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NON-LINEAR MANY-DIMENSIONAL VIBRATIONS OF A MOTORCAR UNDER DISTURBANCE FROM THE ENGINE AND THE ROAD

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The non-linear three-dimensional vibrations of a motorcar with a driver and charge, and the connected with them angular vibrations in the transmission, are treated in this paper. The many-dimensional vibrations are excited from the engine (ICE), and the road.

The motorcar is treated as a rigid body O (Fig.1) with mass M and inertia moments tensor J (towards $OXYZ$). The tyres are treated as a three-dimensional elasto-damping elements (EDE) $E_{kl}U_{kl}V_{kl}W_{kl}$ with stiffness and damping constants c_{xl}, c_{yl}, c_{zl} and b_{xl}, b_{yl}, b_{zl}, and coordinates l_{xl}, l_{yl}, l_{zl}, $(l = 1,4)$.

The motorcar mast 1 is treated as a rigid body $(m_1, J_1, O_1X_1Y_1Z_1)$, rotating around the axis $O_1^*Y_1^* \equiv O_1Y_1$. The mast position is defined by the direction cosines $(\alpha = \cos\varphi'_{n1}, \alpha^* = \sin\varphi'_{n1})$ and the vector $h_{n1} = (e_x^*, e_y^*, e_z^*)^T$. The mast is connected with the car body by torsional EDE E_C. The charge 2 (m_2, J_2, $O_2X_2Y_2Z_2$) translates along the axis $O_2^*Z_2^* \equiv O_2Z_2$. The charge position is defined with the distance l along Z_2^*. The charge is hanged on the mast by EDE E_T.

The driver plus the seat is treated as a rigid body (m_3, J_3, $O_3X_3Y_3Z_3$). It makes relative translation along the axis $O_3^*Z_3^* \equiv O_3Z_3$ by EDE E_B. Its position is defined by the direction cosines $(\gamma = \cos\varphi_{n3}, \gamma^* = \sin\varphi'_{n3})$ and the vector $h_{n3} = (e'_x, e'_y, e'_z)^T$.

The engine plus the gearbox A is treated as a body $(M_A, J_A, O_AX_AY_AZ_A)$, elastically mounted on the motorcar by three-dimensional EDE $E_{Ai}U_{Ai}V_{Ai}W_{Ai}$ $(i = 1,4)$ with stiffness and damping constants c_{Axi}, c_{Ayi}, c_{Azi}, and b_{Axi}, b_{Ayi}, b_{Azi}. The EDE coordinates towards $O_AX_AY_AZ_A$ are l_{Axi}, l_{Ayi}, l_{Azi}, and towards $OXYZ$ are l'_{Axi}, l'_{Ayi}, l'_{Azi}.

The elasto-damping elements (EDE) E_C, E_T, E_B are non-linear. Their characteristics are as follows:

$$\begin{aligned} F_C &= c_C\varphi_C + f_{cC}(\varphi_C) + b_C\dot{\varphi}_C + f_{bC}(\dot{\varphi}_C); \\ F_T &= c_Tq_T + f_{cT}(q_T) + b_T\dot{q}_T + f_{bT}(\dot{q}_T); \\ F_B &= c_Bq_B = f_{cB}(q_B) + b_B\dot{q}_B + f_{bB}(\dot{q}_B), \end{aligned}$$

