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**Measurement of Dynamic Properties of Machine Aggregate
with Variable Parameters and Asynchronous Motor**

1. Introduction

In this contribution the dynamic properties of machine aggregate are examined. Under the notation of machine aggregate, a rotational system is understood, composed of a driving and a loading machine and a power transmission (gear box) between them.

For the correct design and functioning of the machine aggregate it is of importance to analyze the effect of its parts upon the operation of the entire device.

The project of the experimental device draws the attention upon papers [1, 2], dealing with both analytic and analogue solution of aggregates with variable parameters.

The experimental device (Fig. 1.a) consists of the mechanism (*M*) with one degree of freedom (cam), the gear box (*P*) and the asynchronous motor (*D*).

The cam mechanism provides for the defined variable moment of aggregate inertia $I(\varphi)$ and loading moment $M_L(\varphi)$.

The equation of motion for the machine aggregate (see Fig. 1.a) has the form

$$(1) \quad \frac{d}{d\varphi} \left[\frac{1}{2} I(\varphi) \omega^2 \right] = M_D - \frac{M_L(\varphi_M)}{i_P},$$

where φ , ω are angular position and velocity of the main element,

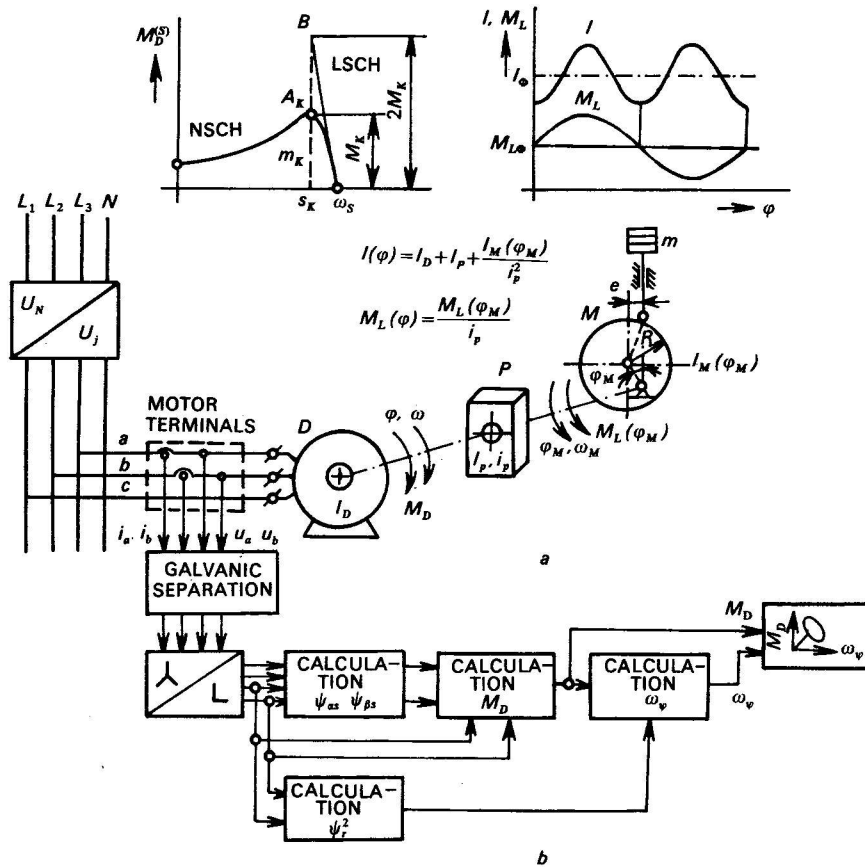


Fig. 1 a - Scheme of the machine aggregate, load of the aggregate and speed-torque characteristic of asynchronous motor

b - Block diagram of the measurement (calculation) of the electromagnetic moment and the absolute slip of asynchronous motor

$$i_P = \frac{\omega}{\omega_M} - \text{transmission ratio,}$$

M_D - electromagnetic (driving) moment of the asynchronous motor.

