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**An Examination of the State of Stress in Coated
Flat Pieces by the Finite Element Method**

1. Introduction

There exist three basic methods for determining of stress in electrodeposits: the strip-bending method [1], the spiral-twisting method [2], and the length change method [3]. In the first method one of the sides of cathode 4 is coated with a lacquer insulation layer 3 and is submerged in electrolyte together with the anode plate 6 (Fig.1). After switching on the electric current a stressed layer 5 is deposited on the cathode in the course of a certain time period and the obtained sample bends. The stress in the deposit is calculated on the basis of the measured values of displacement δ and deposit thickness h_1 [1, 4-6].

In [7] an approach is proposed for solving the following problem: using the results from the experimental sample testing, the Authors determine the state of stress of a piece made of the cathode material, however with an arbitrary form, where this state of stress is caused by the deposition of the same kind of stressed deposit on the piece. The approach is based on the fact that in the theoretical model of a stressed deposit there exist quantities, namely the initial strains in the deposit, which depend only on the materials of the deposit and the substrate and on the technological process of deposition, but not on the form or dimensions of the substrate [8]. This fact has been confirmed experimentally [9]. Then, after the initial strain values for all deposit sample layers in Fig.1 have been determined, according to the method described in [4], and these values have been assigned to the deposit layers on the piece, one can estimate the stress in the coated piece.

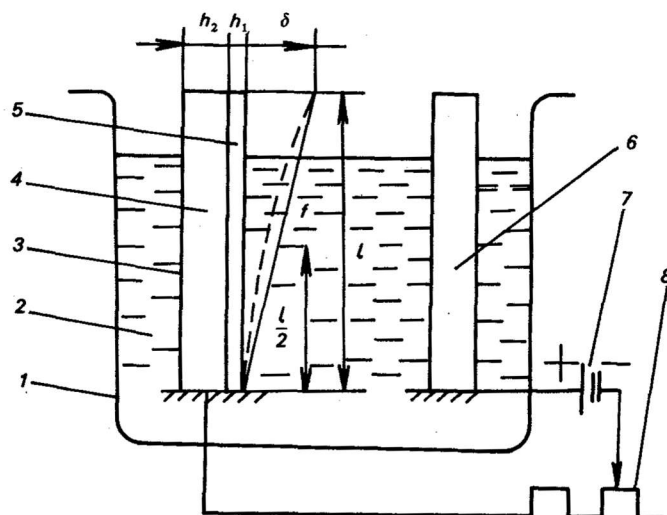


Fig.1. A scheme of the experimental equipment: galvanic bath 1, electrolyte 2, lacquer insulation 3, elastic strip cathode 4, deposit 5, anode 6, electric current source 7, control device 8

This approach has been used in [7] for determining the stress in coated cylindrical pieces.

The present paper aims at widening the application of this approach for flat pieces over whose prismatic or cylindrical lateral surface a stressed coating is deposited (Fig.2). It can be assumed that, as a result of the deposition, a planar state of stress emerges in the so obtained composite disc.

Taking into account the great variety of possible forms of the disc midsurface contour and the availability of versatile software systems for computer realization of the finite element method, it is proposed that this method is used in solving the planar problem of the theory of elasticity for the composite disc. Then it should be theoretically possible to use the existing planar finite elements for discretization of the piece and the deposit; this would, however, lead to great difficulties in practice because of the negligible thickness of the deposit compared to the piece dimensions.

In the technical literature [10, 11] specialized finite elements have been described, designed for modelling states of stress and strain in zones with stress concentration or in media composed of different layers. Knowledge of the character of the state of stress is a

prerequisite for the development of such finite elements. The hybrid method is the most frequently used one in such cases.

The finite elements for modelling the state of stress in media consisting of several layers, as described in literature, are not suitable for the modelling of deposits because the deposit thickness is very small compared to the substrate dimensions and because the stress distribution in the substrate close to the deposit is a complex one. Therefore it was necessary to develop a specific finite element which could be combined with the existing ones and thus enable the modelling of the stressed coating 2 deposited on the piece 1 (Fig.2).

