

ON THE SYNTHESIS OF SKEW-AXES GEARS WITH PLANE REGION OF MESH

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1. Introduction

When profiling kinematically connected surfaces by means of which a transformation of rotations between skewed axes is realized, the basic principles that have to be observed are known as Olivier's principles in the meshing theory literature [1, 2, 3].

We will discuss briefly our opinion about the mathematical modelling of skew-axes gears whose synthesis obeys Olivier's second principle. In this case when forming the gears tooth surfaces the following conditions have to be satisfied: a) the instrumental surfaces which cut the tooth surfaces of one of the gears is a geometric copy of the other gear tooth surfaces, and b) the processes of instrumental and working meshing are equivalent.

2. Mathematical modelling using a pitch point

In this case is synthesized a skew-axes gear pair with guaranteed quality characteristics in one point of contact, and in its vicinity, respectively. This is achieved by choosing a suitable point of the stationary space as a point belonging to the region of mesh, aiming at insuring the optimal set of quality characteristics, such as: accuracy of a realization of the necessary motion transformation function, conditions for effective driving energy utilization, etc. These characteristics, which control the meshing quality in only one contact point, define the technologic strategy for manufacturing of the whole gear set, and its price, respectively.

This mathematical modelling method is based on the following. Some contact point of a pair of conjugate surfaces is a common point of two circles whose centers lie on the axes between which is performed the rotations transformation and whose planes are perpendicular to the described axes. These circles are called by the authors pitch primary circles [4].

3. Mathematical modelling related to the gears quality characteristics in the whole region of mesh

When synthesizing skew-axes gears with linear contact between their tooth surfaces, the advisability of controlling the quality characteristics of the gear set in the whole region of mesh is obvious. Such an approach to the synthesis problem determines the necessity of creating a mathematical model which describes the process of a rotations transformation. Then, however, it is necessary to be defined completely one of the tooth surfaces, one of the gears respectively, which form the conjugate linear contact.

This approach has to be used when technologic, strength or exploitation reasons have been taken into consideration and when the strict definition of definite quality characteristics of the skew- axes gears type is exigent. The same approach is binding if the pitch point place definition is impossible or not needed. The gear pair examined in the paper refers to this case.

4. Mathematical modelling and synthesis of non-orthogonal skew-axes gear-pair, based on a plane region of mesh

4.1. Region of mesh

The synthesis of skew-axes gears transforming rotations between non-orthogonal skewed axes is a scientific problem that has a wide applied significance. The non-orthogonality between the driving and driven shafts in a given transmission leads to some positive effects as: a decrease of the transmission dimensions, an increase of the strength and exploitation conditions of the gear set by the help of an elimination of the nodes from the region of mesh, etc.

The paper studies a skew-axes gear-pair which has the following characteristics:

- the tooth surfaces are generated by Olivier's second principle;
- the rotations transformation is realized between skewed axes with shaft angle not equal to $\frac{\pi}{2}$;
- the working tooth surfaces of one of the gears are planes.

The case when the axes of rotation are orthogonally skewed is known as Wildhaber's gear [5].

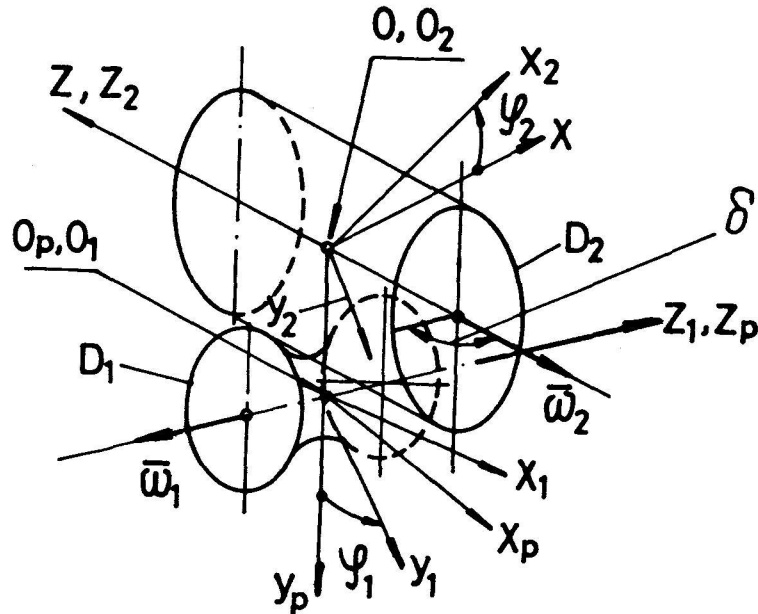


Fig. 1. Kinematic scheme of the gear set

In Fig. 1 is shown a kinematic scheme of the gear set. D_1 and D_2 are the pitch surfaces of the gears which are surfaces of revolution with axes of rotation 1-1 and 2-2, resp. D_2 is a cylinder while D_1 is a surface, such that D_1 and D_2 are kinematically conjugate [6].