Electromagnetic (driving) moment M_D can be expressed as a linearized dynamic characteristic (LDCH) of the asynchronous motor in the form

$$(2) \quad T_D \frac{dM_D}{dt} + M_D = 2M_K \frac{s}{s_K},$$

where T_D is the time constant,

s_K, M_K - values of the slip or moment in the maximum of the corresponding speed-torque characteristic (NSCH) (Fig. 1.a)

$$s = \frac{\omega_s - \omega}{\omega_s} - \text{slip of asynchronous motor,}$$

ω, ω_s - angular velocity or synchronous angular velocity (Fig. 1.a)

As $T_D \equiv 0$, the linearized static characteristic (LSCH) can be expressed from Eq. (2) in the form

$$(3) \quad M_D^s = 2M_K \frac{s}{s_K}.$$

The system of differential equations (1), (2) defines an autonomous nonlinear problem. The aim of the solution is determining of the steady-state motion in the form

$$(4) \quad \omega(t) = \omega_\phi + \omega_\pi(t),$$

$$M_D(t) = M_{D\phi} + M_{D\pi}(t),$$

where the index ϕ denotes the mean value of a quantity and the index π its centered periodic component. The problem is solved analytically (method of consecutive approximations) and on an analogue computer in paper [2].

2. Dynamic characteristics of machine aggregate

One of the dynamic characteristics of machine aggregates is the non-uniformity of their run, expressing the non-uniformity of the angular velocity $\omega(t)$ of the main element of the aggregate or the non-uniformity of the driving moment $M_D(t)$ of the aggregate drive.

In Fig.2 the results of solution for LDCH are shown in the form Eq. (3) for non-uniformity of angular velocity $\omega(t)$ and non-uniformity of electromagnetic (driving) moment $M_D(t)$ versus drive parameter m_K . The quantities define the dynamic characteristics of the machine aggregate in the form $M_D(\omega)$.

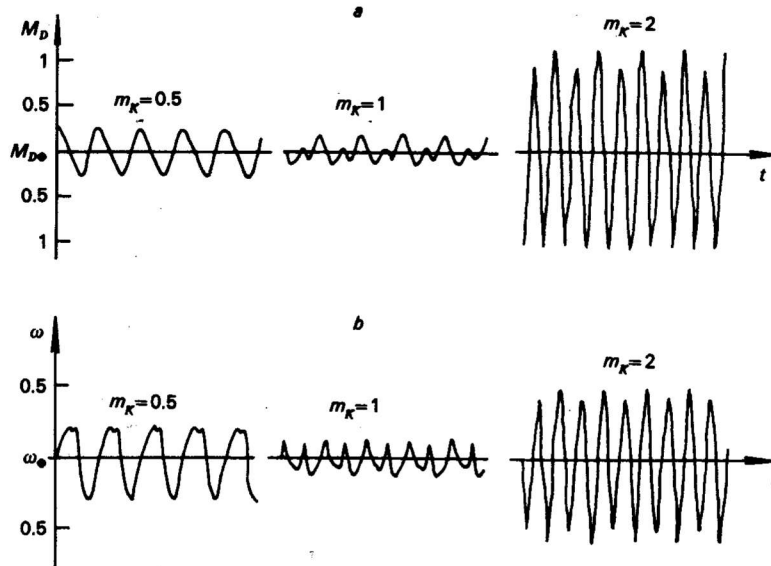


Fig. 2 a - Course of steady-state electromagnetic (driving) moment versus drive parameter $m_k = \frac{M_K}{I \phi \omega_s^2}$
 b - Course of steady-state angular velocity versus drive parameter m_k

As an illustration, results of solution for LDCH Eq.(2) or LSCH Eq.(3) are given in Fig.3.

The Qualitative difference between solutions for LDCH Eq.(2) and LSCH Eq.(3) is evident for the dynamic characteristics of the machine aggregate in the form $M_D(\omega)$.