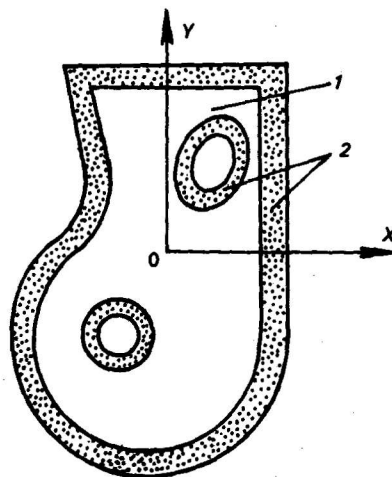


Fig.2 A piece 1 with a stressed deposit 2 coated on it

2. A New finite element for modelling a stressed deposit on a piece

The new finite element I (Fig.3) is connected by its nodes to an ordinary quadrilateral isoparametric finite element II modelling a part of the piece surface layer. The boundary between the piece and the deposit lies along line 2-3.

The deposit is schematized in the form of n separate elastic layers with identical or different mechanical characteristics, joint-connected with their two ends to the AB and CD rods. A constant initial linear strain $\bar{\epsilon}_k$ ($k=1,2,\dots,n$) is assigned to each layer and it is assumed that a homogenous linear state of stress emerges in it. It is also assumed, according to the Bernoulli hypothesis, that the rods AB and CD remain

straight under load, in order to ensure the continuity of the deposit on the boundary between two finite elements. In order not to impede the relative displacement of nodes 1 and 2 of the quadrilateral finite element connected to the *AB* rod, it is assumed as well that the *AB* rod has a zero longitudinal rigidity in section 1-2. The same assumption is made for section 3-4 of the *CD* rod.

To determine the basic relations of the new finite element, the basic equation of the finite element method is used:

$$(I) \quad [K] \{\delta\} = \{F\} + \{F\}_{\varepsilon}^T,$$

where

$[K]$ is the rigidity matrix of the element;

$\{\delta\} = \{u_1, v_1, u_2, v_2, \dots, u_4, v_4\}^T$ is the vector of the generalized node displacements;

$\{F\}_{\varepsilon}^T$ is the vector of the generalized node forces resulting from the assigned initial strain, and

$\{F\}$ is the vector of the total node forces.

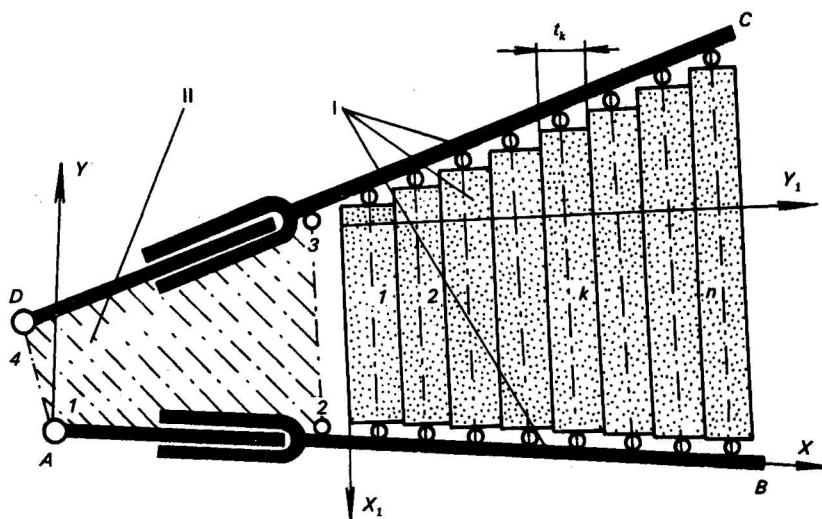


Fig.3. A scheme of the newly developed finite element (I) and the quadrilateral finite element connected with it (II).

The rigidity matrix of the element is obtained through examining the individual states of stress. The node displacement $u_1 = 1, v_1 = 1, \dots, v_4 = 1$ are successively assigned and the change in the length Δl_k of the k -th deposit layer is determined. The normal force is expressed by Δl_k :

$$(2) \quad N_k = \frac{\Delta l_k E_k S_k}{l_k},$$

The node forces $\{U_1, V_1, U_2, V_2, \dots, U_4, V_4\}^T$ which are elements of the rigidity matrix are determined from the equilibrium conditions for the individual states of stress. The generalized node forces resulting from the assigned initial strain are expressed by the normal force $\bar{N}_k = -\bar{\epsilon}_k E_k S_k$ by means of the equilibrium conditions.

