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A MECHANICAL MODEL ON THE DYNAMICS OF SPIROID GEARS*

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Introduction

The up to date level of development of industry and transport is characterized by a considerable comparative growth of the use of gears. The spatial gearing, i.e. gears with skewed axes, are characterized by the greatest constructive variety and by highest investigation complexity. After this reason they are not so well studied yet, and therefore, it is not realized the full amount of their abilities, yet. Among the most up to date gearing with skewed axes take part the Spiroid and Helicon gears. Spiroid gears and gears from type Helicon are a trade mark of Ilinois Tools – Chigago. The first report about them are the publications of the scientists from USA F.Bohle [1], O.Saari [2], W.Nelson [3, 4, 5], D.Dudly [6], in the course of the period 1954-1964. At the end of this period another scientists became to investigate these gears such as A. Georgiev, I. V. Goldfarb [7, 8, 9, 10, 11], V. Ganshin [12] and others.

Since 1980 in the Institute of Mechanics in Bulgarian Academy of Sciences V.Abadjiev started investigations in the field of this class of high reduction gears in the case of arbitrary skewed axes of spiroid worm and spiroid gear and when the flanks of the spiroid worm are a part of the convolute surface [13, 14].

From the carried out algorithms for synthesis and analysis of this gearing are obtained the corresponding calculation processions with an application in synthesis in gears with orthogonal axis of spiroid worm and worm wheel with archimed and involute teeth of the worm as well as for calculation of convolute, involute and archimed Helicon gearings [15].

While analysing the published in the specialized literatute papers dealing with the investigation of Spiroid and Helicon gears it can be estimated that this study in the field of vibration behaviour of these gears is a pioneer one. The aim of this paper is an investigation of the level of vibrations in one

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real spatial Spiroid gearing type Helicon designed to work as a drive in closed rooms in a closed neighbourhood with children.

The mechanical modelling is created on the basis of a double mass dynamic model with twelve degrees of freedom — six for each mass [19, 24]. The influence of the elastic links between the two masses and also the energy mutual linkages between the coordinates of the two bodies are investigated [16, 17, 21, 22, 23].Natural frequencies and amplitudes of motion could be defined by means of computer, experiment in spatial aspect [19]. The obtained results can be used for optimization of the constructive parameters of the spiroid gear and also for improving of harmful resonance effects.

General Concept of Spatial Mechanical Modeling

A Spiroid gear-pair is revealed presented in Fig. 1. The mechanical modelling of the Spiroid gear-pair is created on the basis of a double mass dynamic spatial model with twelve degrees of freedom (Fig. 1). The elastic links and also the energy mutual links between the coordinates are considered [18, 19, 20]. The mechanical model is the base for a creation of a system of twelve non-linear mutually related differential equations from second order [19, 21].

It is convenient to define the position of the rigid body 1 with mass m_1 and rigid body 2 with mass m_2 (see Fig.1) in space using three orthogonal

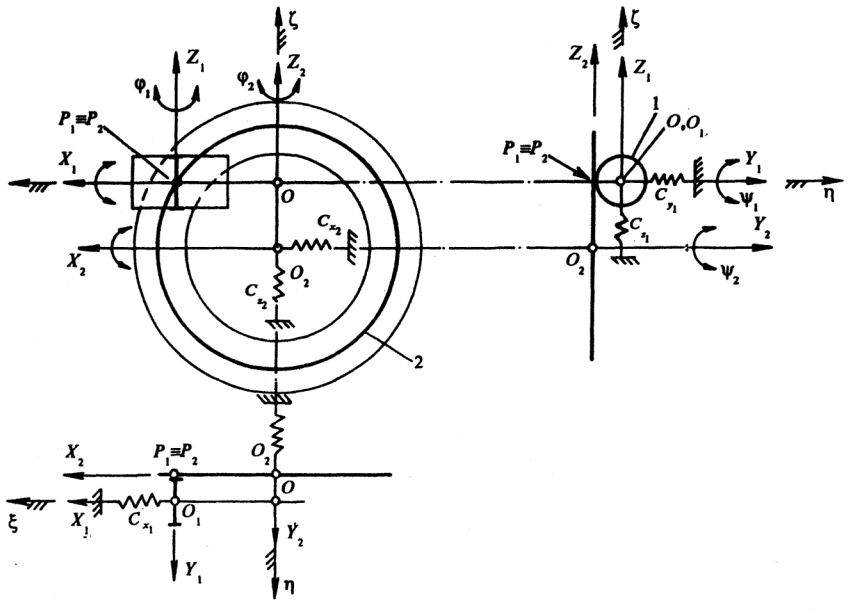

Fig. 1. A double mass dynamic spatial model.

