

V. Galabov

Synthesis of Path Generation Mechanisms

1. Introduction

Path generation mechanisms (PGM) are widely used in machine automation. As a component part of various mechatronic technical devices, generating the geometry of target motion PGM increase the reliability and economic efficiency in large and medium-scale production technological processes. Control of the PGM output link with respect to velocity is independent and most often is managed by microprocessors. Vast literature is devoted to the synthesis of path generation linkages [1, 2, 3, 4]. Their limited kinematic possibilities actually are the reason to pose the problem for synthesis of PGM with higher pairs and to yield initial solutions which exclude the alternative to introduce in advance various geometric, kinematic and dynamic restrictions [5, 6].

In the present paper a differential approach as proposed for the representation and synthesis of PGM in order to create conditions for preliminary introduction of various restrictions and to simplify the procedure for PGM synthesis. The PGM structure is presented as superposition of a provisionally OKC (OKC) and function generator mechanisms (FGM) with closed kinematic chains, connecting the links of the chain, which is rather different from the well-known approach for provisional dividing of the mechanism, thus breaking its kinematic chain into open kinematic chains without repeating links [7, 8].

2. Basic functions and structures of path generation mechanisms

PGM serve for kinematic guidance (leading) of a body in space according to a specified geometric law of motion which in the general case includes generation of a trajectory of a point from the body and its orientation.

Path generation linkages in the general case generate approximately a specified trajectory. Various methods for synthesis of path generation linkages are known, precisely, generating definite finitely separated positions under minimax or mean quadratic deviation from a specified geometric law of motion in intermediate positions [2, 3, 4]. If the deviations are greater than the admitted ones, the whole work done related to the synthesis of path generation linkages has been in vein. In another approach the synthesis of path generation linkages is treated as a partial case of the synthesis of mechanisms with higher pairs using a criterion for maximally contracted centrode (evolute) [9, 10]. This approach allows simultaneously to obtain solutions for several types

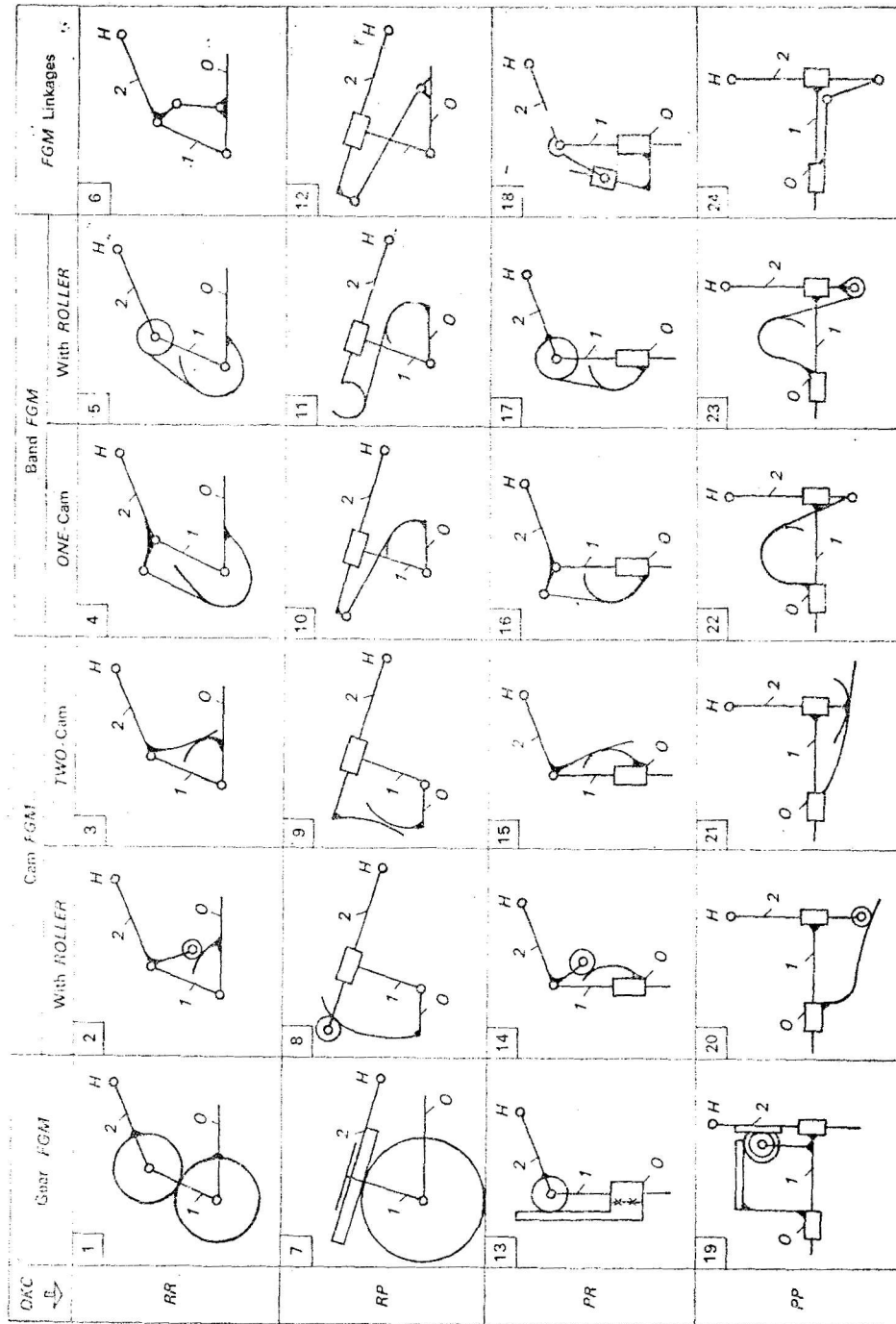


Fig. 1. Different types of path generation mechanisms

of mechanisms and, following certain criteria, to separate one or several perspective solutions, i. e. to accomplish type-dimensional synthesis [11, 12].