After solving the problem by the finite element method, the displacements of the four nodes are obtained. Then the change in the length of each layer Δl_k can be computed and from these the normal stress in the cross-section of the k -th layer can be obtained:

$$(3) \quad \sigma_k = \frac{(\Delta l_k - \bar{\epsilon}_k l_k) E_k}{l_k},$$

The newly created finite element is included under the designation RR2S in the VIMKE program system [12] developed in the Rouse Technical University. The maximum number of layers is limited to 5. Under the made assumptions, in each deposit layer there emerges a homogenous linear state of stress with the following components:

$$\sigma_{x_1} = 0, \quad \sigma_{y_1} = \sigma_{z_1} = 0, \quad \tau_{x_1 y_1} = \tau_{y_1 z_1} = \tau_{z_1 x_1} = 0,$$

where the linear strain ϵ_{x_1} , which is a constant for each layer, is a

linear function of the y_1 coordinate of the layer's midsurface (Fig.3).

Forces act on the finite element modelling the substrate only through the four nodes; as a result a planar state of stress emerges in it. With this schematization a question arises of how accurately the state of stress of the coated piece can be described. This is discussed below.

3. Numerical experiments for testing the finite element

For the purpose of testing the performance of the new RR2S finite element, the following planar problem of the theory of elasticity has

been discussed. Two rectangular elastic disks are welded together (Fig.4). The first one is made of zinc, and the second one - of platinum; their Young's moduli are $E_1 = 0.922 \cdot 10^{11}$ Pa and $E_2 = 1.7 \cdot 10^{11}$ Pa and their Poisson's ratios are $\mu_1 = 0.27$ and $\mu_2 = 0.44$ respectively. The task is set to determine the planar state of stress of the obtained composite disc, which is caused by the emerging of an initial strain $\bar{\epsilon}_x = 5 \cdot 10^{-4}$ identical in all points of the upper disc. The composite disc may be viewed as a theoretical model of a coating 1 deposited on the substrate 2. This planar problem of the theory of elasticity has been solved in [13] by means of the finite element method using the VIMK Eprogram system [12]. The use of a twice finer discretization mesh has led to the confirmation of no less than three significant digits in the stress results. That is why this solution [13] has been accepted as a standard for comparing the new six solutions of the same problem which are shown below.

In solution No. 1 the upper disc (a model of the deposit) has been schematized by means of GLRF rod finite elements (Fig. 5a) and in solutions No.2...6 this has been done by means of the new RR2S finite elements

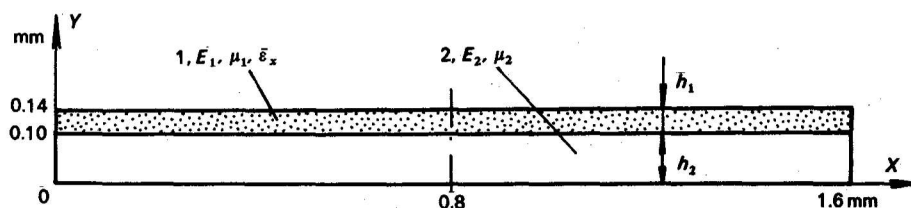


Fig.4. A composite rectangular disc.

(Fig.5b,c). In solutions No.2 and 3 the number of layers in the finite element has been assumed to be $n = 1$ (Fig. 5b) and in solution No.4 , 5 and 6 the assumed value has been $n = 4$ (Fig. 5c). Because of the symmetry only half of the composite disc has been examined. In experiments No.1 , 2 and 4 the half of the lower disc has been

discretized in 60, in experiments No.3 and 5 - in 180, and in experiment No.6 - in 1200 isoparametric rectangle finite elements RR2P (see Fig. 5 and Table 1).