cordinate systems — an immovable one $O\xi\eta\zeta$ and two movable $O_1X_1Y_1Z_1$ and $O_2X_2Y_2Z_2$ where O_1 and O_2 are the mass centres of the two bodies presented in Fig. 1. The kinetic energy of the dynamic system can be defined by equations:

$$
\begin{aligned}
T_1 &= \frac{1}{2}\left[m_1\left(\dot{x}_1^2+\dot{y}_1^2+\dot{z}_1^2\right)+J_{x_1}\dot{\theta}_1^2+J_{y_1}\dot{\psi}_1^2+J_{z_1}\dot{\varphi}_1^2\right],\\
T_2 &= \frac{1}{2}\left[m_2\left(\dot{x}_2^2+\dot{y}_2^2+\dot{z}_2^2\right)+J_{x_2}\dot{\theta}_2^2+J_{y_2}\dot{\psi}_2^2+J_{z_2}\dot{\varphi}_2^2\right] \qquad (1)\\
T &= T_1+T_2,
\end{aligned}
$$

where m_1 and m_2 are masses of two rigid bodies; J_{x_i}, J_{y_i}, J_{z_i} mass moments of inertia, x_i, y_i, z_i, θ_i, ψ_i, φ_i — generalized coordinates q_k.

The following equations describe the links between the coordinates of the three orthogonal coordinate system $O\xi\eta\zeta$, $O_1X_1Y_1Z_1$ and $O_2X_2Y_2Z_2$:

$$
\begin{aligned}
\xi_i &= \xi + x_i - \varphi y_i + \psi z_i,\\
\eta_i &= \eta + \varphi x_i + y_i - \theta z_i, \qquad (2)\\
\zeta_i &= \zeta - \psi x_i + \theta y_i + z_i.
\end{aligned}
$$

Deformations of elastic elements are defined as follows

$$
\begin{aligned}
u_i &= \xi + \psi z_i - \varphi y_i,\\
v_i &= \eta + \varphi x_i - \theta z_i, \qquad (3)\\
w_i &= \zeta + \theta y_i - \psi x_i.
\end{aligned}
$$

The potential energy of the dynamic system can be defined by equation:

$$
\prod = \frac{1}{2}\sum_{i=1}^{n}\left(C_x^i u_i^2 + C_y^i v_y^2 + C_z^i w_i^2\right), \qquad (4)
$$

where C_x^i, C_y^i, C_z^i are the corresponding elasticity coefficients.

After replacement of formulae (2) and (3) in Eq.(4) can be obtained equation (5) defining the potential energy of the single mass vibration sieve.

$$
\begin{aligned}
\prod &= \prod_1 + \prod_2,\\
\prod_1 &= \frac{1}{2}\left[C_{x_1}(x_1+\psi_1 z_{O_1}+\varphi_1 y_{O_1})^2 + C_{y_1}(y_1-\varphi_1 x_{O_1}-\theta_1 z_{O_1})^2 + \right.\\
&\quad \left. C_{z_1}(z_1+\theta_1 y_{O_1}-\psi_1 x_{O_1})^2\right]\\
\prod_2 &= \frac{1}{2}\left[C_{x_2}(x_2+\psi_2 z_{O_2}-\varphi_2 y_{O_2})^2 + C_{y_2}(y_2-\varphi_2 x_{O_2}-\theta_2 zO_2)^2 + \right.\\
&\quad \left. C_{z_2}(z_2+\theta_2 y_{O_2}-\psi_2 x_{O_2})^2\right] \qquad (5)\\
&\quad + \frac{1}{2}\left[C_x(x_2+x_{p_2}+\psi_2 z_{p_2}-\varphi_2 y_{p_2}-x_1-x_{p_1}-\psi_1 z_{p_1}+\varphi_1 y_{p_1})^2 + \right.\\
&\quad + C_y(y_2-\varphi_2 x_{p_2}+y_{p_2}-\theta_2 z_{p_2}-y_1-y_{p_1}-\varphi_1 x_{p_1}+\theta_1 z_{p_1})^2 + \\
&\quad \left. + C_z(z_2-\psi_2 x_{p_2}+z_{p_2}+\theta_2 y_{p_2}-z_1-z_{p_1}+\psi_1 x_{p_1}-\theta_1 y_{p_1})^2\right],
\end{aligned}
$$

where $C_{x_i}, C_{y_i}, C_{z_i}, C_x, C_y, C_z$ are the coefficients of elastisity of handling.