While for LSCH one gets in a steady-state motion only along a line, for LDCH one gets closed trajectories, the form and the size of which depend upon the parameter m_k .

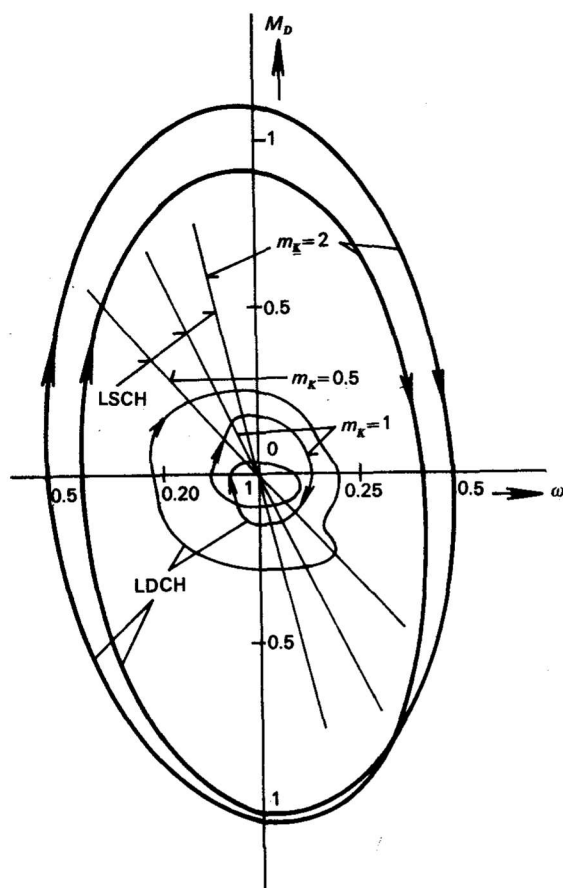


Fig. 3 Dynamic characteristics of the machine aggregate investigated

The equivalence between the dynamic and the real aggregate model is additionally provided for by rigid couplings between aggregate elements and by measurement (calculation) of the electromagnetic (driving) moment $M_D(t)$ and the angular velocity (absolute slip) $\omega_\psi(t)$ of an asynchronous motor, using the method based on the measurement of state variables (currents and voltages) on the motor terminals.

3. Mathematical model of the asynchronous motor

The voltage differential equations of the asynchronous motor in a complex form of the space vectors of both stator and rotor can be expressed in the form

$$(5) \quad \begin{aligned} \vec{U}_s &= R_s \vec{i}_s + \frac{d\vec{\psi}_s}{dt} + jp\omega_s \vec{\psi}_s, \\ \vec{U}_r &= R_r \vec{i}_r + \frac{d\vec{\psi}_r}{dt} + jp\omega_r \vec{\psi}_r, \end{aligned}$$

where

$\vec{U}_s, \vec{U}_r$ - are space vectors of the induced stator (rotor) voltage

$\vec{i}_s, \vec{i}_r$ - space vectors of the stator (rotor) current,

R_s, R_r - true stator (rotor) resistances,

$\vec{\psi}_s, \vec{\psi}_r$ - space vectors of the coupled magnetic flux of the stator (rotor),

ω_s - synchronous angular velocity,

$\omega_{\vec{\psi}_r} = s\omega_s = \omega_s - \omega$ - absolute slip or angular velocity of the vector $\vec{\psi}_r$

p - number of the pole pairs.

For the vectors of coupled magnetic flux it holds that

$$(6) \quad \begin{aligned} \vec{\psi}_s &= L_s \vec{i}_s + L_{sr} \vec{i}_r, \\ \vec{\psi}_r &= L_r \vec{i}_r + L_{sr} \vec{i}_s, \end{aligned}$$

where L_s, L_r are total inductances of the stator (rotor) phase,

L_{sr} - mutual inductance.