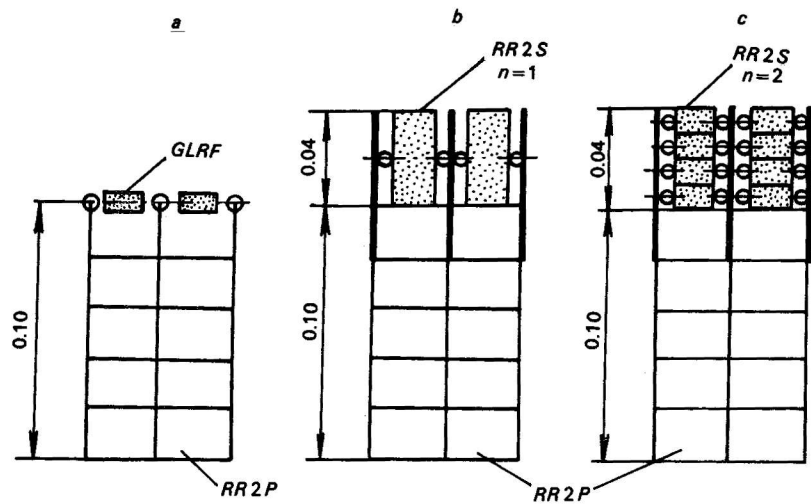


Fig.5. Discretization of a part of the composite disc from Fig. 4. Upper disc discretization : a — by GLRF rod finite elements; b and c — by the newly developed RR2S finite elements at $n = 1$ and $n = 4$ respectively.

Fig. 6 shows the diagrams of stress in the cross-section $x = 0.30$ mm (Fig.4) obtained for the six solutions and for the solution in [13]. Comparing the results for the upper disc 1 it can be seen that solution No.1 provides the roughest approximation $\sigma_x = 25.7$ MPa (Fig.6a) in place of the average value $\sigma_x = 18.7$ MPa obtained in [13] (Fig.6b). This can be explained by the translation of the disc 1 midsurface line to $y=0.10$ mm, the correct position being $y = 0.12$ mm (Fig.4). This shortcoming has been avoided in solution No.2 thanks to the use of the new RR2S finite

elements; thus a value of $\sigma_x = 19.7$ MPa is obtained. If, beside that, the mesh in the x direction is accepted to be three times finer, as this is done in solution No.3 then the above-mentioned value from [13] is obtained.

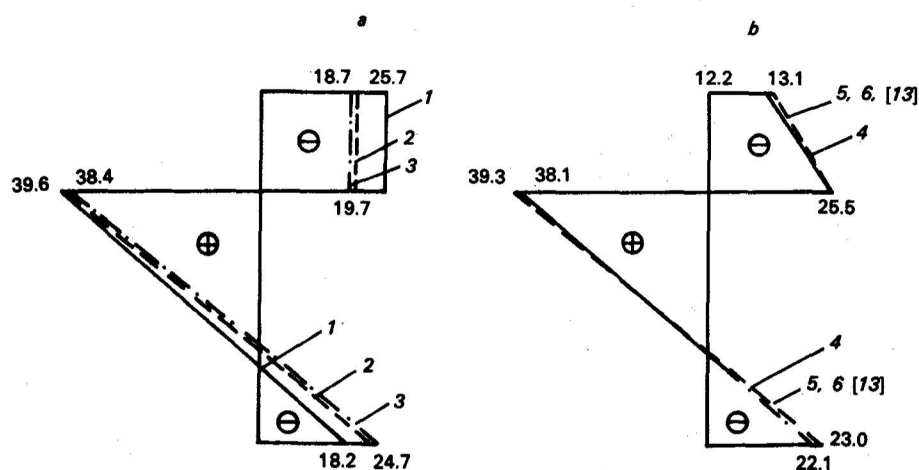


Fig.6. Normal stress σ_x in the cross-section $x=0.30$ mm of the composite disc (Fig.4); values have been obtained from solutions No.1...6 and from [13].

Table 1. Finite elements of the discretization of the examined half of the upper disc.

Solution ■	Finite element type	Number of layers in the finite element	Number of finite elements
1	GLRF	1	10
2	RR2S	1	10
3	RR2S	1	30
4	RR2S	4	10
5	RR2S	4	30
6	RR2S	4	120

The diagrams in Fig.6b show that with the increase of the number of layers n and the number of finite elements in the x direction, into which the disc is divided, solutions No.4, 5 and 6 get closer to the solution from [13].

Fig. 7 shows the diagrams of stresses $\sigma_y(x, 0.1)$ and $\tau_{yx}(x, 0.1)$ with which the two rectangular discs interact along their connecting surface near the end $x = 0$. It can be seen that with the increase of the number of finite elements this diagram also gets closer to the respective diagrams from [13]. Solution No.6 is obtained at the same density of the finite element meshes for the solution in [13] and gives results that are a little less accurate but close to the ones in [13].

4. Application of the new finite element for determining the state of stress of a coated metal-cutting tool

It is known that for the purpose of improving the wear resistance of metal-cutting tools their cutting edges are being coated with hard carbide or nitride layers by electric-arc discharge evaporation in vacuum. A significant stress appears in these coatings influencing their strength and their adhesion to the base metal and thus affecting the tool reliability. With a view to this the determining of stress in a coated tool represents a great theoretical and practical interest.