It is convenient for dynamic systems to use equation of Lagrange from second order presented in the form:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j - \frac{\partial \prod}{\partial q_j}. \tag{6}$$

Using the formulas for coordinate transformation (2), and the Lagrange equation (6) after linearization is obtained a system of six linear differential equations (7). Formulae defining the potential and kinetic energy of the body are presented in (1) and (5).

$$\begin{aligned}
m_1\ddot{x}_1 + (c_{x_1} + C_x)x_1 =& C_x(x_2 + x_{p_2} - \varphi_2 y_{p_2} + \psi_2 z_{p_2} - x_{p_1} + \\
& \varphi_1 y_{p_1} - \psi_1 z_{p_1}) - p, \\
m_1\ddot{y}_1 + (C_{y_1} + C_y)y_1 =& C_y(y_2 + \varphi_2 x_{p_2} + y_{p_2} - \theta_2 z_{p_2} - \varphi_1 x_{p_1} - \\
& y_{p_1} + \theta_1 z_{p_1}) - C_{y_1} x_{01}\varphi_1 - p + \\
& m_1 e_1 \omega_1^2 \sin(\omega_1 t + \alpha), \\
m_1\ddot{z}_1 + (C_{z_1} + C_z)z_1 =& C_z(z_2 - \phi_2 x_{p_2} + \theta_2 y_{p_2} + z_{p_2} + \phi_1 x_{p_1} - \\
& \theta_1 y_{p_1} - z_{p_1}) + C_{z_1} x_{01}\psi_1 - p + \\
& m_1 e_1 \omega_1^2 \cos(\omega_1 t + \alpha), \\
J_{x_1}\ddot{\theta}_1 + (C_y z_{p_1}^2 + C_z y_{p_1}^2)\theta_1 =& -C_y z_{p_1}(y_2 + \varphi_2 x_{p_2} + y_{p_2} - \theta_2 z_{p_2} - \\
& y_1 - \varphi_1 x_{p_1} - y_{p_1}) + C_z y_{p_1}(z_2 - \psi_2 x_{p_2} + \\
& \theta_2 y_{p_2} + z_{p_2} + \psi_1 x_{p_1} - z_1 - z_{p_1}) + \\
& p(z_{p_1} - y_{p_1}), \\
J_{y_1}\ddot{\psi}_1 + (C_x z_{p_1}^2 + C_z x_{p_1}^2)\psi_1 =& C_x z_{p_1}(x_2 + x_{p_2} - \varphi_2 y_{p_2} + \psi_2 z_{p_2} - \\
& x_1 - x_{p_1} + \varphi_1 y_{p_1}) - C_z x_{p_1}(z_2 - \psi_2 x_{p_2} + \\
& \theta_2 y_{p_2} + z_{p_2} - z_1 - \theta_1 y_{p_1} - z_{p_1}) + \\
& C_{z_1} x_{01}(z_1 - x_{01}\psi_1) + p(x_{p_1} - z_{p_1}), \\
m_2\ddot{x}_2 + (C_{x_2} + C_x)x_2 =& -C_x(x_{p_2} - \varphi_2 y_{p_2} + \psi_2 z_{p_2} + \psi_2 z_{p_2} - x_1 \\
& - x_{p_1} + \varphi_1 y_{p_1} - \psi_1 z_{p_1}) - \\
& C_{x_2}(x_2 + \psi_2 z_{02} - \varphi_2 y_{02}) + p + \\
& m_2 e_2 \omega_2^2 \sin(\omega_2 t + \beta - \alpha), \\
m_2\ddot{y}_2 + (c_{y_2} + C_y)y_2 =& -c_y(\varphi_2 x_{p_2} + y_{p_2} - \theta_2 z_{p_2} - y_1 - \\
& \varphi_1 x_{p_1} - y_{p_1} + \theta_1 z_{p_1}) - \\
& C_{y_2}(y_2 + \varphi_2 x_{02} - \theta_2 z_{02}) + p, \\
m_2\ddot{z}_2 + (C_{z_2} + C_z)z_2 =& -C_z(-\psi_2 x_{p_2} + \theta_2 y_{p_2} + z_{p_2} - z_1 +
\end{aligned} \tag{7}$$

$$\begin{aligned}
&\psi_1 x_{p_1} - \theta_1 y_{p_1} - z_{p_1}) - C_{z_2}(z_2 + \\
&\theta_2 y_{02} - \psi_2 x_{02}) + p + m_2 e_2 \omega_2^2 \cos(\omega_2 t + \beta - \alpha),
\end{aligned}$$

