

FLUID MECHANICS

PATH TO CHAOS FOR FLOW-INDUCED VIBRATIONS IN TUBES CONVEYING FLUID

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1. Introduction

The present work discusses numerical results for a dynamical system which describes the vibration of an elastic tube conveying fluid in the presence of motion-limiting constraints.

The computer program used for these calculations was obtained as a modification of the finite-element code employed in [3] to calculate the flow-induced motion of a cantilevered tube with a variable support between the clamped end and the free end. Results of [3] were compared with experimental results of [1] and found to be in good qualitative agreement. It should be noted that in this case no chaos was found in either the numerical or the experimental results.

In [2], the support is replaced by a pair of motion-limiting constraints (see figure 2.1) which compress like linear springs. These springs are positioned on both sides of the tube as shown in figure 2.1 with a clearance gap. The tube is allowed to oscillate and hit the springs, compressing them vertically while sliding past them.

The presence of motion-limiting constraints is modelled by means of a sort of penalty method, which can be done with a small modification of the code developed in [3]. Specifically, if the transverse coordinate of that point on the tube whose longitudinal coordinate is equal to that of the constraints exceeds in magnitude half the gapwidth between the constraints, then a force is exerted on the tube at that point which is proportional to the amount by which the constraint is compressed and which points toward the opposite constraint. The results for this case were compared with experimental results of [4]. In [4]

the motion of the tube is observed to be chaotic for certain parameter values. Similar behaviour is realized in [2] for parameter values similar to those of [4]. In fact two paths to chaos were found. One is obtained by increasing the flow velocity which all other parameters remain constant. The second path occurs when the penalty parameter μ (the stiffness of the springs on the motion-limiting constraints) is allowed to increase while all other parameters remain constant.

The present paper investigates the question of what happens to the solution when the position of the motion-limiting constraint is allowed to vary along the tube. Let x_0 stand for the longitudinal component of the distance between the constraints and the build-in end of the tube (see figure 2.1). Runs were done on the cases $x_0 = 0.50, 0.55, 0.575, 0.60, 0.65$.

Section 2 develops the equations which describe the system, section 3 shows the results and section 4 gives the conclusions.

2. Mathematical modelling

Consider a straight cantilevered inextensible elastic tube of length L conveying fluid under pressure (figure 2.1). The tube is clamped at one end and free at the other, with motion constraints at a distance l_0 from the clamped end. These motion constraints can be rigid or flexible and they are positioned with a clearance d to the tube. Fluid moves through the tube in a direction tangent to the tube's centre line with a relative velocity U which is independent of time and position along the tube. Rotatory inertia is neglected.

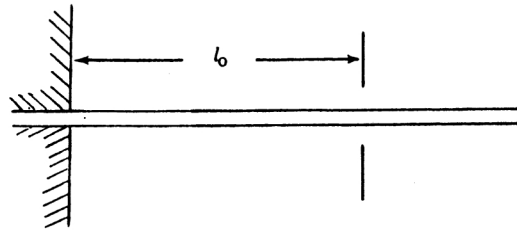


Figure 2.1 . Tube Clamped at One End, Free at the Other with Motion Constraints Whose Position Can Vary Along the Tube

As shown in [3], if the above described tube has motion constraints between its ends, then its motion is governed by the vector equation,

$$(2.1) \quad \frac{\partial}{\partial s} \left((N - p - El\kappa^2) \frac{\partial r}{\partial s} \right) + f = El \frac{\partial^4 r}{\partial s^4} + 2MU \frac{\partial^2 r}{\partial s \partial t} + MU^2 \frac{\partial^2 r}{\partial s^2} + (M + m) \frac{\partial^2 r}{\partial t^2}$$

with the inextensibility constraint,

$$(2.2) \quad \frac{\partial r}{\partial s} \cdot \frac{\partial r}{\partial s} = 1$$

Here, N is the axial force in the tube, p is the compressive axial pressure force at the section, El is the flexural rigidity, f is the body force per unit length, s stands for distance along the tube from the clamped end and

$$(2.3) \quad r(s, t) = w(s, t)i + u(s, t)j$$

stands for the position vector at time t of the material element at s . Unit vectors i and j point respectively in the longitudinal and transverse directions of the undisturbed tube. The curvature κ^2 is given by

$$(2.4) \quad \kappa^2 = \frac{\partial^2 r}{\partial s^2} \cdot \frac{\partial^2 r}{\partial s^2}$$

Finally, M and m are respectively, the mass per unit length of the fluid and the tube.

