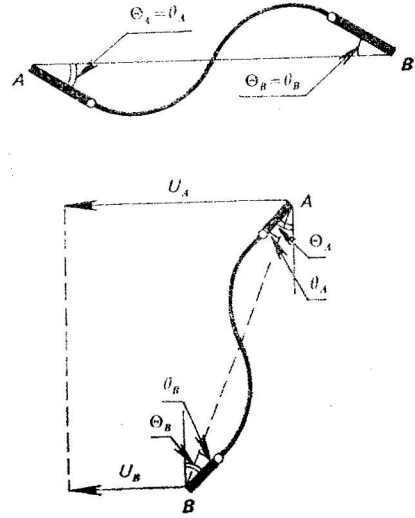


Fig. 3. Displacements at joints and member ends

By combining Eqs (2), (3), (4) a stiffness relation of a member can be formulated for member end moments and rotations in a following form:

$$(5) \quad \begin{Bmatrix} \Delta M_A \\ \Delta M_B \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} \Delta \theta_A \\ \Delta \theta_B \end{Bmatrix}.$$

For a beam member with the horizontal global coordinate axis, no further transformation is necessary to express local member end rotations and moments in the global coordinates. However, a member axis of a column is rotated by lateral column end displacement (Fig. 3). Some transformation is required to express global member end displacements in the local member coordinate. Transformation matrix $[C]$ can be expressed as

$$(6) \quad [C] = \begin{bmatrix} 1 & 0 & -1/l & 1/l \\ 0 & 1 & -1/l & 1/l \end{bmatrix}$$

in which l —total length of a column.

The member end rotations can be expressed by lateral displacement U_A and U_B , and rotations $\Delta \theta_A$ and $\Delta \theta_B$, using transformation matrix $[C]$.

$$(7) \quad \begin{Bmatrix} \Delta \theta_A \\ \Delta \theta_B \end{Bmatrix} = [C] \begin{Bmatrix} \Delta \theta_A \\ \Delta \theta_B \\ U_A \\ U_B \end{Bmatrix} \quad (8) \quad \begin{Bmatrix} \Delta M_A \\ \Delta M_B \\ \Delta P_A \\ \Delta P_B \end{Bmatrix} = [C]^T \begin{Bmatrix} \Delta M_A \\ \Delta M_B \end{Bmatrix}.$$

In an incremental form the transformation of member end forces in the local member coordinates to those in the global coordinates can be written in the form (7). By combining Eqs (5), (7), (8) the stiffness relation of a column can be formulated in a form:

$$(9) \quad \begin{Bmatrix} \Delta M_A \\ \Delta M_B \\ \Delta P_A \\ \Delta P_B \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} \Delta \Theta_A \\ \Delta \Theta_B \\ \Delta U_A \\ \Delta U_B \end{Bmatrix}.$$

The above mentioned relations are obtained in [2].

Formulation is made of an instantaneous structural stiffness matrix on the basis of the compatibility relation of displacement at a joint and the equilibrium of external forces. The structural stiffness matrix has related incremental joint displacements to incremental joint forces.

$$(10) \quad \begin{Bmatrix} \Delta P \\ \Delta M \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} \Delta U \\ \Delta \Theta \end{Bmatrix}$$

in which $[K_{11}]$ — symmetric submatrix of size, number of stories by number of stories; $[K_{21}]$ — submatrix of size, number of joints by number of stories; $[K_{22}]$ — symmetric banded submatrix of size number of joints by number of joints; $\{\Delta P\}$ — vector which contains incremental horizontal forces at each floor level, $\{\Delta M\}$ — vector of incremental bending moments at each joint; $\{\Delta \Theta\}$ — vector of incremental rotation at each joint; $\{\Delta U\}$ — vector of incremental horizontal displacements at each floor level.

Incremental displacements ΔU are computed as a numerical solution of the equation of motion.