A differential approach for synthesis of PGM is possible provided that their structure is subdivided into one provisionally OKC with links and $(n-2)$ FGM, connecting kinematically every second link of OKC [13, 14]. The minimal number of OKC links allowing to generate a plane trajectory is $n=3$. Four structures of three-link OKC are known and they can be denoted by the type of kinematic pairs between the links: *RR*, *RP*, *PR* and *PP*.

Different types of PGM are schematically represented in four lines in Fig. 1 with *RR*, *RP*, *PR* and *PP* structure of a provisionally OKC consisting of links 0, 1 and 2. FGM in the first column is a gear mechanism with circular gears or with non-circular gears if the FGM transfer relation is variable. The FGM in the second column is a cam mechanism with a cam as an input link, fixed to the link 0 and to the follower 2 with a roller. When the cam and the roller change places the result is a profiled-follower mechanism, or, the so-called inverse cam mechanism. The FGM in the third column is with two cams. If they are shaped along the centrodes of the relative motion of link 2 with respect to link 0 the result will be a unique centrode PGM for a provisionally given OKC and trajectory. If the profile of the cam to link 2 is straight-line an inverse cam mechanism with flat input link will be obtained. The FGM in the fourth column is band-type with one cam. If the cam is connected to link 2 then the FGM is band profiled-follower mechanism. The FGM in the fifth column is a band mechanism with roller follower 2. When the cam and the roller change places the result is a band mechanism with profile follower 2 and roller input link. If the circular roller is substituted by a non-circular, a twin-cam band mechanism is obtained. The FGM in the sixth column is a lever one. It can be treated as a partial case of band mechanisms, the cams contracting into points (joints) and the band is replaced by a connecting lever. If the two points are finitely distant then they define the axes of two *R* pairs of the connecting lever, as can be seen in schemes 6, 12 and 24, the FGM being correspondingly a four-bar linkage, a slider-crank mechanism and a double-slider mechanism. If the point in the link 0 plane is infinitely distant, a connecting slide-block appears which transforms the four-bar linkage in Scheme 6 into a slider-crank mechanism, and the slider-crank mechanism in Scheme 12 is transformed into a double-slider mechanism, the sliders being placed between joints. A mechanism with two consecutively placed slide blocks is the one in Scheme 18. The substitution of any *R* pair on Scheme 24 by a *P* pair leads to a wedge mechanism. If the joint from the link 2 plane is infinitely distant it is again replaced by a slider: the four-bar linkage in Scheme 6 is transformed into an inversion of slider-crank mechanism; the slider-crank mechanism in Scheme 12 is transformed into a mechanism with two consecutively placed sliders; the mechanism in Scheme 18 is transformed into a wedge mechanism. Provided that both joints in Scheme 6 are at the same time infinitely distant, the result is an inversion of Scotch-Yoke mechanism. Thus, by introducing sliders in the cases when the synthesized joints are infinitely distant, the well-known seven anisomorphous structures of four-linkage mechanisms are obtained.

For plane guidance of a rigid body we need at least one more moving link 3 of a provisionally OKC connected to link 2 by a joint and one more FGM connecting kinematically links 1 and 3, link 2 serving as a frame. In principle, connection of links 2 and 3 by a slider is possible and the total number of anisomorphous structures of the four-link OKC rises to seven, however, it is rarely applied since the angle orientation is not accomplished by the last kinematic pair. For links 1, 2 and 3, together

with the connecting FGM, the same schemes found in Fig. 1 are valid and the same considerations about their structures hold.

The choice of the provisionally OKC structure depends on the concrete manipulative problem posed and on the possibilities for structural decoupling of motions. OKC with RR structure have universal application. They are suitable for manipulators which generate trajectories close to an arc of a circle, hypocycloid, epicycloid and their derivatives: ellipses, line segments, polygons with rounded vertices or with cusps, etc. Bearing also in mind the favourable constructive and exploitation parameters of R , pairs, OKC with RR structure are to be preferred except in the cases when the trajectory is close to rectilinear with large stroke. Then PR or PP structures are to be preferred allowing structural decoupling of motions. In the case of spiral trajectories or trajectories close to an arc of a circle, RP structures of OKC might be used. A precise choice of a structure can be made only when the problems for dimensioning and dislocating of provisionally OKC have been solved as well as the transfer functions of FGM have been determined according to geometric, kinematic and dynamic restrictions imposed.

3. Provisionally open kinematic chain dimensioning and dislocating

The method for searching circle quadratic points [4] allows to calculate three unknown parameters of a three-link provisionally OKC with RR structure. A version of this method is the method for approximation synthesis with the help of an algorithm for putting in a binary link when three unknown parameters are also calculated [15]. The method of maximally contracted centrode (evolute) which will be developed for the synthesis of PGM is characterized by its geometric clearness and by a simplified algorithm.

Maximally contracted centrode method. In view of the Euler-Chasles theorem, under instantaneous motion of one link with respect to another, there exists an instantaneous rotation centre (IRC) of both points through which the normal to the trajectory of each point from the first link passes, described in the plane of the other link. Therefore, the envelope of the normals to the trajectory is an evolute of the trajectory and at the same time — a locus curve of IRC — centrode. Its conjugate centrode is the normal to the trajectory of the point. Both centrodes in the planes of the two links form a kinematic pair which determines entirely the geometry of motion of the first link with respect to the other one. If the centrode degenerates at a finitely distant point, then IRC has a fixed position in the planes of the two points and defines the position of the axis of a revolute pair (R) connecting the two links in a mobile way. If the centrode degenerates at an infinitely distant point then its practical realization is accomplished by a prismatic pair (P). The method of maximally contracted centrode allows to calculate the values of the parameters of a provisionally OKC from the condition for minimal deviation of the trajectory of the output link of the chain characteristic point H in the plane of the neighbouring link from a circle (or a straight line) which can be judged by the maximal contraction of the evolute of the relative trajectory.