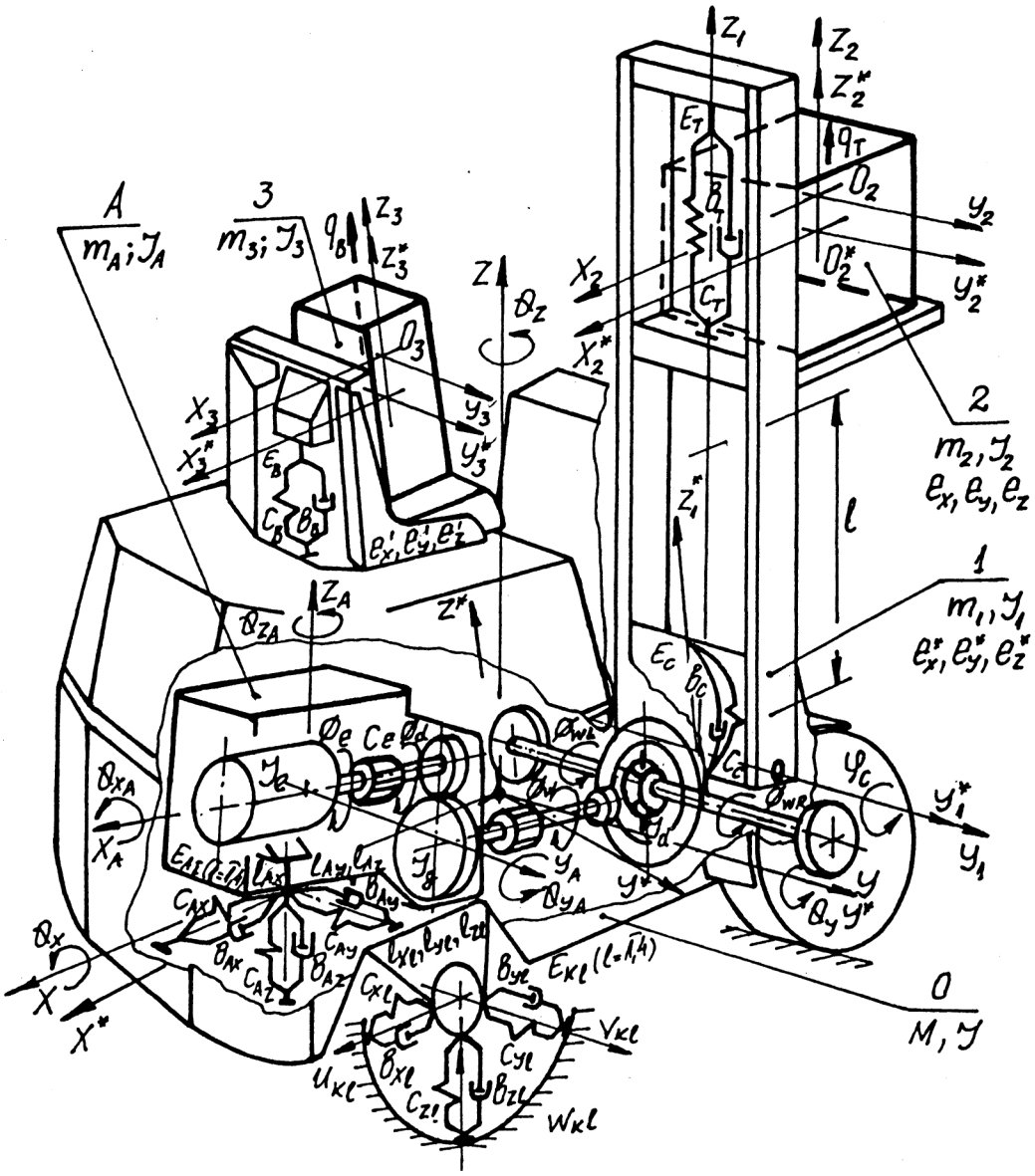


Fig. 1

where c_C, c_T, c_B, and b_C, b_T, b_B are stiffness and damping constants [1].

The transmission is described by rotating masses with coordinates φ_{wL}, φ_{wR}, φ_g, φ_e. The transmission circuit is branched by an automotive differential, and closed by the wheels and the road. The mass moments of inertia are J_e, J_g, J'_g, J_d, J'_d and J_w; the angular velocity ratios are u_g and u_d; the torsional spring and damping constants of couplings are c_e, c_d, and b_e, b_d; the

wheels statical radius of rolling are R_{wL} and R_{wR}.

