

INVESTIGATION OF LOAD DISTRIBUTION IN STATICALLY NON-DETERMINABLE PLANETARY GEARS*

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1. Notation

$1, 3, H, 2I, 2II$ – indices for pinion, body, carrier, first and second satellite;

L, Z, W – indices for bearings, gear meshing, and shafts/axles;

f_e, ψ_e – gear eccentricity and phase of the eccentricity;

f_A, ψ_A – bearing openings deviation from coaxial position and deviation phase;

f_{At} – tangent inaccuracy of satellite axle;

$\Delta s, j$ – tooth width deviation, bearing or meshing windage;

$\lambda = \alpha \cdot F$ (linear), or $\lambda = \beta \cdot F^q$ (nonlinear) – elastic characteristic of the element;

F_n, F_r – normal and radial loading force;

F_{nlim} – borderline loading force (the time of the second satellite meshing);

z_i, α_w – teeth number of the 'i' gear, meshing angle;

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φ_{1r} – position parameter (i.e. relative rotation angle of the pinion stem towards the carrier);

C_L – unequal load distribution coefficient: $C_{Lj} = F_{nj}/\Sigma F_{nj}, j = I, II$ – of each separate power branch; $C_{L\Sigma} = \max_j(C_{Lj})$ – of the planetary gear as a whole; C_{Lz} – of the particular tooth.

2. Introduction

The planetary gears with more than one satellite are statically non-determinable and the inevitable inaccuracies in the production and assembly lead to unequal load distribution between satellites. In certain cases this shortcoming may cause serious damages. In the least it would cause load capacity loss, ineffective material usage, raise in the weight and cost of the planetary gear.

Different equalizing mechanisms with different efficiency are used for overcoming of this problem. The planetary gears with equalizing mechanisms have been more thoroughly studied both theoretically and experimentally. However the statically non-determinable planetary gears (i.e. these without equalizing mechanisms) have their own strong advantages: constructive and technological simplicity, effective and economical material usage, lower costs. This makes the investigation necessary and motivates the present interest.

That is why the goal of the present work is a theoretical investigation of the impact of various factors on the internal load distribution in two satellite statically non-determinable planetary gears. For that purpose, the impact factors have been theoretically analyzed; mechanical and mathematical models, research programme and research automation software have been created; computer aided experimental research work has been done.

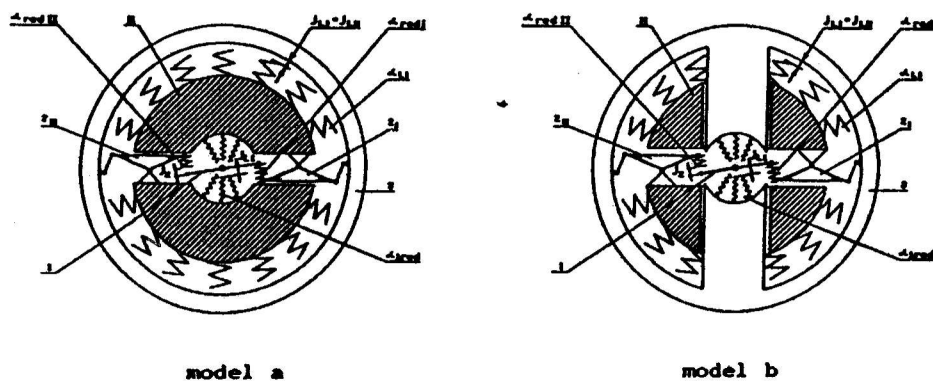


Fig. 1.

3. Impact factors and premises

Generally the impact factors interacting in statically non-determinable planetary gears can be divided into several groups: (a) geometry factors (geometric deviations of the elements i.e. the inaccuracy of the elements – by size and phase, the bearing windages and tooth width deviation); (b) elasticity factors (constant or variable pliabilities, i.e. linear or nonlinear characteristics of the elements); (c) load of the planetary gear; and (d) gear structure (the place of the pinion (1) bearing: in the carrier (H), (see graphic model 'a', Fig. 1.), or in the body (3), (see graphic model 'b', Fig. 1). The gear structure factor leads to a different inaccuracy and elasticity models.

The models have been constructed under the following assumptions: (a) the problem is considered as a plane; (b) only 'zero-' and low frequency factors are taken into consideration; (c) the inner and outer meshing forces for the satellite (2) are assumed equal; and (d) the problem is considered to be static. The possibility for a static consideration without detriment to reliability is conditioned by the experimental results published by different authors [1]. The results obtained under different conditions show that the dynamic effects in the planetary gears with rotational speed less than 3000, 1/min (most widespread case) are commensurable with the experimental method error.