Determination of the centrode by the differential geometry apparatus. The trajectory τ of the point H is specified by parametric equations $x_H = x_H(t)$, $y_H = y_H(t)$ and the rotation of link 1 with respect to link 0, described by the equation $\varphi = \varphi(t)$. The trajectory $\tilde{\tau}$ of the point H is determined in the initial position 0 of the plane $\tilde{x}\tilde{y}$ of link 1 with axes $\tilde{x} \parallel x$ and $\tilde{y} \parallel y$, intersecting at the rotation centre between the links 0 and 1:

$$(1) \quad \begin{bmatrix} \tilde{x}_H \\ \tilde{y}_H \end{bmatrix} = [R_{-\varphi}] \begin{bmatrix} x_H - x_O \\ y_H - y_O \end{bmatrix},$$

where

$$(2) \quad [R_{-\varphi}] = \begin{bmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{bmatrix}$$

is a rotation matrix of the angle $(-\varphi)$.

After a consecutive differentiation we obtain

$$(3) \quad \begin{bmatrix} \dot{\tilde{x}}_H \\ \dot{\tilde{y}}_H \end{bmatrix} = [R_{-\varphi}] \begin{bmatrix} \dot{x}_H + (y_H - y_O) \dot{\varphi} \\ \dot{y}_H - (x_H - x_O) \dot{\varphi} \end{bmatrix},$$

$$(4) \quad \begin{bmatrix} \ddot{\tilde{x}}_H \\ \ddot{\tilde{y}}_H \end{bmatrix} = [R_{-\varphi}] \begin{bmatrix} \ddot{x}_H + 2\dot{y}_H \dot{\varphi} + (y_H - y_O) \ddot{\varphi} - (x_H - x_O) \dot{\varphi}^2 \\ \ddot{y}_H - 2\dot{x}_H \dot{\varphi} - (x_H - x_O) \ddot{\varphi} - (y_H - y_O) \dot{\varphi}^2 \end{bmatrix}.$$

The parametric equations of the centrode $\tilde{C}$ in the initial position of the plane of link 1 are determined by means of the expressions (1), (2), (3) and (4) as equations of an evolute

$$(5) \quad \begin{aligned} \tilde{x}_C(t) &= \tilde{x}_H(t) - p(t) \dot{\tilde{y}}_H, \\ \tilde{y}_C(t) &= \tilde{y}_H(t) + p(t) \dot{\tilde{x}}_H, \end{aligned}$$

where

$$(6) \quad p(t) = (\dot{\tilde{x}}_H^2 + \dot{\tilde{y}}_H^2) / (\dot{\tilde{x}}_H \ddot{\tilde{y}}_H - \dot{\tilde{y}}_H \ddot{\tilde{x}}_H).$$

The curvature radius of τ is determined by the expression

$$(7) \quad \tilde{\rho}_H = (\dot{\tilde{x}}_H^2 + \dot{\tilde{y}}_H^2)^{1.5} / (\dot{\tilde{x}}_H \ddot{\tilde{y}}_H - \dot{\tilde{y}}_H \ddot{\tilde{x}}_H)$$

or as a distance of the point H to the corresponding centre C of curvature

$$(8) \quad \tilde{\rho}_H = [(\tilde{x}_H - \tilde{x}_C)^2 + (\tilde{y}_H - \tilde{y}_C)^2]^{0.5}.$$

Determination of the centrode by small displacement method. The trajectory τ is specified by subsequent close standing positions $H_j(x_{Hj}, y_{Hj})$ of the point H , to which correspond values φ_j of the angle φ ($j=0, 1, 2, \dots, n$). If it is assumed that the coordinates of point $O(x_O, y_O)$ are known, then it is possible to find a sequence of points $\tilde{H}_j(\tilde{x}_{Hj}, \tilde{y}_{Hj})$ in the initial position of the plane of link 1 using the expressions (1) and (2) by adding a subscript j after the letters H and φ . Through each three subsequent points H_{j-1}, H_j, H_{j+1} a circle, passes as follows:

$$(9) \quad D_j = \begin{bmatrix} \tilde{x}_H^2 + \tilde{y}_H^2 & \tilde{x}_H & \tilde{y}_H & 1 \\ \tilde{x}_{H_{j-1}}^2 + \tilde{y}_{H_{j-1}}^2 & \tilde{x}_{H_{j-1}} & \tilde{y}_{H_{j-1}} & 1 \\ \tilde{x}_{H_j}^2 + \tilde{y}_{H_j}^2 & \tilde{x}_{H_j} & \tilde{y}_{H_j} & 1 \\ \tilde{x}_{H_{j+1}}^2 + \tilde{y}_{H_{j+1}}^2 & \tilde{x}_{H_{j+1}} & \tilde{y}_{H_{j+1}} & 1 \end{bmatrix}$$

with center

$$(10) \quad x_{Cj} = \frac{(D_{12})_j}{2(D_{11})_j}; \quad y_{Cj} = \frac{-(D_{13})_j}{2(D_{11})_j}$$

and radius

$$(11) \quad l_{2j} = [(\tilde{x}_{Cj}^2 + \tilde{y}_{Cj}^2 + (D_{14})_j / (D_{11})_j)]^{0.5},$$

where $(D_{11})_j$, $(D_{12})_j$, $(D_{13})_j$ and $(D_{14})_j$ are the subdeterminants of the determinant D_j . If $n = \infty$, the points $\tilde{C}_j$ describe the evolute $\tilde{c}$ of the trajectory τ .

Criterion for maximally contracted centrode. The centres $\tilde{C}_j$ of the circles passing through three neighbouring points $\tilde{H}_{j-1}$, $\tilde{H}_j$, $\tilde{H}_{j+1}$ have centroid $\tilde{C}$ with coordinates

$$(12) \quad \tilde{x}_C = \frac{1}{N} \sum_j \tilde{x}_{Cj}; \quad \tilde{y}_C = \frac{1}{N} \sum_j \tilde{y}_{Cj}$$

where for an open trajectory $N = n - 1$, $j = 1, 2, 3, \dots, n - 1$, while for a closed trajectory $N = n + 1$, $j = 0, 1, 2, \dots, n$. For each pair of coordinates $\tilde{x}_O, \tilde{y}_O$ in a domain of the plane xy specified beforehand, corresponding coordinates $\tilde{x}_C, \tilde{y}_C$ of the point C are determined in the plane $\tilde{x}\tilde{y}$ and a mean-arithmetic deviation, is formed as follows:

$$(13) \quad \Delta c = \frac{1}{N} \sum_j [(\tilde{x}_{Cj} - \tilde{x}_C)^2 + (\tilde{y}_{Cj} - \tilde{y}_C)^2]^{0.5}$$

of the points $\tilde{C}_j$ from the point $\tilde{C}$. This pair of coordinates $\tilde{x}_O^*, \tilde{y}_O^*$, respectively $\tilde{x}_C^*, \tilde{y}_C^*$, for which $\Delta c_{\min} = \min \Delta c$ is obtained, defines the lengths

$$(14) \quad l_1 = [\tilde{x}_C^{*2} + \tilde{y}_C^{*2}]^{0.5}; \quad l_2 = 0.5(l_{2\min} + l_{2\max})$$

of links 1 and 2, where $l_{2\min} = \min \overline{C^* \tilde{H}_j}$, $l_{2\max} = \max \overline{C^* \tilde{H}_j}$ ($j = 0, 1, 2, \dots, n$) and their initial position with respect to τ .

An exact solution of the problem posed is obtained only in partial cases when the centrode contracts into a point ($\Delta c_{\min} = 0$). In the general case $\Delta c_{\min} \neq 0$, and the generation of τ is connected with deviations of the angle φ from its specified values φ_j . A set of solutions is possible for which these deviations are admissible. Among all possible solutions a final choice can be made on the basis of criteria for optimality as maximal angle stroke under minimal non-uniformity of velocities and accelerations of link 1 with respect to link 2. Provided that they are satisfied, fast operation, reliability and precision of control could be steadily increased. Other optimality criteria could be related to the dimensions of a provisionally OKC, the angle stroke of link 2 with respect to link 1 which should be agreed with the possibilities of FGM and so on.

Determination of centrodes by matrix-geometric apparatus. The parameters of a provisionally OKC can be calculated without variation of the coordinates of point O , by specifying in addition the equation $\psi = \psi(t)$ describing the rotation of link 2 with respect to link 1, which as a rotation of the FGM follower with respect to the frame, is of substantial significance for the type-dimensional synthesis of FGM and PGM as a whole.

A coordinate system xy and the following displacement matrices are introduced:

$$(15) \quad [U_{j0}]_1 = \begin{bmatrix} c \varphi_{j0} & -s \varphi_{j0} & x_{Cj} - x_{C0} c \varphi_{j0} + y_{C0} s \varphi_{j0} \\ s \varphi_{j0} & c \varphi_{j0} & y_{Cj} - x_{C0} s \varphi_{j0} - y_{C0} c \varphi_{j0} \\ 0 & 0 & 1 \end{bmatrix};$$

$$(16) \quad [U_{j0}]_2 = \begin{bmatrix} c(\varphi_{j0} + \psi_{j0}) & -s(\varphi_{j0} + \psi_{j0}) & x_{C_j} - x_{C_0}c(\varphi_{j0} + \psi_{j0}) + y_{C_0}s(\varphi_{j0} + \psi_{j0}) \\ s(\varphi_{j0} + \psi_{j0}) & c(\varphi_{j0} + \psi_{j0}) & y_{C_j} - x_{C_0}s(\varphi_{j0} + \psi_{j0}) - y_{C_0}c(\varphi_{j0} + \psi_{j0}) \\ 0 & 0 & 1 \end{bmatrix},$$

from initial position 0 into an arbitrarily chosen position j of links 1 and 2, respectively, in the plane of link 0. φ_{j0} and ψ_{j0} correspondingly denote the rotation angles of link 1 with respect to link 0 and of link 2 with respect to link 1 under the displacement of point H from its initial position H_0 into position H_j . The short denotations of sin and cos are s and c .

The point O is the centre of rotation of link 1 with respect to link 0 for which the matrix (15) holds with the substitution $x_{C_0} = x_{C_1} = x_0$ and $y_{C_0} = y_{C_1} = y_0$. The matrix (15) is changing in form but the numerical values of its elements remain the same whence

$$(17) \quad \begin{aligned} x_{C_j} - x_{C_0}c\varphi_{j0} + y_{C_0}s\varphi_{j0} &= x_0 - x_0c\varphi_{j0} + y_0s\varphi_{j0}, \\ y_{C_j} - x_{C_0}s\varphi_{j0} - y_{C_0}c\varphi_{j0} &= y_0 - x_0s\varphi_{j0} - y_0c\varphi_{j0}. \end{aligned}$$

Analogously, if in the matrix (16) we substitute $x_{C_0} = x_{H_0}$, $y_{C_0} = y_{H_0}$ and $x_{C_j} = x_{H_j}$, $y_{C_j} = y_{H_j}$, it changes its form but the numerical values of its elements remain the same whence

$$(18) \quad \begin{aligned} x_{C_j} - x_{C_0}c(\varphi_{j0} + \psi_{j0}) + y_{C_0}s(\varphi_{j0} + \psi_{j0}) &= (c_{13})_{j0}, \\ y_{C_j} - x_{C_0}s(\varphi_{j0} + \psi_{j0}) - y_{C_0}c(\varphi_{j0} + \psi_{j0}) &= (c_{23})_{j0}, \end{aligned}$$

where

$$\begin{aligned} (c_{13})_{j0} &= x_{H_j} - x_{H_0}c(\varphi_{j0} + \psi_{j0}) + y_{H_0}s(\varphi_{j0} + \psi_{j0}), \\ (c_{23})_{j0} &= y_{H_j} - x_{H_0}s(\varphi_{j0} + \psi_{j0}) - y_{H_0}c(\varphi_{j0} + \psi_{j0}). \end{aligned}$$

