

CONTINUUM MECHANICS

THEORETICAL INVESTIGATIONS OF RESIDUAL STRAIN IN LEFT VENTRICLE*

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Introduction

Numerous models dealing with the strained and stressed states of the left ventricle have been proposed in recent years (see review of Yin [1]). In most of them, the unloaded left ventricle is commonly taken as the reference state in which the strain and the stress are zero everywhere. This approach leads to a non-uniform strain and stress distribution across the ventricular wall, decreasing from the endocardium to epicardium. This effect is more pronounced when nonlinear material properties of the left ventricle are used [2]. Based on experimental evidence, Fung [3] proposed the idea of residual stress, defined as the stress in a body when all external loads are removed. Later, Omens and Fung [4] tried to assess the residual strain in rat left ventricle. Their results showed that the circumferential residual strain was negative in the endocardial region which suggests a compressive residual stress component and a more uniform end-diastolic stress distribution across the wall thickness.

The present investigation is concerned with a theoretical model of the residual strain in an ellipsoidal left ventricle.

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Model assumptions

The present finite deformation model of the left ventricle is based on the following assumptions:

— The left ventricle is represented by a thick-walled ellipsoid of revolution with a constant thickness which retains the ellipsoidal geometry during deformation.

— The unloaded state is defined as one in which the pressure in the left ventricle cavity is zero. This state is achieved at end-systole. The two semiaxes and the wall thickness can be measured from clinical ventriculography.

— The stress-free state is obtained after a cut of the left ventricle along the meridian. This state is characterized by a circumferential opening angle θ_0 . In this work a residual angle

$$(1) \quad \theta_r = 2\pi - \theta_0$$

is used instead of the opening angle θ_0 .

The loaded state of the left ventricle is achieved under pressurization caused by the incoming blood during heart diastole. This analysis is confined only to the end-diastolic state of the left ventricle, dimensions of which could be easily measured from ventriculography.

— The ventricular wall is divided into 10 thin layers, thus the assumptions of the membrane theory are taken into account.

— The material of the left ventricle is incompressible.

Kinematics

Based upon the model assumptions, the geometry of the unloaded and loaded states are known from clinical data. In this way, the first step is to determine the long semiaxes A , the short semiaxes B , the thickness H and the residual opening angle θ_0 of stress-free state, which are needed four conditions for.

Using data from [4] about the endocardial circumferential residual stretch ratio λ_{2i} and the opening angle θ_0 , the quantities B and θ_r could be determined. The long semiaxes is supposed to have the same length as in the unloaded state, i.e.

$$(2) \quad A = a$$

As a fourth relation the incompressibility condition is used, resulting in a cubic equation for the stress-free thickness.

Once having the geometry of the stress-free state determined, the deformations of the thin-walled ellipsoids can be described by the following expressions

$$(3) \quad r = r(R), \quad \vartheta = \frac{2\pi}{\theta_r} \theta, \quad z = z(R)$$

for radial, circumferential and meridional directions, respectively. The principal stretch ratios in meridional, circumferential and radial directions are, respectively,

$$(4) \quad \lambda_1 = \frac{ds}{dS}, \quad \lambda_2 = \frac{2\pi}{\theta_r} \frac{r}{R}, \quad \lambda_3 = \frac{h}{H},$$

where ds and dS are the deformed and stress-free meridian arc lengths. Using the incompressibility condition and eqs. (4), the first eq. (4) can be expressed as an ordinary differential equation

$$(5) \quad \frac{dr}{dR} = \left\{ \frac{1 + \frac{A^2}{B^2} \cdot \frac{R^2}{B^2 - R^2}}{1 + \frac{a^2}{b^2} \cdot \frac{r^2}{b^2 - r^2}} \right\}^{1/2} \cdot \frac{\theta_r}{2\pi} \cdot \frac{1}{\lambda_3} \cdot \frac{R}{r},$$

where small letters represent the deformed states and capital letters represent the stress-free state.