The vector of generalized displacements, describing the motion of the examined mechanical system, has the following form:

$$\begin{aligned}(1)\qquad &\psi = (x, y, z, \theta_x, \theta_y, \theta_z, \varphi_c, q_T, q_B,\\ &x_A, y_A, z_A, \theta_{Ax}, \theta_{Ay}, \theta_{Az}, \varphi_{wL}, \varphi_{wR}, \varphi_g, \varphi_e)^T_{19\times 1}.\end{aligned}$$

The engine's work is described by disturbing force H_A and disturbing torque M_A, towards $O_4X_4Y_4Z_4$, and torque M_A^*, applied on the engine rotor and the wheels. They are represented in complex vector form, as follows:

$$\begin{aligned}H_A &= \mathrm{Re}\left\{\kappa_H \sum_{k=1}^{n} H_k^* e^{jk\varphi_e}\right\}, \quad H_k^* = (H_{Ax}^*, H_{Ay}^*, H_{Az}^*)^T;\\ M_A &= \mathrm{Re}\left\{\kappa_M \sum_{k=0}^{n} M_k^* e^{jk\varphi_e}\right\}, \quad M_k^* = (M_{Ax}^*, M_{Ay}^*, M_{Az}^*)^T;\\ (2)\qquad M_A^{(e)*} &= \mathrm{Re}\left\{\kappa_M \sum_{k=0}^{n} M_{Axk}^{(e)*} e^{j(k\varphi_e+\varphi_o)}\right\};\\ M_{wL} &= \mathrm{Re}\left\{\kappa_{wL} \sum_{k=0}^{n} M_{wLk}^* e^{jk\varphi_{wL}}\right\};\\ M_{wR} &= \mathrm{Re}\left\{\kappa_{wR} \sum_{k=0}^{n} M_{wRk}^* e^{jk\varphi_{wR}}\right\};\end{aligned}$$

where H_k^*, M_k^* and M_{wL}, M_{wR} are the amplitudes of the k-th harmonic, and κ_H and κ_M are coefficients of engine charge [1, 2, 4].

A three-dimensional kinematic excitation appears when the motorcar moving on the road

$$(3.1)\qquad \mu_{kl} = (U_{kl}, V_{kl}, W_{kl})^T \quad l = 1, 4,$$

non-determined or determined:

$$(3.2)\qquad \mu_{kl} = \mathrm{Re}\left\{\sum_{k=1}^{s} \mu_{kl}^* e^{jpk*t}\right\}, \quad l = 1, 4,$$

where μ_{kl}^* is the amplitudes vector, and p_k^* is excitation frequency [2,3,4].

The differential equations of motion of the examined mechanical system, after cutting the members of second and higher degree towards the generalized coordinates, have the following matrix form:

$$(4)\qquad A\ddot{\psi} + B\dot{\psi} + C\psi = -S\tilde{\dot{\psi}} - R(\psi, \dot{\psi}) + Q_F + Q_K,$$

where the matrices $A = [a_{ij}]$, $B = [b_{ij}]$, $C = [c_{ij}]$ describe the linear properties of inertia, damping and elasticity. The matrices are compound of sub-matrices, as follows:

$$A = \begin{bmatrix} A_0 & A_{CT} & A_B & 0 & 0 \\ A_{CT}^T & A_Q & 0 & 0 & 0 \\ A_B^T & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & A_A & 0 \\ 0 & 0 & 0 & 0 & A_T \end{bmatrix}_{19\times19}$$

(5)

$$C = \begin{bmatrix} C_0 & 0 & 0 & -C_{0A} & C_R \\ 0 & C_Q & 0 & 0 & 0 \\ 0 & 0 & c_B & 0 & 0 \\ -C_{0A}^T & 0 & 0 & C_A & 0 \\ C_R^T & 0 & 0 & 0 & C_T \end{bmatrix}_{19\times19}$$

(6) $A_O = [a_{ij}^{(O)}]_{6\times6}$; $a_{ij}^{(O)} = a_{ji}^{(O)}$; $a_{11}^{(O)} = a_{22}^{(O)} = a_{33}^{(O)} = M + \sum_{i=1}^{3} m_i$;

$a_{15} = -a_{24} = \sum_{i=1}^{3} m_i e_{iz}^*$; $a_{16}^{(O)} = -a_{34}^{(O)} = -(\sum_{i=1}^{3} m_i e_{iy}^*)$;

$a_{26}^{(O)} = -a_{35}^{(O)} = \sum_{i=1}^{3} m_i e_{ix}^*$; $a_{44}^{(O)} = J_{xx}^{(O)} + \sum_{i=1}^{3} J_x^{(i)} + \sum_{i=1}^{3} m_i(e_{iy}^{*2} + e_{iz}^{*2})$;