A system of eight linear algebraic equations is formed, written as a matrix

$$(19) \quad [A][Z] = [B],$$

where

$$(20) \quad [A] = \begin{bmatrix} 1 - c\varphi_{10} & s\varphi_{10} & c\varphi_{10} & -s\varphi_{10} & -1 & 0 & 0 & 0 \\ -s\varphi_{10} & 1 - c\varphi_{10} & s\varphi_{10} & c\varphi_{10} & 0 & -1 & 0 & 0 \\ 1 - c\varphi_{20} & s\varphi_{20} & c\varphi_{20} & -s\varphi_{20} & 0 & 0 & -1 & 0 \\ -s\varphi_{20} & 1 - c\varphi_{20} & s\varphi_{20} & c\varphi_{20} & 0 & 0 & 0 & -1 \\ 0 & 0 & c(\varphi_{10} + \psi_{10}) & -s(\varphi_{10} + \psi_{10}) & -1 & 0 & 0 & 0 \\ 0 & 0 & s(\varphi_{10} + \psi_{10}) & c(\varphi_{10} + \psi_{10}) & 0 & -1 & 0 & 0 \\ 0 & 0 & c(\varphi_{20} + \psi_{20}) & -s(\varphi_{20} + \psi_{20}) & 0 & 0 & -1 & 0 \\ 0 & 0 & s(\varphi_{20} + \psi_{20}) & c(\varphi_{20} + \psi_{20}) & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$(21) \quad \begin{aligned} [B] &= [0 \ 0 \ 0 \ 0 \ -(c_{13})_{10} \ -(c_{23})_{10} \ -(c_{13})_{20} \ -(c_{23})_{20}]^T; \\ [Z] &= [x_0 \ y_0 \ x_{C_0} \ y_{C_0} \ x_{C_1} \ y_{C_1} \ x_{C_2} \ y_{C_2}]^T \end{aligned}$$

and solution

$$(22) \quad [Z] = [A]^{-1}[B].$$

The first four equations of the system (21) are obtained from equations (17) by specifying $j=1$ and $j=2$. The last four equations are obtained from equations (18) by specifying $j=1$ and $j=2$. For each following triplet of positions 1, 2, 3; 2, 3, 4; ..., $n-2$, $n-1$, n the expressions from 19 to 22 hold true and all numeric indices of the coordinates and angles increase by one. Thus, for every triplet of positions the coordinates of the point O are determined, which in the general case do not coincide with those for another triplet of positions, and the coordinates of the point C are found, which in the general case do not coincide with those for another triplet of positions either. For $n=\infty$ the locus curves of the points O and C in the planes of links 0, 1 and 1, 2, resp., define centrodes which, considered as kinematic pairs, will accomplish the specified laws of motion in a precise manner. An approximate solution will be obtained if the centrodes are replaced by their gravity centres O^* and C_0^* , thus defining the positions of R pairs of a provisionally OKC. The satisfaction of the functions $\varphi(t)$ and $\psi(t)$ might be obtained with considerable deviations, nevertheless acceptable, provided they do not exceed admissible limits, specified in advance. Quite a few partial cases exist where the centrodes, as locus curves of the points O and C , contract into a point. Let the trajectory be an ellipse with a large semi-axis a , a small semi-axis b , with parametric equations $x_H=a \cos t$, $y_H=b \sin t$. The angles $\varphi=-t$, $\psi=2t$. Irrespective of the boundaries of the parameter t , contraction of the centrodes at the points $O(0, 0)$ and $C(0, (a+b)/2)$ will be obtained. Contraction of the centrodes could also result in the case of straight line, hypo-cycloid, epicycloid and other trajectories.

4. Synthesis of connecting function generator mechanisms

Determination of the FGM transfer functions. Transfer functions are uniquely determined for a specified structure, dimensions and dislocation of a provisionally OKC. For this purpose, one can apply differential calculus, kinematic and analytic geometry or a matrix-geometric apparatus.

Transfer functions as geometric derivatives of the displacement function are determined in a quite known though not the easiest way, namely the differential calculus apparatus. Tables in Fig. 2 give mathematical expressions with numbers 0, 1 and 2, serving to write the displacement function and the first two transfer functions, resp., of FGM kinematically connecting the two output links 0 and 2 of each of the four three-link provisionally OKC. The displacement function is represented in a parametric form. First the output FGM coordinate is written as a displacement of link 2 with respect to link 1 as a function of the coordinates of point H . After differentiating twice the expressions of the output and input coordinates and certain transformations, particularly for RR and RP schemes of a provisionally OKC, the first and second transfer functions are finally yielded.

A matrix-geometric method for the determination of transfer functions can be built up onto an already elaborated vector-matrix mathematical model for synthesis of cam profiles [19]. We introduce a first differential relative displacement matrix of the relative instantaneous displacement

$$(23) \quad [\dot{U}_0] = \begin{bmatrix} 0 & \psi' - 1 & 0 \\ 1 - \psi' & 0 & l_1 \psi' \\ 0 & 0 & 0 \end{bmatrix}$$

and a second differential relative displacement matrix of the relative instantaneous displacement

Synthesis of Path Generation Mechanisms

Diagram	N ₂	Transfer functions
	0	$\psi \equiv \varphi_{21} = \arccos \frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1 l_2}$ $\varphi \equiv \varphi_{01} \equiv -\varphi_{10} = \arcsin \left(\frac{l_2}{\sqrt{x^2 + y^2}} \sin \psi \right) - \arctg \frac{y}{x}$
	1	$\psi' = \frac{d\psi}{d\varphi} = \lambda \frac{\cos \varphi_{10} + y' \sin \varphi_{10}}{\cos (\varphi_{10} + \varphi_{21}) + y' \sin (\varphi_{10} + \varphi_{21})} + 1; \quad \lambda = \frac{l_1}{l_2}$
	2	$\psi'' = \frac{d^2\psi}{d\varphi^2} = \frac{\lambda^2 + (1 - \psi')^3 + \lambda (1 - \psi')(2 - \psi') \cos \psi}{-\lambda \sin \psi}$ $+ \frac{y'' l_2 [\lambda \sin \varphi_{10} + (1 - \psi') \sin (\varphi_{10} + \psi)]^3}{-\lambda \sin \psi}$
	0	$S = \sqrt{x^2 + y^2 - h^2}; \quad \varphi \equiv \varphi_{01} \equiv -\varphi_{10} = -\arctg \frac{y}{x} - \arctg \frac{S}{h}$
	1	$S' = \frac{dS}{d\varphi} = \frac{(x + yy')(x^2 + y^2)}{S(y - xy') - h(x + yy')}$
	2	$S'' = \frac{d^2S}{d\varphi^2} = \frac{S^2 S'^3}{x^2 + y^2} \left\{ \left[\frac{1 + y'^2 + yy''}{(x + yy')^3} + \frac{1 - (h/S)^2}{(x + yy')(x^2 + y^2)} \right] (y + xy') \right.$ $\left. + \frac{xy''}{(x + yy')^2} + \frac{2h}{S(x^2 + y^2)} \right\}$
	0	$\psi \equiv \varphi_{21} = \arccos \frac{x}{l}; \quad \sigma \equiv \sigma_{01} \equiv -\sigma_{10} = l \sin \psi - y$
	1	$\psi' = \frac{d\psi}{d\sigma} = \frac{1}{x + y'(y - \sigma_{10})}$
	2	$\psi'' = \frac{d^2\psi}{d\sigma^2} = \frac{(y - \sigma_{10})[1 + y''(y - \sigma_{10})] - xy'}{[x + y'(y - \sigma_{10})]^3}$
	0	$S \equiv S_{21} = y; \quad \sigma \equiv \sigma_{01} \equiv -\sigma_{10} = -x$
	1	$S' = \frac{dS}{d\sigma} = -y'$
	2	$S'' = \frac{d^2S}{d\sigma^2} = y''$