The electromagnetic (driving) moment of the asynchronous motor is the vector defined by the relation

$$(7) \quad M_D = \frac{3}{2}p(\vec{\psi}_s \times \vec{i}_s) = -\frac{3}{2}p(\vec{\psi}_r \times \vec{i}_r) = \frac{3}{2}p \frac{k}{\sigma L_r} (\vec{\psi}_r \times \vec{\psi}_s),$$

where σ is the leakage coefficient,

k - the stator coupling factor.

Equations (6) and (7) describe three possible modes of measuring the electromagnetic moment of the asynchronous motor:

- 1 - by measuring vectors $\vec{\psi}_s$, $\vec{\psi}_r$,
- 2 - by measuring vectors i_s , U_m ,
- 3 - by measuring vectors i_s and U_s ,

where U_m magnetic voltage vector.

The first two methods require constructionally modified electric engines and they are not suitable for our purpose. The third method initiates from measuring the vector of stator currents i_s and of the vector of stator voltage U_s . Both stator quantities are well measurable without interfering with the asynchronous motor construction.

4. Mathematical description of the method of electromagnetic moment measurement

The relation for electromagnetic (driving) moment of the asynchronous motor in the coordinate system of the stator $(\alpha, \beta, 0)$ is considered in the form

$$(8) \quad M_D = \frac{3}{2} p (\psi_{\alpha s} i_{\beta s} - \psi_{\beta s} i_{\alpha s}),$$

where $\psi_{\alpha s}$, $\psi_{\beta s}$ are components of the vector of the stator magnetic flux;
 $i_{\alpha s}$, $i_{\beta s}$ - components of the stator current;
 p - number of pole pairs.

For components of the magnetic flux the relations hold

$$(9) \quad \psi_{\alpha s} = \int_0^t (U_{\alpha s} - R_s i_{\alpha s}) dt,$$

$$(10) \quad \psi_{\beta s} = \int_0^t (U_{\beta s} - R_s i_{\beta s}) dt,$$

where R_s is the true resistance of stator winding,

$U_{\alpha s}$, $U_{\beta s}$ - vector components of stator voltage.

The components of the current and voltage are determined from the relation ships

$$(11) \quad U_{\alpha s} = U_a - U_0, \quad U_{\beta s} = \frac{\sqrt{3}}{3} (U_b - U_c)$$

where U_a , U_b , U_c are instantaneous voltage values of the threephase supply system of the asynchronous motor (voltages are picked up on the motor terminals) Fig. 1.b.

$$(12) \quad U_0 = \frac{1}{3} (U_a + U_b + U_c),$$

$$(13) \quad i_{\alpha s} = i_a, \quad i_{\beta s} = \frac{\sqrt{3}}{3} (i_a + 2i_b),$$

where i_a , i_b are instantaneous current values of the threephase supply system of the asynchronous motor.

5. Measurement (calculation) of absolute slip

For the calculation (measurement) of the absolute slip (angular velocity) the relation for the electromagnetic (driving) moment in the components of the rotating system (x , y , 0) of the rotor can be used:

$$(14) \quad M_D = -\frac{3}{2} p \left(\psi_{xr} i_{yr} - \psi_{yr} i_{xr} \right).$$

Provided that the asynchronous motor is with a squirrel cage, then $U_{xr} = U_{yr} = 0$ and for the components of the rotor current we have

$$(15) \quad i_{xr} = \frac{p\omega \vec{\psi}_{yr} - \dot{\psi}_{xr}}{R_r},$$

$$i_{yr} = \frac{p\omega \vec{\psi}_{xr} - \dot{\psi}_{yr}}{R_r},$$

The electromagnetic (driving) moment is defined by the relation (for $\dot{\psi}_{xr} = \dot{\psi}_{yr} = 0$)

$$(16) \quad M_D = \frac{3}{2} p^2 \frac{\omega_{\vec{\psi}}}{R_r} \psi_r^2,$$

where $\psi_r^2 = \psi_{xr}^2 + \psi_{yr}^2$ is coupled magnetic flux of the rotor.