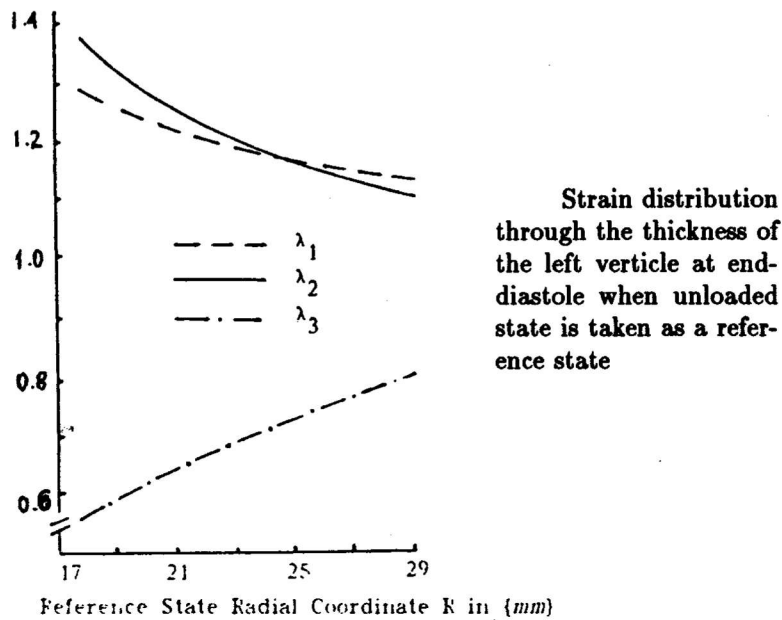


Fig. 1.

Results and discussion

The stress-free geometry was calculated with $\theta_0 = 45^\circ$, $\lambda_{2i} = 0.97$ and $\lambda_{2e} = 0.92$ taken from [4]. Supposing normal left ventricle in unloaded state with $a = 3.46$ cm, $b = 1.79$ cm and $h = 1.296$ cm, the following dimensions were calculated in stress-free state: $B = 2.1$ cm and $H = 1.238$ cm at $\lambda_{2i} = 0.92$; $B = 2.223$ cm and $H = 1.272$ cm at $\lambda_{2i} = 0.97$. One can see, that in stress-free state the thickness decreases not more than 5%.

Stretch ratios in the loaded state are calculated using dimensions of the left heart that eject 70 ml of blood and shortens the long semiaxes by 26% during systole.

The distribution of the stretch ratios $\lambda_1, \lambda_2, \lambda_3$ through the wall at the equator when the unloaded state is taken as a reference state is shown in Fig. 1. It is apparent that the strain distribution is highly nonlinear. For instance, $\lambda_2 = 1.38$ at the endocardium and $\lambda_2 = 1.1$ at the epicardium, so the innermost muscle fibers are elongated 1.25 times more than the outermost ones.

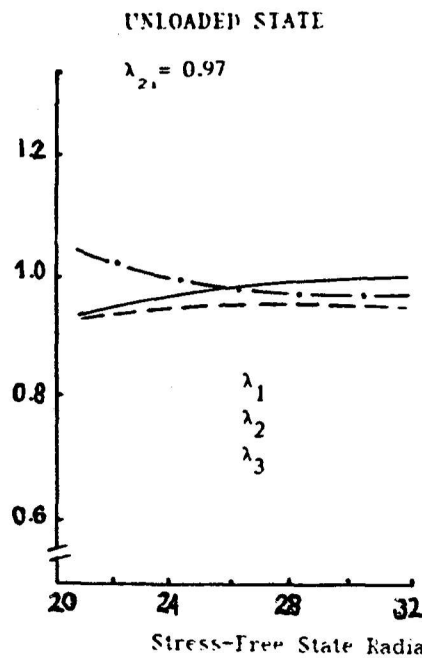


Fig. 2a.