As an example we shall discuss here the problem of determining the state of stress of the toothing of a disk cutter whose peripheral surface has been vacuum-coated with a stressed titanium nitride coating. The

cutter has 64 teeth and an outer diameter $D=100\text{mm}$. Accounting for the sector symmetry, only a part of the cutter is examined including one of its teeth (Fig.8). The basic parameters of the tooth are: $q = 1\text{mm}$, $h = 2\text{mm}$, $\alpha = 8^\circ$, $\theta = 64^\circ$, $\eta = 58^\circ$.

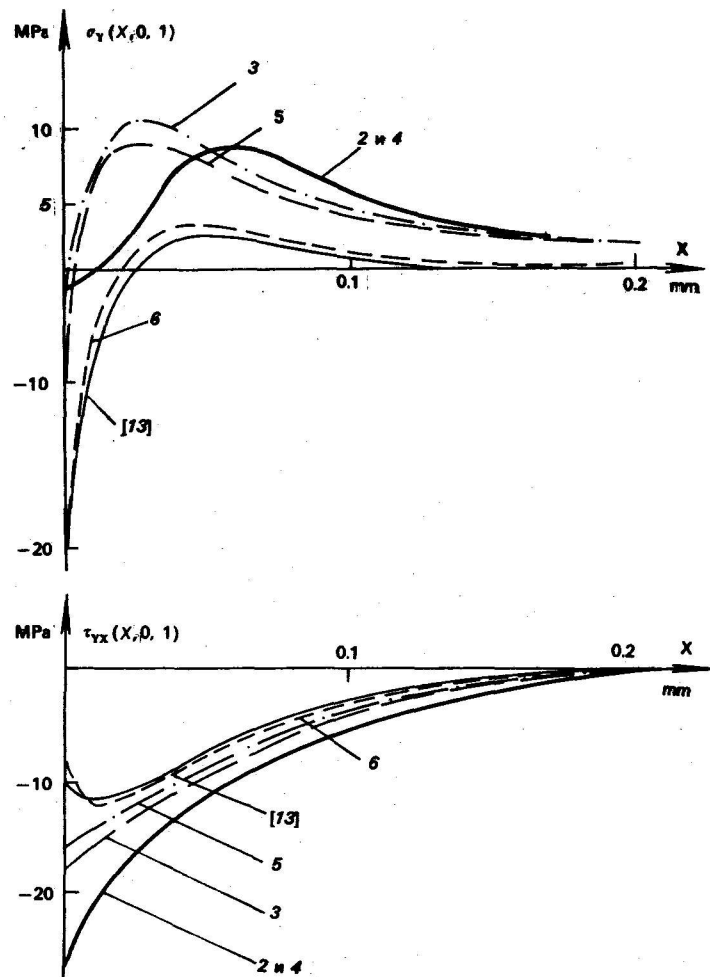


Fig.7. Diagrams of the stress of interaction between the two rectangular discs in Fig.4; values have been obtained from solutions No.2...6 and from [13].