$a_{45}^{(O)} = \sum_{i=1}^{3} m_i e_{ix}^* e_{iy}^* - J_{xy}^{(O)}$; $a_{46}^{(O)} = \sum_{i=1}^{3} m_i e_{ix}^* e_{iz}^* + \sum_{i=1}^{3} J_{xz}^{(i)} - J_{xz}^{(O)}$;

$a_{55}^{(O)} = J_{yy}^{(O)} + \sum_{i=1}^{3} J_y^{(i)} + \sum_{i=1}^{3} m_i(e_{ix}^{*2} + e_{iz}^{*2})$; $a_{56}^{(O)} = -\sum_{i=1}^{3} m_i e_{iy}^* e_{iz}^* - J_{yz}^{(O)}$;

$a_{66}^{(O)} = J_{zz}^{(O)} + \sum_{i=1}^{3} J_z^{(i)} + \sum_{i=1}^{3} m_i(e_{ix}^{*2} + e_{iy}^{*2})$;

$e_{3x}^* = e_x'$; $e_{3y}^* = e_y'$; $e_{3z}* = e_z'$; $e_{2y}^* = e_y + e_y^*$;

$e_{1x}^* = e_{1x}' + e_x^*$; $e_{1x}' = \alpha^* l$; $e_{1y}^* = e_y^*$; $e_{1z}^* = e_{1z}' + e_z^*$; $e_{1z}' = \alpha l$;

$e_{2x}^* = e_{2x}' + e_x^*$; $e_{2x}' = \alpha e_x + \alpha^* e_z$; $e_{2z}^* = e_{2z}' + e_z^*$; $e_{2z}' = -\alpha^* e_x + \alpha e_z$;

$J_x^{(i)} = \alpha^2 J_{xx}^{(i)} + \alpha^{*2} J_{zz}^{(i)}$; $J_z^{(i)} = \alpha^{*2} J_{xx}^{(i)} + \alpha^2 J_{zz}^{(i)}$; $J_{xz}^{(i)} = \alpha^* \alpha (J_{zz}^{(i)} - J_{xx}^{(i)})$;

$$J_x^{(3)} = \gamma^2 J_{xx}^{(3)} + \gamma^{*2} J_{zz}^{(3)};\ J_z^{(3)} = \gamma^{*2} J_{xx}^{(3)} + \gamma^2 J_{zz}^{(3)};\ J_{xz}^{(3)} = \gamma^* \gamma (J_{zz}^{(3)} - J_{xx}^{(3)});$$

$$A_{CT} = [a_j^{(CT)}]_{6\times 2};\quad a_{11}^{(CT)} = \sum_{i=1}^{2} m_i e'_{iz};\quad a_{31}^{(CT)} = \sum_{i=1}^{2} m_i e'_{ix};$$

$$a_{41}^{(CT)} = -(\sum_{i=1}^{2} m_i e'_{iz} e^*_{iy});\quad a_{51}^{(CT)} = \sum_{i=1}^{2} m_i (e^*_{ix} e'_{ix} + e'_{iz} e^*_{iz});$$

$$a_{61}^{(CT)} = -(\sum_{i=1}^{2} m_i e'_{ix} e^*_{iy});\quad a_{12}^{(CT)} = m_2 \alpha^*;\quad a_{32}^{(CT)} = m_2 \alpha;$$

$$a_{42}^{(CT)} = -m_2 \alpha e^*_{2y};\quad a_{52}^{(CT)} = (\alpha^* e^*_{2z} - \alpha e^*_{2x}) m_2;\quad a_{62}^{(CT)} = -m_2 \alpha^* e^*_{2y};$$

$$A_Q = \begin{bmatrix} m_c^* & m_q^* \\ m_q^* & m_2 \end{bmatrix};\qquad \begin{aligned} m_c^* &= J_{yy}^{(1)} + m_1 l^2 + m_2 (e_x^2 + e_z^2); \\ m_q^* &= -m_2 e_x; \end{aligned}$$