The investigation is carried out using the coordinate systems shown in Fig. 1, namely $S_0(x, y, z)$ and $S_p(x_p, y_p, z_p)$ are stationary ones, $S_1(x_1, y_1, z_1)$ and $S_2(x_2, y_2, z_2)$ are firmly connected with D_1 and D_2 , resp., i.e. S_1 and S_2 rotate round the axes Oz and O_pz_p with angular velocities ω_1 and ω_2 . $\delta = \angle(\omega_1, \omega_2)$. $\varphi_i (i = 1, 2)$ is the parameter of meshing.

The equations of the plane Σ_2 of the tooth are (Fig. 2):

$$(1) \quad \begin{aligned} x_2 &= u \sin \alpha \\ y_2 &= r_2 + u \cos \alpha \\ z_2 &= \tau \end{aligned}$$

Here r_2 is the radius of the pitch cylinder D_2 , α is the pressure angle, u and τ are the coordinates of the tooth. These parameters satisfy the conditions: $u > 0$ for the addendum flank, $u < 0$ for the dedendum flank, $u = 0$ if a point of the plane Σ_2 is

simultaneously a point of D_2 ; $\tau \in [-\frac{b_2}{2}, \frac{b_2}{2}]$, b_2 is the face width. $\alpha \in (0, \frac{\pi}{2})$ describes the surface Σ_2 contacting with Σ_1 when ω_1 and ω_2 are the angular velocities shown in Fig. 1 and when ω_1 is the input rotation angular velocity. $\alpha \in (-\frac{\pi}{2}, 0)$ describes the opposite surface Σ_2 of the same tooth.

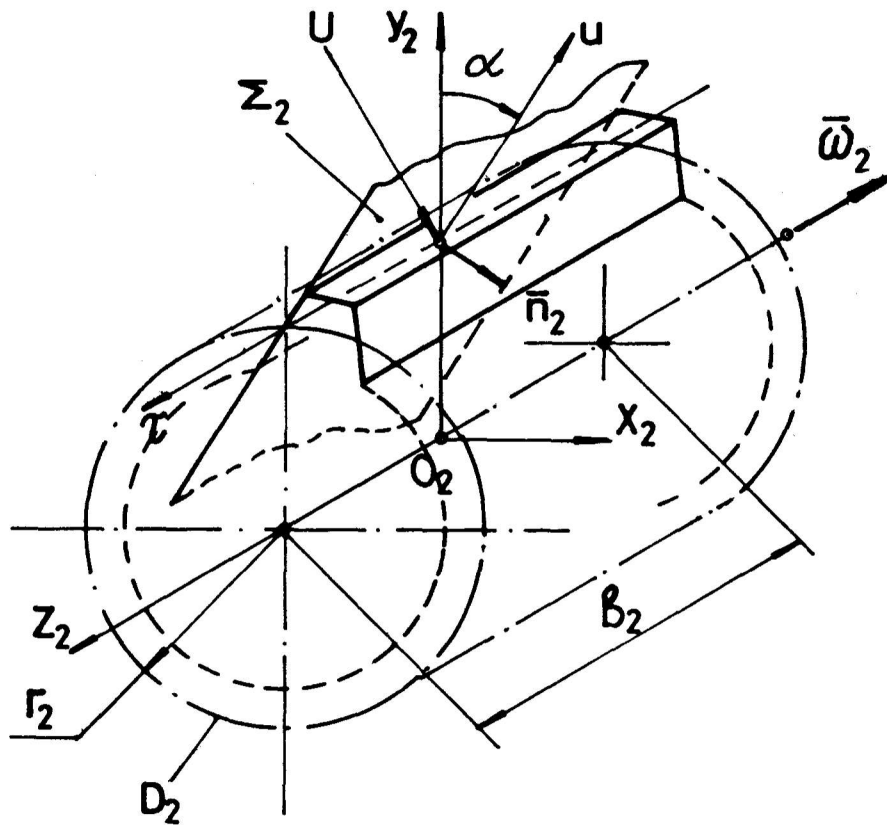


Fig. 2. Spur gear with plane teeth

To synthesize a skew-axes gear-pair with regard to the whole region of mesh, it is necessary to know the equation of meshing. In the most common case this equation is [7]

$$(2) \quad v_{12} \cdot n_2 = 0.$$

Here v_{12} is the velocity vector of the relative sliding between Σ_1 and Σ_2 ; n_2 is the normal vector to Σ_2 in the contact points of Σ_1 and Σ_2 .

