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ON THE MATHEMATICAL MODELING OF IMPACT IN A SYSTEM OF BODIES

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In previous publications [1, 2] the impact is considered as a sequence of impacts between bodies of the system. According to the problems which have to be studied there are two approaches which can be used: phenomenological and analytical. For the analytical approach itself two main approaches are possible: deterministic and stochastic. The first one gives only a linear graph of binary impacts for a set of initial conditions of the system. When the second one is used there are variety of possible methods to be used. One important group of methods is based on stochastic tests approach, i.e. instead natural experiments numerical experiments based on statistics of variables with respect to the natural experiments is the possibility to calculate the value of impulses exchanged in particular binary impacts.

When stochastic methods are used the graph of the impact is not a linear one but a tree with a lot of routes from the beginning to the end of the impact. An important characteristic of each route is its probability to take place. The distribution of the probabilities with respect to the routes can be used to define the stability of the system with respect to the impact [2].

Thus, the test of a hypothesis

$$p_\nu \geq p_* \tag{1}$$

where p_* = given level of probability,

p_ν = frequency of a route with suffix ν which takes place in natural as well as in numerical experiments. One of the most effective method in a situation when there is lack of information about the system is Wald's Sequential Analysis [3]. Following this method the complex hypothesis (1) can be presented by two simple for $q_\nu = 1 - p_\nu$:

$$H_0 \quad q_\nu = q_0 = 1 - p_* \tag{2}$$

$$H_1 \quad q_\nu = q_1 = 1 - p_{**} \tag{3}$$

where $q_0 < q_1$ and $p_{**} < p_*$,
and $q_1 - q_0 = \Delta q = p_* - p_{**}$ is defined on the basis of acceptable tolerances.

The appearance of the route ν in "i" test is a random variable x_i. Let $f_o(x_i)$ be the probability that this route does not take place if the hypothesis H_1 is true. Let α be the probability to reject the hypothesis H_0 when it is true, and β be the probability to accept it when the hypothesis H_1 is true. Then, after "m" tests the following ratio of plausibility can be written

$$\frac{q_{1m}}{q_{0m}} = \frac{f_1(x_1)}{f_0(x_1)} \cdot \frac{f_1(x_2)}{f_0(x_2)} \cdots \frac{f_1(x_m)}{f_0(x_m)} \tag{4}$$

According to the Sequential Analysis if after "n" tests

$$\frac{q_{1n}}{q_{0n}} \leq \frac{\beta}{1-\alpha} \tag{5}$$

the hypothesis H_0 is accepted and if

$$\frac{q_{1n}}{q_{0n}} \geq \frac{1-\beta}{\alpha} \tag{6}$$

the hypothesis H_1 is accepted.

If both conditions (5) and (6) are not satisfied then

$$\frac{\beta}{1-\alpha} < \frac{q_{1n}}{q_{0n}} < \frac{1-\beta}{\alpha} \tag{7}$$

takes place and tests are continued.

If the number of cases when the route ν does not take place in "n" tests is $\alpha_{\nu n}$ the condition (7) can be transformed into

$$b_n < \alpha_{\nu n} < \alpha_n \tag{8}$$

where

$$a_n = \frac{\ln \frac{1-\beta}{\alpha} + n . \ln \frac{1-q_0}{1-q_1}}{\ln \frac{q_1}{q_0} - \ln \frac{1-q_1}{1-q_0}}; \quad b_n = \frac{\ln \frac{\beta}{1-\alpha} + n . \ln \frac{1-q_0}{1-q_1}}{\ln \frac{q_1}{q_0} - \ln \frac{1-q_1}{1-q_0}}.$$

When (8) is satisfied the tests should continue. If $\alpha_{\nu n} \leq b_n$ the hypothesis $p_\nu \geq p_*$ should be accepted and when $\alpha_{\nu n} \geq a_n$ it should be rejected and $p_\nu \leq p_{**}$ is to be accepted.

The method presented above in combination with a set of levels of probability for different levels of stability of the system of bodies with respect to the impact would be useful in many cases of study of different systems by means of numerical experiments. This opportunity will help to avoid expensive natural experiments.

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