3. Formulation of the equation of motion on the basis of the equilibrium condition of all forces at each joint.

4. Numerical integration of the ordinary differential equations. The governing equation of motion is given by

$$(11) \quad [M]\{\ddot{X}\} + [C]\{\dot{X}\} + \{k(X)\} = -[M]\ddot{y}(t),$$

where $[M]$ and $[C]$ — mass and damping matrices; $\{X\}$, $\{\dot{X}\}$, $\{\ddot{X}\}$, $\{k(X)\}$ — relative displacement, velocity, acceleration and restoring force vectors; $\ddot{y}(t)$ — ground acceleration.

The above described equations are solved using Newmark's step-by-step integration method, with parameters corresponding to the linear acceleration scheme. Existing experience shows that a good approximation to the actual dynamic responses is attained only if the time step is chosen as a fraction of the smallest period of the system. In the linear variation scheme this step must satisfy the stability requirements. In structures with elastic behaviour good results are obtained if a step is used equal to $T_n/10$, T_n being the smallest period. In inelastic structures restrictions in the step size are more stringent because the contributions of higher modes are normally significant for the response; inherited errors due to round-off become important and above all changes of branches in the evaluation of the inelastic parameters must be precisely defined. Otani recommends use of a time step equal to $T_n/20 - T_n/50$ (1).

In this paper we use $T_n/20$ because the predicted structural response has been practically invariant with smaller time steps. However, more investigation on this topic is needed, since in the definition of the time step the relation between structural periods and the prevailing periods of the ground motion, i. e. those corresponding to the maximum energy content in the record are not considered. More detailed information about Newmark's method and a comparative analysis with other similar methods can be found in [3, 4].

Calculation of energy response

1. The energy balance equation

The energy balance equation at any time can be expressed as

$$(12) \quad E_K + E_D + E_P + E_H = E_I$$

The terms of the equation represent kinetic energy E_K , energy dissipated due to viscous damping E_D , elastic potential energy E_P , hysteretic energy E_H and the total input energy E_I , respectively.

This equation is obtained from Eq. (11) after multiplying by relative velocity vector both its sides and integrating over the duration time in (5).

At the end of response, both E_K and E_P are zero, so that Eq. (12) becomes

$$(13) \quad E_D + E_H = E_I$$

The energy input E_I as the load effect can be represented at its final state:

$$(14) \quad E_I = -[M] \int_0^t \ddot{y}(t) \{ \dot{X} \} dt.$$

Both energy dissipated E_D and input energy E_I can be determined through numerical integration of Eq. (11). Therefore, E_H can be determined from Eq. (13) at the end of seismic excitation.

2. The behaviour factor

In this study the behaviour factor is defined as the ratio of the total hysteretic energy to the total input elastic energy

$$(15) \quad K = E_H / E_I.$$

Numerical example

Two-spans three storey r. c. frame (Fig. 4) is investigated. It is designed in accordance with Bulgarian aseismic code [6] for seismic load, corresponding to seismicity of 8th degree, design acceleration 0.15 g and medium ground conditions. The fundamental natural period of the structure is 0.43 s. The input seismic ground motion is represented by accelerogram-NS component of the Bucharest record of Vrancea 1977 earthquake. The record is scaled to $a_{\max} = 0.15g$ — case 1 and $a_{\max} = 0.27g$ — case 2.