Fig. 2. Transfer functions as geometric derivatives of the displacement function

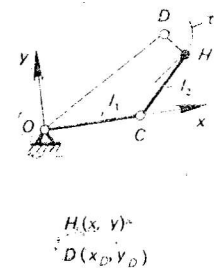
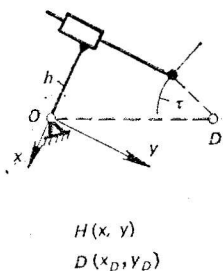
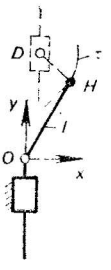
Diagram	№	Transfer functions
 RR $H(x, y)$ $D(x_D, y_D)$	1	$\psi' = \frac{x + yy'}{x + yy' - l_1}$
	2	$\psi'' = \frac{[\psi'(\psi' - 2)(l_1 - x) + x] x_D - (\psi'^2 - 1)^2 yy_D - l_1 \psi'^2 (l_1 - x)}{yx_D + (l_1 - x)y_D - l_1 y}$
	$y' \neq 0$	
	$y' = 0$	$\psi'' = \frac{\psi'(\psi' - 2)(l_1 - x)y' + (\psi' - 1)^2 y}{x + yy' - l_1}$
 RP $H(x, y)$ $D(x_D, y_D)$	1	$S' = x + yy'$
	2	
	$y' \neq 0$	$S'' = \frac{(x - S') x_D + yy_D + S'^2}{y_D - y}$
	$y' = 0$	$S'' = y + (2S' - x)y'$
 PR $H(x, y)$ $D(x_D, y_D)$	1	$\sigma' = x + yy'; \quad \psi' = \frac{1}{\sigma'}$
	2	
	$y' \neq 0$	$\sigma'' = \frac{(x_D - 2\sigma') x + yy_D + \sigma'^2}{y - y_D}$
	$y' = 0$	$\sigma'' = xy' - y$
		$\psi'' = -\frac{\sigma''}{\sigma'^3}$

Fig. 3. Transfer functions provided a matrix-geometric method

$$(24) \quad [\dot{U}_0'] = \begin{bmatrix} -(\psi' - 1)^2 & \psi'' & l_1 \psi'^2 \\ -\psi'' & -(\psi' - 1)^2 & l_1 \psi'' \\ 0 & 0 & 0 \end{bmatrix}$$

with respect to a coordinate system Oxy in the plane of link 1 with axis x directed to the point C .

For each position of a provisionally OKC the corresponding normal n to the trajectory τ passes through the point H with radius-vector $\bar{h}$ and curve centre D with radius-vector $\bar{d}$ (Fig. 3). The radius of the curve $\rho_D = |\bar{h} - \bar{d}|$, up to second derivative order, is constant and hence

$$(25) \quad (\bar{h} - \bar{d})^T (\bar{h} - \bar{d}) = \rho_D^2,$$

$$(26) \quad (-\bar{d}')^T (\bar{h} - \bar{d}) = 0,$$

$$(27) \quad (-\bar{d}'')^T (\bar{h} - \bar{d}) + (-\bar{d}')^T (-\bar{d}') = 0.$$

Since $\bar{d}' = [\dot{U}_0'] \bar{d}$ and $\bar{d}'' = [\ddot{U}_0'] \bar{d}$, taking into consideration the expressions (23) and (24) and substituting $\bar{h} = [x_D \ y_D \ 1]^T$, $\bar{d} = [x_D \ y_D \ 1]^T$ in equations (26) and (27) respective equations

$$(28) \quad ax_D + by_D = c;$$

$$(29) \quad dx_D + ey_D = f$$

are obtained with coefficients

$$(30) \quad \begin{aligned} a &= (\psi' - 1)y & ; & & d &= \psi''y - \psi'(\psi' - 2)(l_1 - x) + x; \\ b &= l_1\psi' - (\psi' - 1)x & ; & & e &= \psi''(l_1 - x) + (\psi' - 1)^2y; \\ c &= l_1\psi'y & ; & & f &= l_1[\psi''y - \psi'^2(l_1 - x)]. \end{aligned}$$

From equation (28) ψ' is determined by specifying $x_D/y_D = -y'$ for $x_D = y_D = \infty$ with the aim to simplify the expression for ψ' (Fig. 3, line 1). Equation (29) serves to determine ψ'' for $y'' \neq 0$ (line 2). When the trajectory has a straight-line section ($y'' = 0$), the expression for ψ'' is simplified (line 3).

For RP structure of a provisionally OKC $\psi' = -S'/l_2$ and $\psi'' = -S''/l_2$, $l_1 = l_2 = \infty$ are specified, and the expressions for ψ' and ψ'' are correspondingly transformed for the transfer functions S' and S'' of the respective FGM.