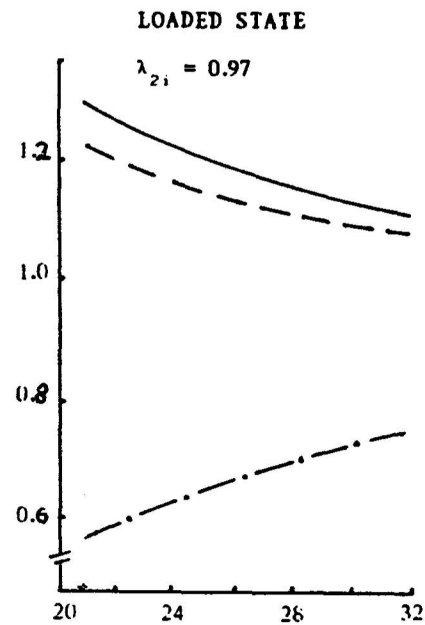


Fig. 2b.

When the residual stress is assumed, the strain distribution across the wall in unloaded state is nonuniform. As can be seen in Fig. 2a, the endocardial fibers are

compressed ($\lambda_2 = 0.97$) and the epicardial fibers are elongated ($\lambda_2 = 1.04$). The neutral line $\lambda_2 = 1.0$ appears at 28% from the endocardial surface. When the endocardial residual strain increases to $\lambda_2 = 0.92$, as shown in Fig. 3.a, the neutral line translates to 70% of the wall thickness. It should be noted that the meridional strain in both cases is negative and almost a constant through the thickness.

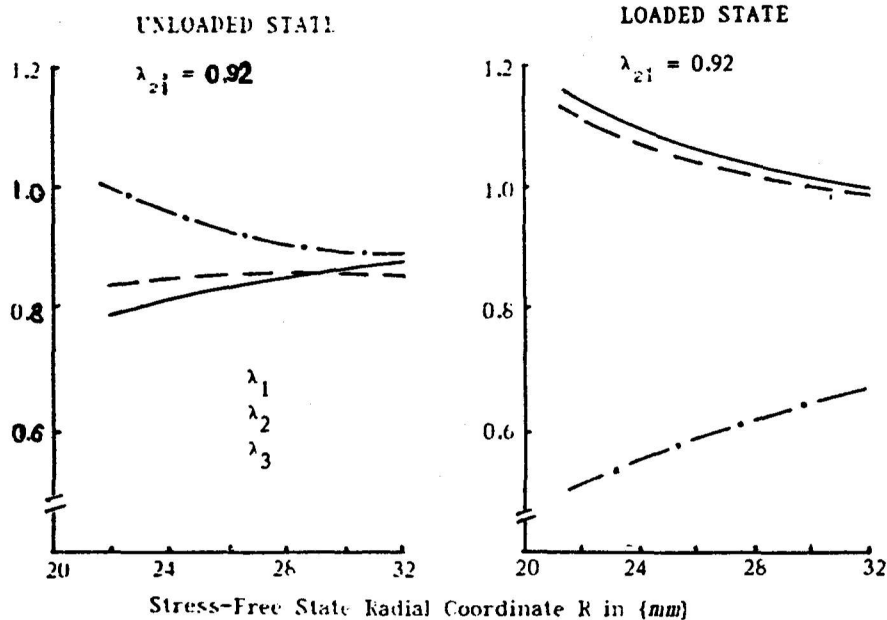


Fig. 3a.

Fig. 3b.

Figures 2.b and 3.b show the distribution of the principal stretch ratios in loaded (end-diastolic) state. The use of the opened up ellipsoid as the zero-strain state reduces the degree of the strain, developed due to a passive inflation of the left ventricle. For example, at $\lambda_{2i} = 0.97$ the circumferential stretch at the endocardium is 1.17 times larger than that one at the epicardium, at $\lambda_{2i} = 0.92$ it is 1.13 times larger. Thus, the presence of the residual strain leads to a more uniform distribution of the strain field in the left ventricular wall. This can play an important role in describing the stress field at end-diastole and during the systole.

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