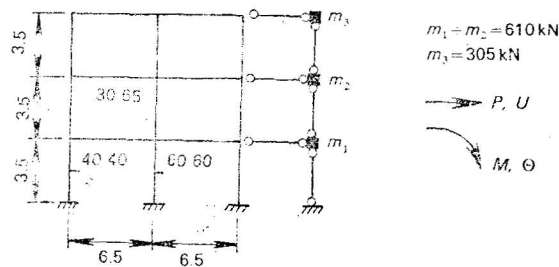


Fig. 4. Frame model

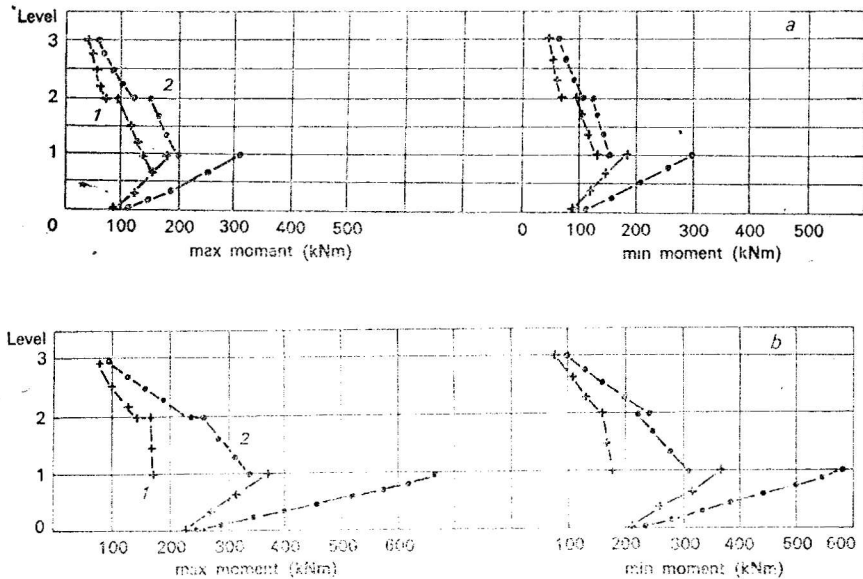


Fig. 5. Maximum flexural response of members as a unit-length simple member to accelerogram No 1
 a — exterior columns; b — interior column
 1 — nonlinear analysis; 2 — linear analysis

Table 1
 Numerical results

Acc.	Base shear force		Base moment		Displ.		Energy		Behaviour factor
	max min (kN)		max min (kNm)		max min (cm)		(kJ)		
	L	NON	L	NON	L	NON	E_H	E_I	$k=E_H/E_I$
1 Vo 15	+488 -550	+369 -357	+3831 -4211	+2644 -2611	+2.0 -2.2	+1.7 -1.8	8.7	10.	0.87
2 Vo 27	927 -1045	+599 -632	+7287 -8009	+4056 -4598	+4.1 -3.8	+25. -13.	130.	36.	3.5

Vo 15 — accelerogram Vrancea $A_{\max} = 0.15$ g.
 Vo 27 — accelerogram Vrancea $A_{\max} = 0.27$ g.

The time-history structural analyses are performed both in elastic and inelastic range. The variation of some results for the frame under investigation and accelerogram-case 1 is shown in Fig. 5. Both linear and nonlinear responses are considered. The maximum values of the base-shear force, base moment and maximum displacements for this frame and accelerogram (case 1 and case 2) are given in Table 1.

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In the same table the values of the hysteretic energy and maximum input energy are presented. The values of the behaviour factor are also given in Table 1. The time-history of the input, dissipated and hysteretic energy is shown in Figs 9, 10.

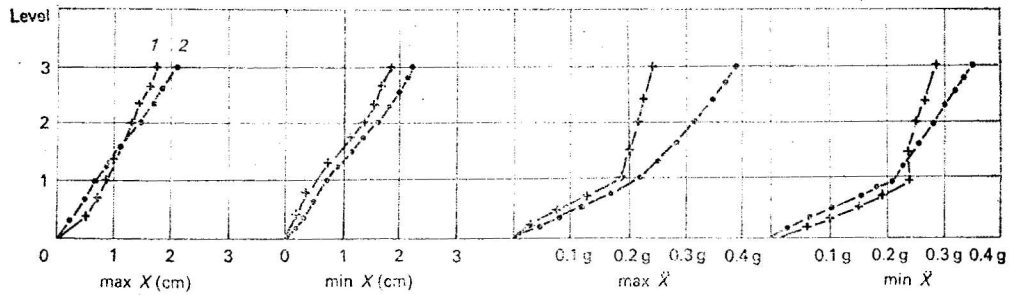


Fig. 6. Maximum displacement and acceleration response
1 — nonlinear response; 2 — linear response