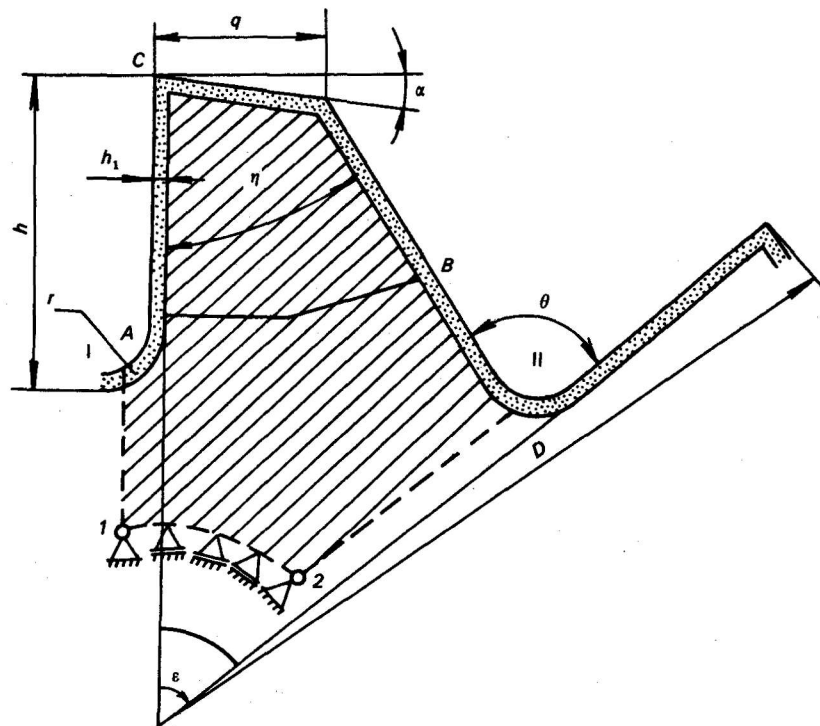


Fig.8. An examined part of the coated disk cutter.

For the purpose of solving the problem, some preparatory work has been done, as described below. In the Scientific Centre of Vacuum Technologies and Equipment at the Rouse Technical University, P. Hovsepyan PhD and M. Tenev, post-graduate student, have coated a thin elastic steel strip with a layer of titanium nitride in a vacuum chamber [14]. The displacement value f and the corresponding layer thickness

value h_1 (analogously to Fig.1) have been measured. On the ground of these data and using relation (20) from [4]¹⁾, the average value of the initial strain in the examined deposit has been calculated and found to be $\bar{\epsilon} = 0.0172$.

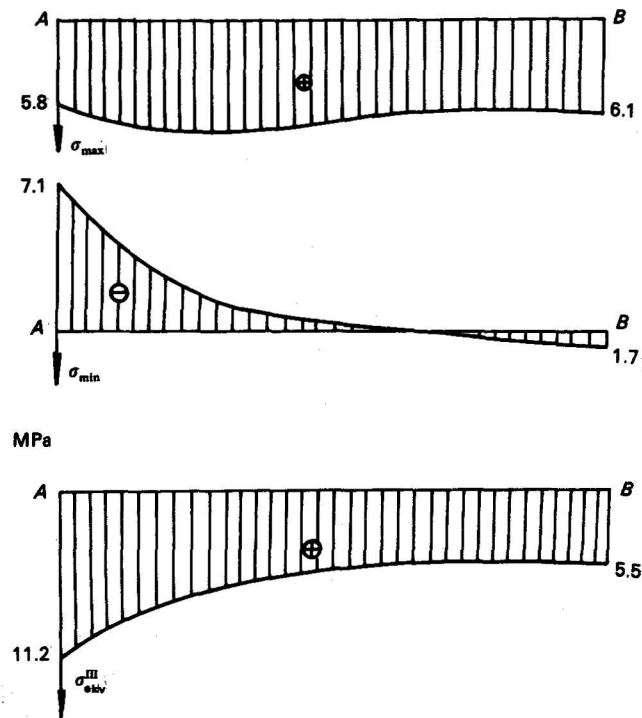


Fig.9. Variation of the main stresses $\sigma_{\max}$ and $\sigma_{\min}$ and of the equivalent stress σ_{eq} along line AB in Fig.8

1) The TiN vacuum coating has an isotropic behaviour [15] and may be examined by use of the relations valid for isotropic electrodeposits.