In terms of the customary nondimensional quantities,

$$(2.5) \quad \begin{aligned} x &= \frac{s}{L}, \quad \eta = \frac{u}{L}, \quad \zeta = \frac{w}{L}, \quad \Pi = \frac{r}{L} = \zeta i + \eta j, \quad Z = \frac{L^2}{El}(N - p) \\ \tau &= \left(\frac{El}{M + m} \right)^{1/2} \cdot \frac{t}{L^2}, \quad \rho = \left(\frac{M}{El} \right)^{1/2} LU, \quad \beta = \left(\frac{M}{M + m} \right)^{1/2} \end{aligned}$$

equation (2.1) becomes

$$(2.6) \quad ((Z - \Pi_{xx} \cdot \Pi_{xx}) \Pi_x)_x + F = \Pi_{xxxx} + 2\rho\beta\Pi_{x\tau} + \rho^2\Pi_{xx} + \Pi_{\tau\tau}$$

where

$$(2.7) \quad F = \frac{L^3 f}{El}$$

The inextensibility constraint takes the form,

$$(2.8) \quad \Pi_x \cdot \Pi_x = 1$$

As in [3], the associated boundary conditions are,

$$(2.9) \quad \begin{aligned} \Pi(0, t) &= 0, \quad \Pi_x(0, t) = i \\ \Pi_{xx}(1, t) &= \Pi_{xxx}(1, t) = 0, \quad Z(1, t) = 0 \end{aligned}$$

Equations (2.6) and (2.8) form a system of three scalar equations in the three unknowns ζ , η and Z . In order to effect a reduction to two equations in two unknowns and make the problem numerically tractable, one introduces as in [3] the penalty parameter λ through the assumption,

$$(2.10) \quad Z = \lambda \left((\Pi_x \cdot \Pi_x)^{1/2} - 1 \right)$$

To obtain the effect of the motion constraints, one chooses,

$$(2.11) \quad F = \begin{cases} \mu(d - \eta(\hat{x}, t))\delta(x - \hat{x}) & \text{if } \eta(\hat{x}, t) \geq d \\ -\mu(d + \eta(\hat{x}, t))\delta(x - \hat{x}) & \text{if } \eta(\hat{x}, t) \leq -d \\ 0 & \text{if } |\eta(\hat{x}, t)| < d \end{cases}$$

Here, $(x_0, 0)$, where $x_0 = \frac{l_0}{L}$ is the point above and below which the motion constraints are positioned, $\hat{x}$ is the relative length of that part of the tube which lies between the clamped end and the point on the tube with horizontal coordinate x_0 , δ is the Dirac delta function and μ is a sort of second penalty parameter whose magnitude is chosen to reflect the flexibility of the motion constraints. The Finite Element method has been employed for the solution of the boundary value problem in the same manner as is described in [2] and [3].

3. Result and discussion

This work takes as its starting point the case of $\rho = 8.05$, $\mu = 10^3$, $\beta = 0.4625$, $d = 0.025$ and $x_0 = 0.5$ in which case the solution is periodic. The value of the flow velocity ρ is chosen to reflect an experimental flow velocity of [4]. After fixing ρ at 8.05, the position x_0 of the motion constraints was allowed to vary along the tube. It appears that, when x_0 is increased gradually starting from $x_0 = 0.5$, the solution changes from periodic to quasiperiodic and ultimately to chaotic. At $x_0 = 0.55$ the solution is still periodic and at $x_0 = 0.575$ is quasiperiodic. At $x_0 = 0.6$ and $x_0 = 0.65$ the solution becomes chaotic. For each run of the code, the time history of the transverse component of the position vector of the second node in the ten node finite element approximation was recorded. This is close to where the experimental signal of [4] was taken. A similar record was also made for the motion of the free end. The power spectra at the second and tenth nodes were in all cases practically identical, as one would expect. Power spectrum plots were made with logarithmic ordinate scales. The phase portraits were plotted as $\dot{\eta}(0.2, \tau)$ versus $\eta(0.2, \tau)$ for the second node.