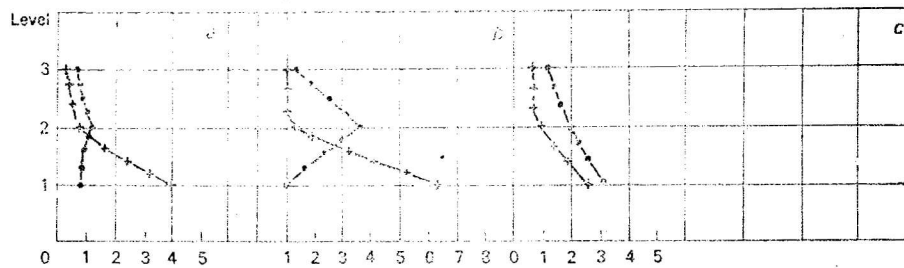


Fig. 7. Member ductilities response to accelerogram No 1 — $a_{\max} = 0.15$ g
a) exterior columns; b) interior column;
(1 — bottom end; 2 — top end)
c) beams (1 — left end; 2 — right end)

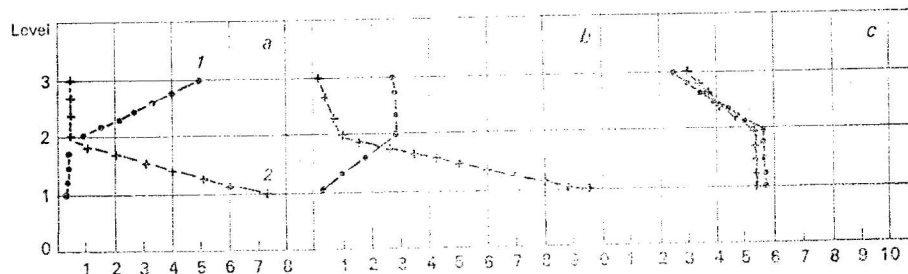


Fig. 8. Member ductilities response to accelerogram No 2 — $a_{\max} = 0.27$ g
a) exterior columns; b) interior column
(1 — bottom end; 2 — top end)
c) beams (1 — left end; 2 — right end)

The graph in Fig. 6 demonstrates that for the accelerogram-case 1, the lateral displacement diagrams have closed values and shapes for both linear and nonlinear analyses. From accelerogram of one intensity to another the maximum displacements

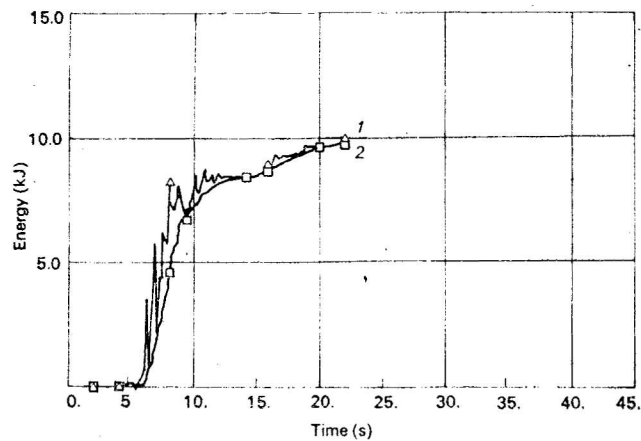


Fig. 9. Linear energy response
(1 — total input energy; 2 — dissipated vis. energy)