4. Mechanical and mathematical models

The inaccuracy models created by authors permit for analytical expressions for the initial and current windage in one of the meshes to be deduced. By means of projecting the translations which result from separate geometric deviations on the $1-2_{II}$ meshing line, the following analytic functions for meshing windage j_z have been worked out:

in the case of carrier bearing of the pinion:

$$(1) \quad j_z = 2 \cos \alpha_W (f_{e2I} - f_{e2II}) \sin(\varphi_{1r} z_1 / z_2 - \psi_{e2}) + \cos \alpha_W (\Delta s_{2I} - \Delta s_{2II}) - \\ 2f_{e1} \sin(\varphi_{1r} - \psi_{e1} + \alpha_W) + 2f_{e3} \sin(\varphi_{1r} z_1 / z_3 - \psi_{e3} + \alpha_W) + \\ 2f_{A1} \sin(\psi_{A1} + \alpha_W) + 2 \cos \alpha_W (f_{A2II} + j_{L2II} - j_{L1} - j_{L3})$$

and in the case of body bearing of the pinion:

$$(2) \quad j_z = 2 \cos \alpha_W (f_{e2I} - f_{e2II}) \sin(\varphi_{1r} z_1 / z_2 - \psi_{e2}) + \cos \alpha_W (\Delta s_{2I} - \Delta s_{2II}) - \\ 2f_{e1} \sin(\varphi_{1r} - \psi_{e1} + \alpha_W) + 4f_{e3} \sin(\varphi_{1r} z_1 / z_3 - \psi_{e3} + \alpha_W) + \\ 2f_{A1} \sin(\varphi_{1r} z_1 / z_3 + \psi_{A1} + \alpha_W) + \\ 2 \cos \alpha_W (f_{A2II} + j_{L2II} - j_{L1} - 2j_{L3})$$

These functions model the initial (current) meshing windage in $1-2_{II}$ (or $1-2_I$) in dependence of the geometric deviations ($f_i, \psi_i, j_{Li}, \Delta s_{2j}$) and the position parameter φ_{1r} .

There are two considerations in the elastic models: preceding and following the borderline moment (i.e. the moment at which the second satellite meshes). The deformation translations are reduced to the pinion axle 1 and to the meshes 1 – 2_I and 1 – 2_{II} (Fig.1). Using elastic, static and geometric equations for the two bearing cases the following functions have been worked out:

$$(3) \quad A.F_{nlim} + (B + C).F_{nlim} = j_Z(f_i, \psi_i, \Delta s, j_{Li}, \varphi_{1r})$$

$$(4) \quad A.(F_{nI} - F_{nII}) + B.(F_{nI} - F_{nII})^q + C.(F_{nI}^q - F_{nII}^q) = j_Z(f_i, \psi_i, \Delta s, j_{Li}, \varphi_{1r})$$

where A , B and C are constants, depending on the elastic characteristics of the elements. B and C also depend on the bearing type:

$$(5) \quad A = 2\alpha_{W1} \cdot \cos \alpha_W + 4\alpha_{W2} \cdot \cos \alpha_W + \alpha_{Z12} + \alpha_{Z23}$$

in the case of carrier bearing of the pinion:

$$(6) \quad B = 2 \cos \alpha_W \cdot \beta_{jL1}$$

$$(7) \quad C = \cos \alpha_W \cdot \beta_{jL2} \cdot (2 \cos \alpha_W)^q + \cos \alpha_W \cdot \beta_{jL3}$$

and in the case of body bearing of the pinion:

$$(8) \quad B = 2 \cos \alpha_W \cdot \beta_{jL2} + \cos \alpha_W \cdot \beta_{jL3}$$

$$(9) \quad C = \cos \alpha_W \cdot \beta_{jL2} \cdot (2 \cos \alpha_W)^q$$

The functions given above have been worked out with nonlinear elastic characteristics ($\lambda = \beta.F^q$, where $\beta = \text{const}$, $q = 2/3$) of the bearings. When the elastic characteristics, meshing windages (from 1 or 2) and gear load ($F_n = F_{nI} + F_{nII}$) are given F_{nlim} is worked out from 3, and F_{nI} , F_{nII} from 4.