Then, using (1), equation (2) written in coordinate system S appears to be

$$(3) \quad [(\cos \delta - i_{21})y - a_W \cos \delta] \cos(\alpha + \varphi_2) + [(\cos \delta - i_{21})x - z \sin \delta] \sin(\alpha + \varphi_2) = 0,$$

where $i_{21} = 1/i_{12} = |\omega_2|/|\omega_1|$ is the speed ratio, a_W is the offset, x, y, z are the coordinates of a contact point which is described by

$$(4) \quad \begin{aligned} x &= r_2 \sin \varphi_2 + u \sin(\alpha + \varphi_2) \\ y &= r_2 \cos \varphi_2 + u \cos(\alpha + \varphi_2) \\ z &= \tau \end{aligned}$$

The set of (3) and (4) represents the equations of the contact lines of Σ_1 and Σ_2 in S , i.e. the region of mesh. It is evident that the region of mesh is a plane and the contact lines are straight-lines.

4.2. Analysis of the region of mesh

The analysis of the region of mesh from a point of view of the character and position of the contact lines is an essential step in mathematical modelling taking into account the region of mesh characteristics.

The existence of nodes in the region of mesh has influence on the gears efficiency, the wear and tear, the strength and the hydrodynamic loading capacity. The total or partial elimination of the nodes from the region of mesh is related to the synthesized tooth surfaces geometry, and to construction and dimensions of the gear set.

The nodes may be divided into two groups.

4.2.1. Ordinary nodes.

It is known [7] that on the tooth surfaces may exist the so-called ordinary nodes which are the common points of contact or intersection of the contact lines. In such points the oil film between the tooth surfaces is broken and as a result a loss of pressure in the film and consecutive negative effects are present.

The condition of ordinary nodes existence is of the form [7]

$$(5) \quad \frac{\partial}{\partial \varphi_2} [v_{12} \cdot n_2] = 0$$

and, in the concrete case, appears to be

$$(6) \quad [(\cos \delta - i_{21})y - a_W \cos \delta] \sin(\alpha + \varphi_2) = [(\cos \delta - i_{21})x - z \sin \delta] \cos(\alpha + \varphi_2)$$

4.2.2. Nodes of undercutting.

If the nodes of undercutting in the region of mesh occur, the process of Σ_1 undercutting means [7] that the fillet surface Σ_J generated by the tip cutting edge

takes a part of the working surface Σ_1 . Then the fillet surface and the working surface have no common tangent but intersect.

The absolute velocity vector of a contact point belonging to Σ_1 may be written as

$$(7) \quad v_1 = v_{e,1} + v_{r,1},$$

where $v_{e,1}$ and $v_{r,1}$ are circumferential and relative velocity vectors, resp.

It is known [7] that if a point $M \in \Sigma_1$ is a point of undercutting, then the condition

$$(8) \quad v_{r,1} = 0$$

is fulfilled. Consequently,

$$(9) \quad v_1 = v_{e,1}.$$

Having in mind that $d\varphi_1 = i_{12}d\varphi_2$, equation (9) written in coordinate system S_1 is:

$$(10) \quad \begin{aligned} dx &= \cos \delta (a_W - y) i_{12} d\varphi_2 \\ dy &= (\cos \delta x - \sin \delta z) i_{12} d\varphi_2 \\ dz &= \sin \delta (y - a_W) i_{12} d\varphi_2 \end{aligned}$$

Thus, using (3) and (10) the following analytic relation is obtained:

$$(11) \quad \frac{\{i_{21}[(\cos \delta - i_{21})y - a_W \cos \delta] - (a_W - y)(1 - i_{21} \cos \delta)\} \sin(\alpha + \varphi_2)}{\{(\cos \delta - i_{21})(\cos \delta x - \sin \delta z) + (\cos \delta - i_{21})x - z \sin \delta\} \cos(\alpha + \varphi_2)} =$$

Equation (11) describes the points of undercutting in the region of mesh.

The obtained relations serve for optimizing the design process. Surely, it is necessary to create more criteria which control for example the values of relative radius of curvature, the sliding velocity values, etc.

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