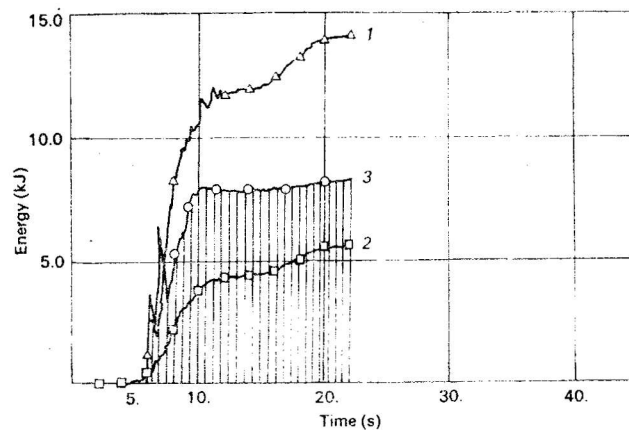


Fig 10. Nonlinear energy response
(1 — total input energy; 2 — dissipated vis. energy; 3 — hysteresis energy)

have very different values (Table 1). Also the ductility factors for the structure elements have very different values for both cases (Figs. 7 and 8). Thus, the ductility factors are between 2-3 times greater for columns in the second case. It should be noted that rotation ductility is taken as a measure of the amount of plastic deformations which the structure must develop. It is defined, for each member end, as the ratio of

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the maximum absolute value attained during the inelastic response to the yield rotation. The yield rotation defines the elastic limit in the moment-rotation diagram, and corresponds to the beginning of reinforcement yielding. Column and beam ductilities deve-

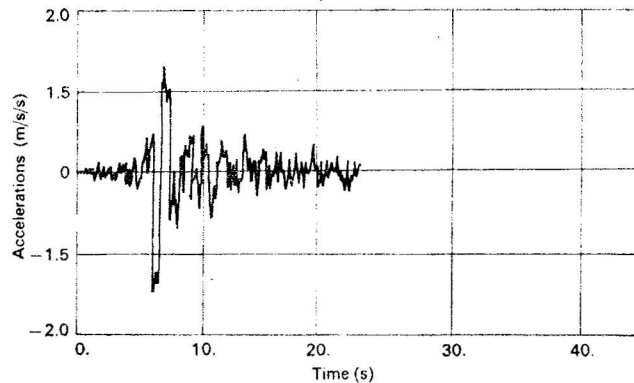


Fig 11. Accelerograma Vrancea—No 1 — $a_{\max} = 0.15 \text{ g}$

loped during the response to accelerogram-case 1 are shown in Fig. 7. It can be seen that plastic deformations are developed preferably by interior column in the first story. But in the second case many plastic deformations occur in all members. We have performed more accurate analysis, using the programme, developed in (4). It becomes clear that in the small time interval (between 5.49 s and 7.4 s of seismic excitation) many plastic hinges occur in the frame—in beams and in columns at first story. The frame stiffness is significantly reduced. Due to the additional effects of the axial forces a structural instability occurs. In this case we have undesirable column-sideway mechanism. In this study the behaviour factor (Table 1) demonstrates the above mentioned behaviour. In the case 1 the structure does not collapse. The behaviour factor is 0.85. The maximum values of the base-shear force and moment in the nonlinear time-history analysis are about 32% less than these in linear analysis in case 1.

Conclusions

The results obtained in this study could be summarized as follows:

1. The comparative analysis presented in this paper points out significant quantitative and qualitative differences between the computed responses of one and same frame, subjected to earthquake with different maximum acceleration.
2. The maximum acceleration is one of the important characteristics of the earthquake which has the quantitative effect on the linear and nonlinear response.
3. The simple structural model in this programme is useful to know the behaviour of the structures only in a general context.
4. The relationship between hysteretic energy and total input energy is important index of aseismic safety of the structure.
5. The reinforced concrete members have different deformational capacities which result in different ductilities. The aseismic structures should have sufficient ductility, which is one of the objectives of the aseismic design. On the other hand, the dynamic

response of structures to actual earthquakes will show the precise relationship between ductility and earthquake.

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