The relation for the absolute slip (angular velocity) of the asynchronous motor is considered in the form

$$(17) \quad \omega_{\vec{\psi}} = \frac{2}{3} \frac{R_r}{p^2} \frac{M_D}{\psi_r^2},$$

For the magnetic flux components of the rotor in the coordinate system of the stator ($\alpha, \beta, 0$) the following relations hold

$$(18) \quad \begin{aligned} \psi_{\alpha r} &= \frac{L_r}{L_{sr}} \psi_{\alpha s} - \frac{L_s L_r - L_{sr}^2}{L_{sr}} i_{\alpha s}, \\ \psi_{\beta r} &= \frac{L_r}{L_{sr}} \psi_{\beta s} - \frac{L_s L_r - L_{sr}^2}{L_{sr}} i_{\beta s}, \end{aligned}$$

The inductances L_s, L_r, L_{sr} are obtained by a measuring at the asynchronous motor in short circuit run and in idling.

For ψ_r^2 the following relation holds

$$(19) \quad \psi_r^2 = \psi_{\alpha r}^2 + \psi_{\beta r}^2.$$

The equations (8) and (17) determine the dynamic characteristics of the machine aggregate in the form $M_D(\omega_{\vec{\psi}})$.

6. Implementation of moment and slip pick-up

The implementation of the moment and the angular velocity (slip) pick-up of the asynchronous motor, the circuit diagrams of the individual blocks of the pick-up (see Fig. 1.b), except for the section circuits, were made on the hybrid analog computer MEDA 43 HA.

In Fig. 4 and Fig. 5 are shown courses of steady-state electromagnetic (driving) moment and slip (angular velocity) versus time, determined by measurement.

Fig. 6 shows the dynamic characteristic of the machine aggregate (see Fig. 1.a) in the steady-state of motion for a periodically variable

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moment of inertia and loading moment.