$$\begin{aligned}
J_{x_2}\ddot{\theta}_2(C_y z_{p_2}^2 + C_z y_{p_2}^2)\theta_2 = &C_y z_{p_2}(y_2 + \varphi_2 x_{p_2} + y_{p_2} - y_1 - \\
&\varphi_1 x_{p_1} - y_{p_1} + \theta_1 z_{p_1}) - C_z y_{p_2}(z_2 - \\
&\psi_2 x_{p_2} - \theta_1 y_{p_1} + z_{p_2} + \psi_1 x_{p_1} - z_1 - z_{p_1}) + \\
&p(y_{p_2} - z_{p_2}) + C_{y_2 z_{O_2}}(y_2 + \varphi_2 x_{02} - \theta_2 z_{02}) - \\
&C_{z_2 y_{O_2}}(z_2 + \theta_2 y_{02} - \psi_2 x_{02}),
\end{aligned}$$

$$\begin{aligned}
J_{y_2}\ddot{\psi}_2(C_x z_{p_2}^2 + C_z x_{p_2}^2)\psi_2 = &-C_x z_{p_2}(x_2 + x_{p_2} - \varphi_2 y_{p_2} - \psi_1 z p_1 - \\
&x_1 - x_{p_1} + \psi_1 y_{p_1}) + C_z x_{p_2}(z_2 + \\
&\theta_2 y_{p_2} + z_{p_2} - z_1 + \psi_1 x_{p_1} - \theta_1 y_{p_1} - z_{p_1}) + \\
&p(z_{p_2} - x_{p_2}) - C_{x_2 z_{O_2}}(x_2 + \psi_2 z_{02} - \varphi_2 y_{02}) + \\
&C_{z_2 x_{O_2}}(z_2 + \theta_2 y_{02} - \psi_2 x_{02}),
\end{aligned}$$

$$\begin{aligned}
J_{z_2}\ddot{\varphi}_2(C_x y_{p_2}^2 + C_y x_{p_2}^2)\varphi_2 = &C_x y_{p_2}(x_2 + x_{p_2} + \varphi_1 y_{p_1} + \psi_2 z p_2 - \\
&x_1 - x_{p_1} - \psi_1 z_{p_1}) - C_y x_{p_2}(y_2 + \\
&y_{p_2} - \theta_2 z_{p_2} - y_1 - \varphi_1 x_{p_1} - y_{p_1} + \theta_1 z_{p_1}) + \\
&p(x_{p_2} - y_{p_2}) - C_{y_2 x_{O_2}}(y_2 + \varphi_2 z_{02} - \theta_2 z_{02}) + \\
&C_{x_2 y_{02}}(x_2 - y_{02}\varphi_2 + \psi_2 z_{02}),
\end{aligned}$$

where:

— e_1, e_2 are coefficient of eccentricity of the input and output axle;

— ω_1, ω_2 are the excitement frequencies;

— p is an impulse force created by errors in the region of mesh.

In order to be obtained the generalized forces it is necessary to define the virtual labour δA following the equation:

$$\begin{aligned}
\delta A &= p(\delta\xi + \delta\eta + \delta\zeta) \\
&= p(x_2 + x_{p_2} + \psi_2 z_{p_2} - \varphi_2 y_{p_2} - x_1 - x_{p_1} - \psi_1 z_{p_1} + \varphi_1 y_{p_1} + \\
&\quad y_2 + \varphi_2 x_{p_2} + y_{p_2} - \theta_2 z_{p_2} - y_1 - \varphi_1 x_{p_1} - y_{p_1} + \theta_1 z_{p_1} + \\
&\quad z_2 - \psi_2 x_{p_2} + z_{p_2} + \theta_2 y_{p_2} - z_1 + \psi_1 x_{p_1} - z_{p_1} - \theta_1 y_{p_1}).
\end{aligned}$$

It is well known [24] that the generalized forces are in fact the coefficients in front of the variations of the generalized coordinates in the equation of the virtual labour. The impulse force p can be defined using the following equation:

$$p = v_k \sqrt{m.C}, \tag{8}$$

where $v_k = \omega_1\sqrt{2\delta_t.a}$ is a worm peripheral velocity, $a = \frac{\rho_2}{\rho_1}(\rho_1 + \rho_2)(\rho_1, \rho_2$ are radii of the profile curves in the pitch point), $m = m_1.m_2/(m_1 + m_2)$ is an adduced mass and assuming $\delta_t = \Delta t$.

3. Conclusion

This paper solves the problems of the mechanical modelling of a double mass dynamic vibration system in spatial aspect. The following expected conclusions can be defined:

1. The character of vibration movement is most important for the working mode of the gearing.

2. We can do optimization of the constructive parameters and also improve harmful resonance effects using the obtained results after carried out computer experiment.

3. In further investigations we can do a suitable adjustment of the double mass dynamic system in order to obtain a desirable monofrequency spatial vibration movement of the dynamic system and to recrease the vibration movement and vibration state of the gear-set.

4. The obtained results can be applied in gears with skewed axes.

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