Selected results are presented in figures 3.1, 3.2, 3.3 . The top picture shows the time history of the transverse component of the position vector of

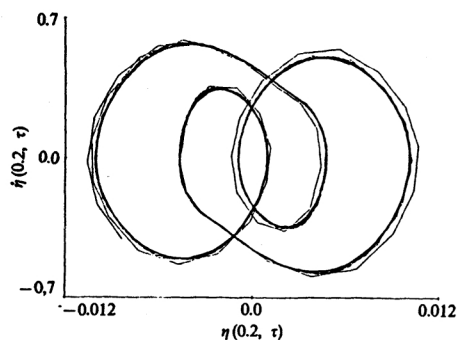
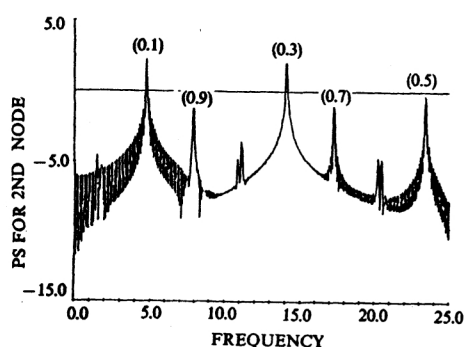
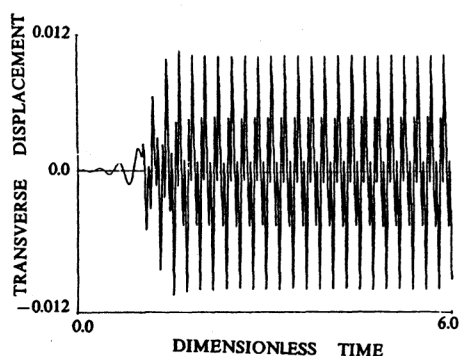


Figure 3.1. Transverse Displacement, Power Spectrum and Phase Portrait of the 2nd Node, when $x_0 = 0.5$

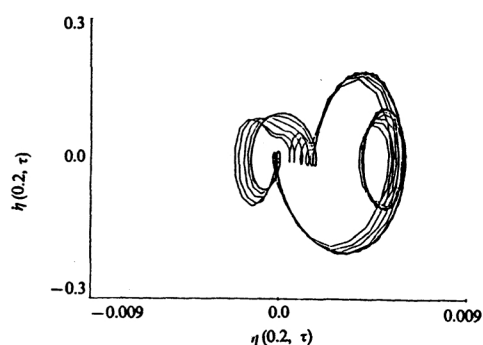
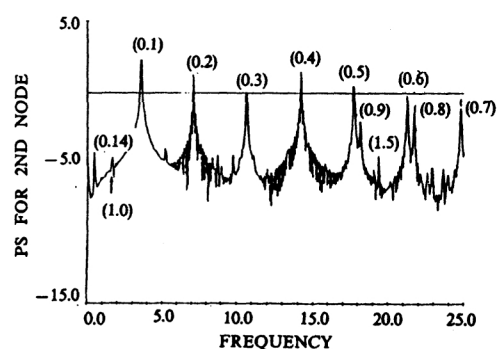
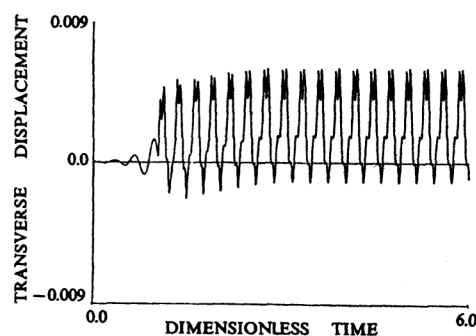


Figure 3.2. Transverse Displacement, Power Spectrum and Phase Portrait of the 2nd Node, when $x_0 = 0.575$

the second node. The middle picture shows the power spectrum of the time history and the bottom picture displays the phase portrait.