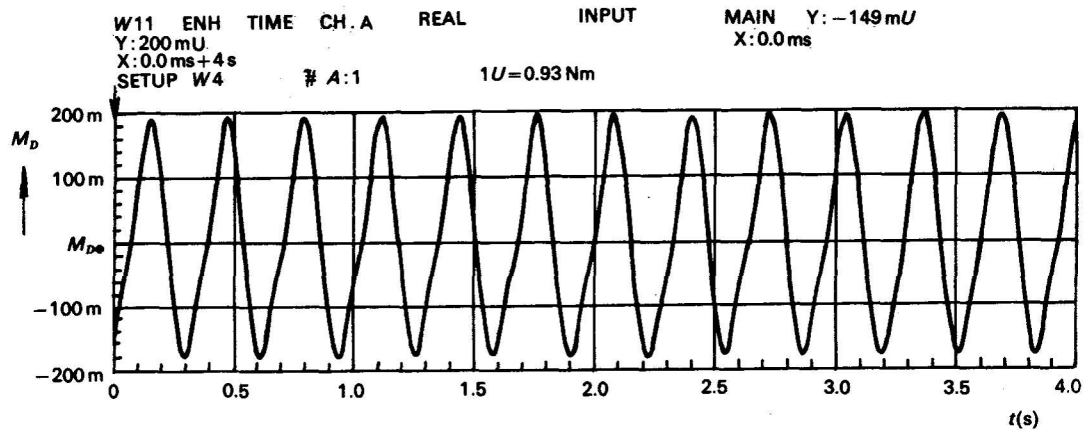


Fig. 4 Electromagnetic (driving) moment time hystory

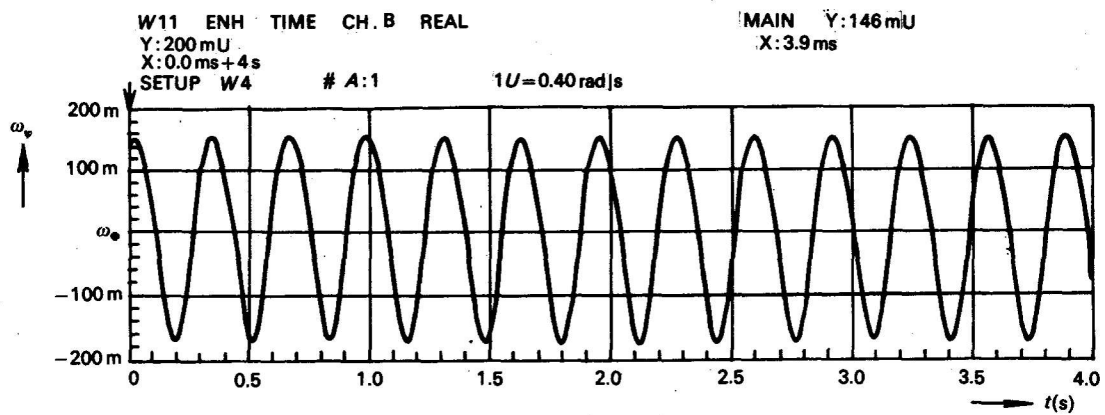


Fig. 5 Slip (angular velocity) time hystory

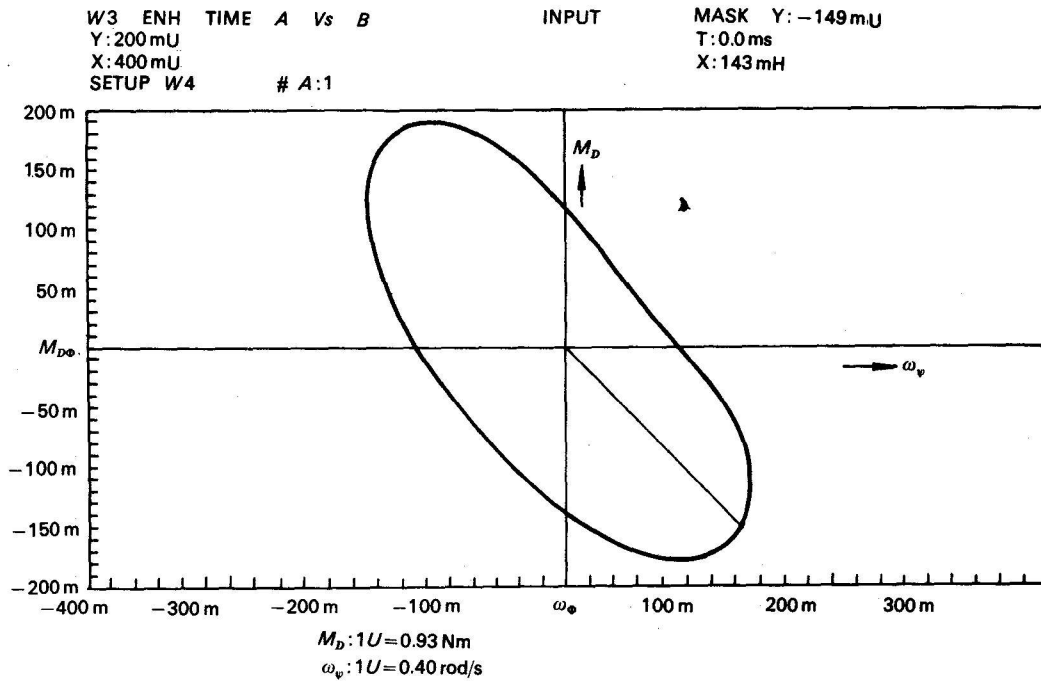


Fig. 6 Dynamic characteristics of the machine aggregate - determined by measurement

7. Conclusions

The results presented in this paper represent only a small part of the results achieved in the field of the dynamics of rotating machine aggregate, which have been obtained at the author's work place in the recent years.

This paper gives a survey of some important results of the theory and of the measurement of some dynamic properties of rotating machine aggregate with a variable moment of inertia and dynamic characteristic of the asynchronous motor with a squirrel-cage.

R e f e r e n c e s

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