The transfer functions $\psi' = 1/\sigma'$ and $\psi'' = -\sigma''/\sigma'^3$ of FGM of a provisionally OKC with PR structure are obtained by substituting the output coordinate S for the RP structure by the input coordinate σ for the PR structure, as well as the coordinates of the points H and D in the expressions for S' and S'' .

When applying the matrix-geometric method, differentiation at the stage of finding the derivative matrices of the instantaneous displacement is much more easier than the differentiation used to determine the transfer functions by means of differential calculus apparatus. Differentiation is completely avoided in finding the FGM transfer functions provided that a kinematic method using IRC, is built up. Two provisional links are introduced (Fig. 4) 3 and 4 $\equiv 0$, kinematically connected by a provisional joint at the center of a curve to the trajectory τ of point H for a finitely distant center D ($y'' \neq 0$) or by a provisional slider for an infinitely distant center D ($y'' = 0$). Link 3 is jointly connected with link 2 at the point H , while link 4 is connected with

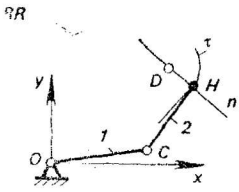
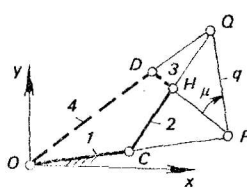
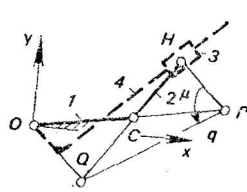
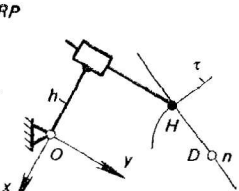
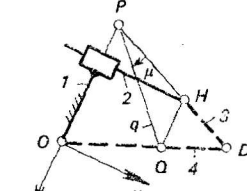
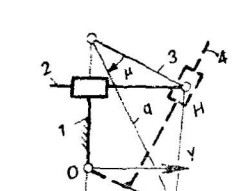
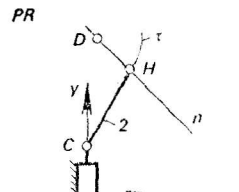
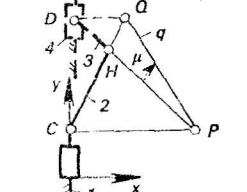
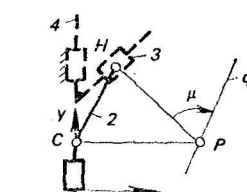
Known	Provision functions	
	$y'' \neq 0$ (arc)	$y'' = 0$ (straight line)
 $H(x, y); C(x_c, y_c)$ $y'; y''$ $k_1 = y_c/x_c$ $k_2 = (y - y_c)/(x - x_c)$ $k_n = -1/y'$	 $k_1 = y_D/x_D$ $x_P = (y - k_n x)/(k_1 - k_n);$ $x_Q = (k_2 x - y)/(k_2 - k_1);$ $k_q = (y_Q - y_P)/(x_Q - x_P);$ $\psi' = x_P/(x_P - x_f)$	 $k_1 = k_n$ $y_P = k_1 x_P$ $y_Q = k_1 x_Q$ $\text{tg } \mu = (k_q - k_n)/(1 + k_q k_n)$ $\psi'' = \psi' (1 - \psi')/\text{tg } \mu$
 $H(x, y)$ $y'; y''$ $k_n = -1/y'$	 $k_1 = y_D/x_D$ $x_P = x + y y'; x_Q = y/k_1;$ $k_q = y_Q/(x_Q - x_P);$ $S' = x_P;$	 $k_1 = k_n$ $y_Q = y$ $\text{tg } \mu = (k_q - k_n)/(1 + k_q k_n)$ $S'' = S'/\text{tg } \mu$
 $H(x, y); C(O, y_c)$ $y'; y''$ $k_2 = (y - y_c)/x$ $k_n = -1/y'$	 $x_P = x + y' (y - y_c);$ $x_Q = (y_D - y_c)/k_2; y_Q = y_D$ $k_q = (y_Q - y_P)/(x_Q - x_P)$ $\text{tg } \mu = (k_q - k_n)/(1 + k_q k_n);$	 $y_P = y_c$ $k_q = k_2$ $\psi' = 1/x_P; \psi'' = -\psi'^2/\text{tg } \mu$

Fig. 4. Transfer functions provided a kinematic method using IRC

link 1 at the point O by a provisional joint if link 1 performs rotation or by a provisional slider if link 1 performs translation.

For a provisionally OKC with RR structure an arbitrarily oriented coordinate system Oxy is introduced in the plane of link 1 and we determine subsequently: the angle coefficient k_4 of the straight line defining the position of link 4; the coordinates of the relative IRC P ; the angle coefficient k_q of the colinear axis q ; the angle μ which the normal n forms to the trajectory τ at the point H and the first two transfer functions.

Following the same sequence of actions, we come to the transfer functions S' and S'' of FGM for a provisionally OKC with RP structure. Aiming to simplify the mathematical model, a coordinate system Oxy is introduced in the plane of link 1 with y axis oriented to the direction of remoteness of the point H from link 2 with respect to the slider from the relative frame 1.

For a provisionally OKC with PR structure a coordinate system is introduced in the plane of link 1 with y axis coinciding with the sliding axis of link 1. We determine subsequently: the position of IRC P and Q , the angle coefficient k_q of the colinear axis q ; the angle μ which the normal n forms to the trajectory τ at the point H and the first two transfer functions ψ' and ψ'' .