Figure 3.1 shows results obtained for the case of $x_0 = 0.5$. The oscillation is periodic with fundamental frequency $f = 4.688$. The phase portrait is characteristic of a periodic time history.

Figure 3.2 shows results obtained for the case of $x_0 = 0.575$. The

oscillation is quasiperiodic with two fundamental frequencies, $f_1 = 1.660$ and $f_2 = 3.516$. Table 3.1 shows the indexing of the remaining peaks. The phase portrait is characteristic of a quasiperiodic time history.

Table 3.1: Indexing of Peaks of the power spectrum in figure 3.2

(n,k)	$f_{(n,k)}$	$nf_1 + kf_2$
(0,14)	0.439($\equiv 49.561$)	49.224(aliasing)
(1,0)	1.660	1.660
(0,1)	3.516	3.516
(1,1)	5.176	5.176
(0,2)	7.080	7.032
(0,3)	10.596	10.548
(0,4)	14.160	14.064
(0,5)	17.676	17.580
(0,9)	18.408($\equiv 31.592$)	31.644(aliasing)
(1,5)	19.336	19.240
(0,6)	21.240	21.096
(0,8)	21.680($\equiv 28.32$)	28.128(aliasing)
(0,7)	24.756	24.612

Figure 3.3 shows results obtained for the case of $x_0 = 0.65$. The oscillation of the tube is chaotic. The transition from quasiperiodic to chaotic solutions takes place somewhere between $x_0 = 0.575$ and $x_0 = 0.6$. The power spectrum in figure 3.3 is characteristic of wide-band chaos. The phase portrait appears very irregular, as expected.

4. Conclusions

It is evident from the above results that the change in position of the motion-limiting constraints affects the type of solution. The transition from periodicity to chaos through quasiperiodicity is clear from the results. The comparison of the results to experimental data of previous experiments suggests that this model and numerical solution gives a very good approximation to the physical behaviour of this dynamical system.

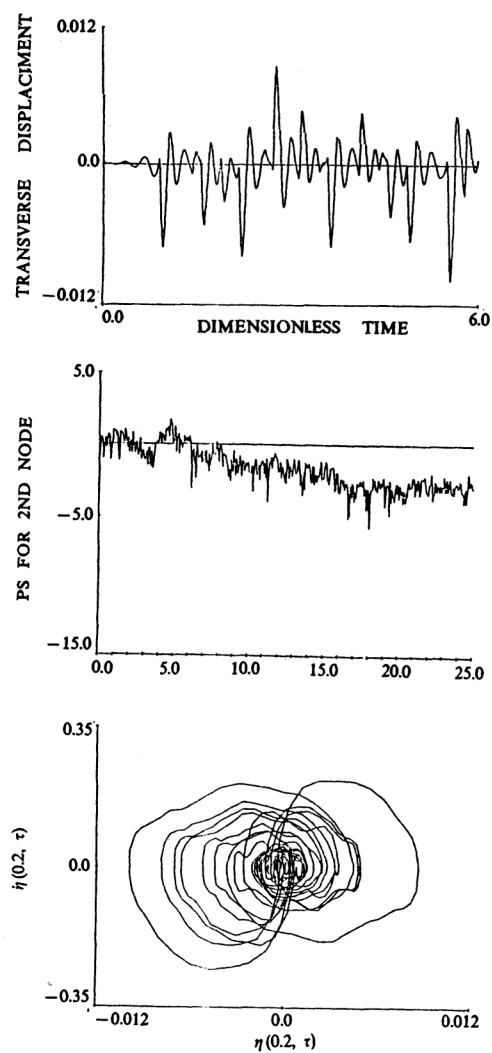


Figure 3.3. Transverse Displacement, Power Spectrum and Phase Portrait of the 2nd Node, when $x_0 = 0.65$

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