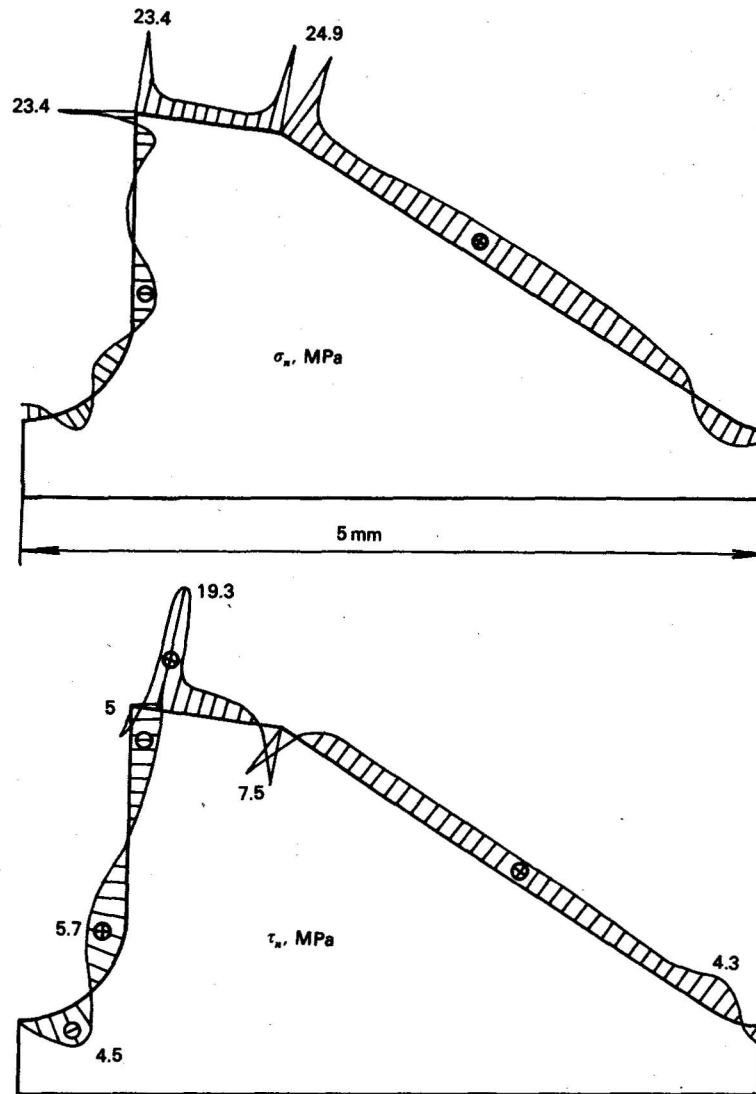


Fig.10. Diagrams of: a — normal stress σ_n , and b — shear stress τ_n in the contact surface with a normal n

The same initial strain value has been assigned to the deposit coated on the disc cutter and the state of stress of its examined part has been determined, using the finite element method (Fig.8). This part has been discretized into 270 quadrilateral isoparametric finite elements of the RR2P type and the deposit has been discretized into 38 finite elements of the newly developed dRR2S type. The solution has been calculated by means of the VIMKE program system [12] at the following data [15]: $E_1 = 2.3 \cdot 10^{11} \text{ Pa}$, $E_2 = 2 \cdot 10^{11} \text{ Pa}$, $\mu_1 = 0.2$, $\mu_2 = 0.3$. Furthermore, the following kinematic boundary conditions are satisfied:

- the lower two nodes 1 and 2 are firmly supported;
- all nodes in the lower edge are immobile in the radial direction;
- identity is ensured of the radial and circumferential displacements of each two nodes determined by the same radius and situated along the end radial sections I-1 and II-2; this is done by use of specialized finite elements.

The problem solution gives the stress values in the deposit and inside the tooth.

In the cross-sections of the deposit there emerges a normal compression stress whose values vary in a narrow range (from 3872 to 3953 MPa). According to data from the literature [15-18] the Tin hard coating formed by physical vapour deposition can stand such stress values.

The stress in the tooth is much smaller. Fig. 9 shows the distribution diagrams of main stresses $\sigma_{\max}$, $\sigma_{\min}$ and equivalent stress σ_{eq} along the line AB in Fig.8. The equivalent stress has its highest value in the tooth root (point A in Fig.8).

A great practical interest lies in the force interaction between the coating and the tooth. In view of this Fig. 10 shows the diagrams of normal and shear stress σ_n and τ_n along the contact surface with a normal n between the coating and the tooth. High values of normal extension stress and of shear stress appear near the salient angles of the tooth. From Fig.10 it can be seen that the highest σ_n and τ_n stress values appear near the point C (Fig.8) where the danger of breaking the adhesion between the coating and the base metal is the greatest. During cutting, this area is subjected to the greatest cutting force and the tool wear-resistance depends on the resultant state of stress there [14].

5. Conclusion

In the present paper a new specialized finite element is offered, designed for modelling stress in a deposit coated on the lateral surface of a flat piece. Actually, a method has been developed for determining the state of stress of the viewed type of coated pieces, basing on the

testing of a coated experimental sample [19,20]. The use of the great capacity of the existing program systems for the automated problem solution by the finite element method has been an important advantage.

Due to the simplifying assumptions made in the development of the new finite element, a simplicity of the solution and a trustworthy stress presentation has been achieved, except for a certain small inaccuracy in the determination of normal stress along the connection surface between the coating and the substrate. Therefore, a further interest may lie in developing and including in the program systems of other specialized finite elements modelling more precisely the state of stress in the deposit and its interaction with the substrate.

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