Determination of the basic function generator mechanism dimensions. The determination of the basic FGM dimensions is an optimization problem related to the satisfaction of a number of criteria concerning the reliability against wedging, the degree of normal force increment, the pressure angle, profile curve of the higher kinematic pair, the instantaneous and integral efficiency, and so on, related to the PGM design and exploitation. The domain β of variation of the basic parameters of cam FGM (Fig. 1, schemes 2 and 8) is determined from the condition for the pressure angle θ to satisfy the condition $|\theta| \leq \theta_p$, since the maximally admissible pressure angle θ_p influences both the cam dimensions and geometric properties and the increment degree of the normal force and efficiency. The Flocke graphic method [16] allows to avoid solving complicated transcendental equations, which are inevitably reached in the general case under analytic solution of this problem. An explicit solution [17] is obtained only for some parabolic and simple harmonic functions for cam mechanisms with axial slider. Then, approximate formulae or nomograms are applied, constructed for a limited number of laws of motion. Even if we neglect the inexact determination of the domain by Flocke method, its application is quite difficult in the cases when the length of the FGM frame is specified. That is why a method for automated determination of the domain is recommended, where the domain β of the basic dimensions of the cam mechanism is determined by using the centrode in the output link plane [18]. Since the band mechanism cam profile (Schemes 4, 10) is an evolute of the cam profile of the respective cam mechanism, then the domain β is related to both mechanisms. The values of the basic dimensions for which the evolute maximally contracts should also be searched for in domain β in order to use a gear FGM.

Domain β of the admissible variation of the basic dimensions of cam FGM with flat output link is limited by the condition for convexity of the cams and by their admissible maximal dimensions. In domain β a set of values of the basic dimensions is demanded which should satisfy preliminarily imposed geometric, kinematic and dynamic solutions depending on the particularly specified problem concerning the PGM design and exploitation.

Profiling of the function generator mechanism cams. FGM kinematically con-

nects link 2 with frame 0 of a provisionally OKC with link 1 serving as a relative frame. In order to accomplish an adequate choice of the FGM type, it is necessary to find first perspective variants of each type of mechanism, which, in principle, could solve the problem posed. This means to synthesize a number of mechanisms and to separate these variants by means of comparison. This aim can be achieved by using various methods and algorithms for synthesis of different types of mechanisms since the theoretic apparatus for synthesis is concretely developed for the different types of mechanisms separately. This means to do a lot of work for the synthesis of multiple variant solutions of FGM by using the obtained transfer functions and taking into consideration the restriction conditions introduced.

An integral approach for FGM type-dimensional synthesis can be built up on the kinematic equivalence of mechanisms [20] by using the kinematic method for synthesis by IRC [11, 12] or the matrix-geometric method [19]. The conjugate profiles of the centrode FGM are unique for a given law of motion of the point H and a provisionally OKC, while the cam profiles of cam and band mechanisms depend on the accepted basic FGM dimensions, which allows optimizations.

5. Conclusion

The separability of motion with respect to geometry and time allows to separate the problem for planning the target function "motion-space" as a purely geometric one from the general problem for PGM control in space and time.

On-line control is possible up to PGM second derivatives by using the most suitable form of transfer functions, preliminarily obtained by three methods.

The differentiated approach for representation of PGM as a set of a provisionally OKC and FGM, connecting the links of a provisionally open kinematic chain, allows to create simplified algorithms for synthesis and control of PGM with introduction of various geometric, kinematic and dynamic restrictions.

It is worth noting the efficiency of the proposed matrix-geometric method for the determination of centrodes which serves to solve a substantial part of the quite difficult problem about the dimensioning and dislocation of a provisionally OKC.

References

1. Beyer, R. *Kinematische Getriebesynthese*. Berlin—Gottingen, 1953.
2. Artobolevsky, I. I., N. I. Levitskii, S. A. Cherkudinov. *Synthesis of Flat Mechanisms*. M., Fizmatgiz, 1959.
3. Hartenberg, R. S., J. Denavit. *Kinematic Synthesis of Linkages*. N. Y., McGraw-Hill book Co., 1964.
4. Sarkisian, Y. L. *Approximation Synthesis of Mechanisms*. M., Nauka, 1982.
5. Perju, D. Mechanisms with variable link length. *Proc. of the Seventh World Congress on the TMM*, 1987.
6. Konstantinov M. S., P. I. Genova, V. G. Pirgozliev. *Mechatronic devices with path generation cam mechanisms*. — *Proc. of the 6th Congress on Mechanics*, 1989.
7. Brat, V. Maticova metoda kinematickeho reseni prostorovych mechanismi s nizsimi kinematickymi dvojicemi. — *Rozpravy SSVAV*, roc. 75/2, NCSAV, Praha, 1965.
8. Litvin, F., R. Jamalov, K. Petrov. *Analysis of overconstrained spatial mechanisms*. — *Theor. Appl. Mech.*, 1977, No 3, 34-43.
9. Galabov, V. B. *Synthesis of elementary plane linkages*. — *Mashinostroene*, 1981, No 1.
10. Id. *Precision criterion and synthesis of plane linkages*. — *Proc. of the Seventh World Congress on the TMM*, 1987.

Synthesis of Path Generation Mechanisms

11. Id. A generalized design approach and a common synthesis method for mechanisms with rotating input link. — *Theor. Appl. Mech.*, 1982, No 3, 11-19.
12. Id. A generalized method for kinematic analysis and synthesis of plane mechanisms. *Ibid.* 1984, No 2, 32-43.
13. Id. Structure, synthesis and application of *Q*-manipulators. — *Proc. of the Sixth World Congress on the TMM*, 1983.
14. Id. Stereotype motions and kinematic control of *Q*-manipulators with constant transfer relations. — *NTK "PR'84"*, 1984.
15. Peissach, E. E. Optimization-quadratic synthesis of flat lever mechanisms. — *Mashinovedenie*, 1986, No 5.
16. Flocke, K. A. Zur Konstruktion von Kurvenscheiben bei Verarbeitungsmaschinen. — *VDI-Forschungsheft*, No 345, Berlin, 1931.
17. Chen F. V. *Mechanics and Design of Cam Mechanisms*. Pergamon press, N. Y., 1982.
18. Galabov V. B., V. Draganov. Automated determination of the basic dimensions of cam mechanisms with roller output link. *Scientific session VMEI "Lenin" '89*. 1989.
19. Id. Matrix-geometric method for synthesis of cam profiles. — *Proc. of the Sixth Congress on Mechanics*, 1989.
20. Id. Kinematic equivalence of plane linkages. — *Proc. of the Fifth Congress on Mechanics*, 1985.

Received on March 3, 1990