5. Numeric experiments and results

The impact of the factors mentioned above has been studied by computer aided experiments. For that purpose an appropriate software was created. The algorithm for the numeric solution of 3 and 4 comprises:

1. CHOOSING of the (a) independent variable $X := \{F_n, \varphi_{1r}, \dots\}$, (b) discrete interval for X , $k := \{1, 2, \dots, N\}$, (c) impact factors $\Phi := \{F_n, \varphi_{1r}, j_{Li}, f_i, \psi_i, \alpha_i, \dots\}$, (d) impact factor levels $\Phi := \{A, B\}$, (e) values for the rest of the parameters $P := \{F_n, \varphi_{1r}, j_{Li}, f_i, \psi_i, \alpha_i, \dots\}$;

2. FOR $i := \{1, 2, \dots, N\}$ COMPUTING of the functions $Y_{\Phi i} := \{C_L\}$, $Y_{Ak} = Y_A(X_k)$, $Y_{Bk} = Y_B(X_k)$ - for explicit functions and $Y_A(X_k, Y_{Ak}) = 0$, $Y_B(X_k, Y_{Bk}) = 0$ - for implicit functions;

3. PRINTING/DRAWING of the functions $Y_A = Y_A(X)$, $Y_B = Y_B(X)$.

The numeric solutions of 3 and 4 at different independent variables and different impact factors are worked out and some of the results of the conducted computer aided experiments are given in Fig. 2-8. The load distribution coefficient C_L is used for the quantitative measure of the unequal load distribution (Y-axis on the diagrams).

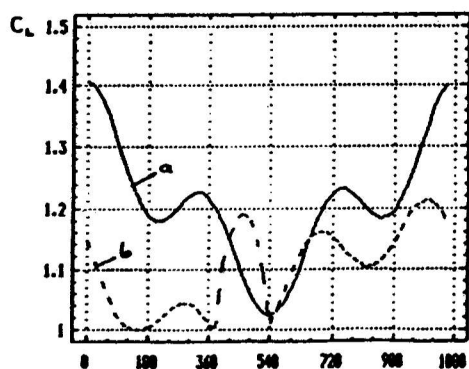


Fig. 2

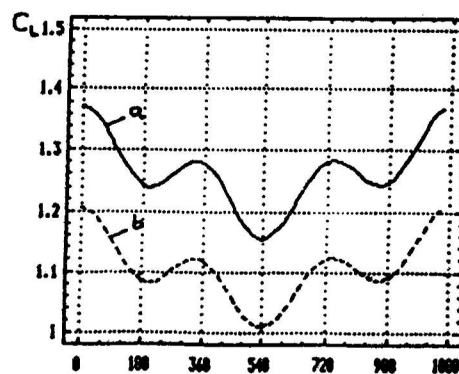


Fig. 3

Fig. 2 presents the geometry deviations impact on C_L (Y-axis) versus φ_{1r} (X-axis). Curve 'a' represents the most adverse combination of the geometry deviations $f_i, \psi_i, j_{Li}, \Delta s_{2j}$. Curve 'b' represents random combination of the geometry deviations.

Fig. 3 presents the differences in C_L (Y-axis) versus φ_{1r} (X-axis) in the case of: 'a'-absence and 'b'-presence of bearing windages of the central elements (pinion and carrier bearings). The advantages of the second case are obvious.

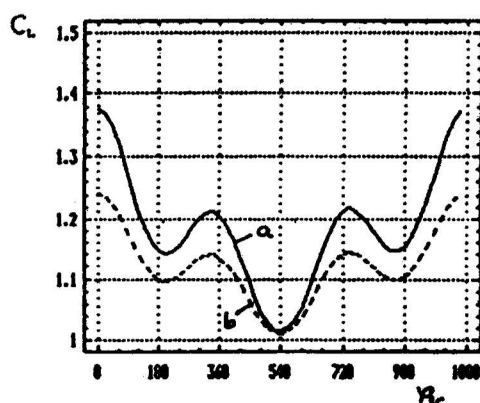


Fig. 4

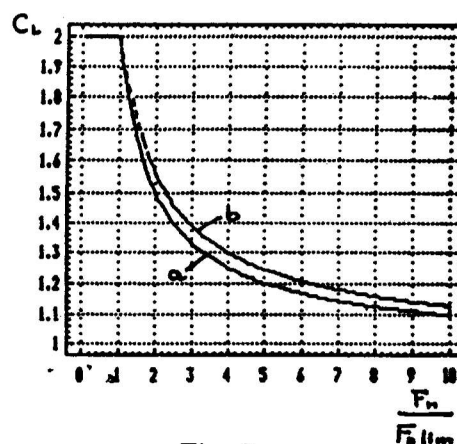


Fig. 5

Fig. 4 presents the impact of: 'a'-'hard' and 'b'-'soft' elastic characteristics of pinion shaft and satellite axes on the C_L (Y-axis) versus φ_{1r} (X-axis). The favorable impact of the 'soft' elasticity is evident.

Fig. 5 presents the change of C_L (Y-axis) versus relative load F_n/F_{nlim} (X-axis) when the elastic characteristics are: 'a'-linear and 'b'-nonlinear.

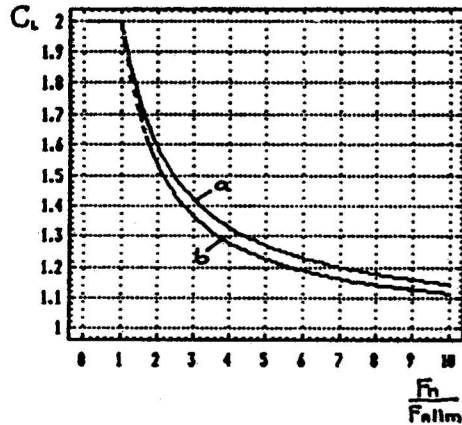


Fig. 6

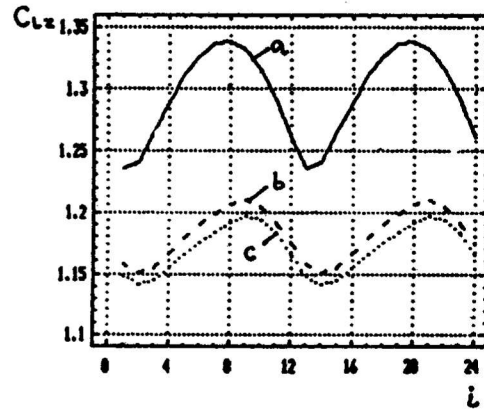


Fig. 7

Fig. 6 presents the change of C_L (Y-axis) versus relative load F_n/F_{nlim} (X-axis) when the pinion bearing place is: 'a'-in the carrier and 'b'-in the body.

Fig. 7 presents the effective load distribution coefficients for strength computations of the pinion for: 'b'-bending ($C_{Lz\text{eff},F}$) and 'c'-contact ($C_{Lz\text{eff},H}$). In the computations the load distribution coefficient specters of the particular tooth (see Fig. 8.) are taken into consideration for each tooth (X-axis, i-tooth number). This method is described in [1].

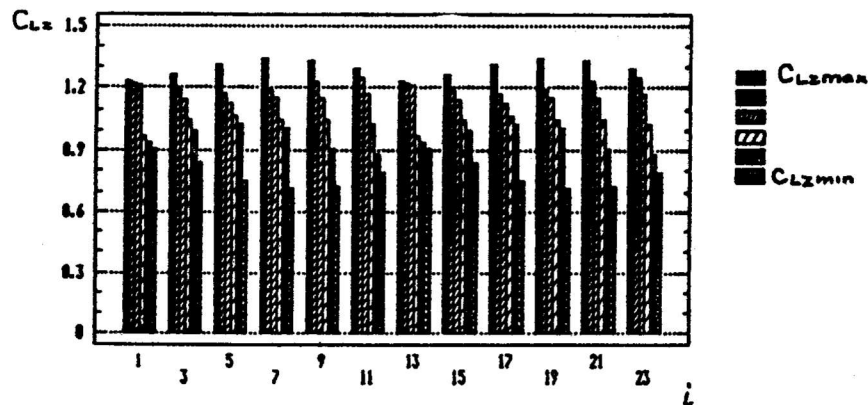


Fig. 8

Fig. 8 illustrates the load distribution coefficient specters of particular tooth of the pinion C_{Lz} (particular loads are given in descending order), for a concrete load case. Every bar in the cluster of the specific tooth represents particular load distribution

coefficient The histogram of C_{Lz} is calculated throughout the cinematic cycle for every tooth (X -axis, odd teeth are given only).

6. Conclusion

Obviously effective load distribution coefficients (curve 'b' and curve 'c') are lower then the maximum load distribution coefficients (curve 'a') for every tooth

$$(C_{Lz\text{eff},F} < C_{Lz\text{max}}, C_{Lz\text{eff},H} < C_{Lz\text{max}}).$$

The significant reserves for more efficient material usage are evident. Their utilization based on the work of the authors [2] is a question of the future.

REFERENCES

- [1] ARNAUDOV, K. Theoretical and experimental research on internal load distribution in planetary gears. Attainment of academic rank, Sofia, Bulgarian Academy of Sciences, 1990.
- [2] ARNAUDOV, K. et al. Investigation of internal load distribution in statically non-determinable planetary gears. Report on contract NI TN-214/92 with the Ministry of Education and Sciences, I part, 1993.

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