$$A_P = [a_j^{(P)}]_{6\times 1};\quad a_1^{(P)} = m_3 \gamma^*;\quad a_3^{(P)} = m_3 \gamma;\quad a_4^{(P)} = -m_3 \gamma^* e'_y;$$

$$a_5^{(P)} = m_3 (\gamma^* e'_z - \gamma e'_x);\quad a_6^{(P)} = -m_3 \gamma^* e'_y;$$

$$A_A = [a_{ij}^{(A)}]_{6\times 6};\ a_{11}^{(A)} = a_{22}^{(A)} = a_{33}^{(A)} = M_A;\ a_{44}^{(A)} = J_{xx}^{(A)};\ a_{45}^{(A)} = -J_{xy}^{(A)};$$

$$a_{46}^{(A)} = -J_{xz}^{(A)};\ a_{55}^{(A)} = J_{yy}^{(A)};\ a_{56}^{(A)} = -J_{yz}^{(A)};\ a_{66}^{(A)} = J_{zz}^{(A)};\ a_{ij}^{(A)} = a_{ji}^{(A)};$$

$$A_T = [a_{ij}^{(T)}]_{4\times 4};\quad a_{11}^{(T)} = a_{22}^{(T)} = J_w + 0.25(J_d + u_d^2 J'_d);$$

$$a_{33}^{(T)} = J_g + u_g^2 J'_d;\quad a_{44}^{(T)} = J_e;\quad a_{12}^{(T)} = 0.25(J_d + u_d^2 J'_d);$$

(7) $$C_{OA} = [c_{ij}^{(OA)}]_{6\times 6};\quad c_{11}^{(OA)} = \sum_s c_{Axs};\ c_{22}^{(OA)} = \sum_s c_{Ays};\ c_{33}^{(OA)} = \sum_s c_{Azs};$$

$$c_{15}^{(OA)} = \sum_s c_{Axs} l_{Azs};\ c_{16}^{(OA)} = -(\sum_s c_{Axs} l_{Ays});\ c_{24}^{(OA)} = -(\sum_s c_{Ays} l_{Azs});$$

$$c_{26}^{(OA)} = \sum_s c_{Ays} l_{Axs};\ c_{34}^{(OA)} = \sum_s c_{Azs} l_{Ays};\ c_{35}^{(OA)} = -(\sum_s c_{Azs} l_{Axs});$$

$$c_{51}^{(OA)} = \sum_s c_{Axs} l'_{Azs};\ c_{61}^{(OA)} = -(\sum_s c_{Axs} l'_{Ays});\ c_{42}^{(OA)} = -(\sum_s c_{Ays} l'_{Azs});$$

$$c_{62}^{(OA)} = \sum_s c_{Ays} l'_{Axs};\ c_{43}^{(OA)} = \sum_s c_{Azs} l'_{Ays};\ c_{53}^{(OA)} = -(\sum_s c_{Azs} l'_{Axs});$$

$$c_{44}^{(OA)} = \sum_s (c_{Ays} l_{Azs} l'_{Azs} + c_{Azs} l_{Ays} l'_{Ays});\ c_{55}^{(OA)} = \sum_s (c_{Azs} l_{Axs} l'_{Axs} +$$

$$+ c_{Axs} l_{Azs} l'_{Azs});\ c_{66}^{(OA)} = \sum_s (c_{Axs} l_{Ays} l'_{Ays} + c_{Ays} l_{Axs} l'_{Axs});$$

$$c_{46}^{(OA)} = -(\sum_s c_{Ays} l_{Axs} l'_{Azs});\ c_{45}^{(OA)} = -(\sum_s c_{Azs} l_{Axs} l'_{Ays});$$

$$c_{56}^{(OA)} = -(\sum_s c_{Axs} l_{Ays} l'_{Azs});\ c_{54}^{(OA)} = -(\sum_s c_{Azs} l'_{Axs} l_{Ays});$$

$$c_{64}^{(OA)} = -(\sum_s c_{Ays} l'_{Axs} l_{Azs});\ c_{65}^{(OA)} = -(\sum_s c_{Axs} l'_{Ays} l_{Azs});$$

$C_O = [c_{ij}^{(O)}]_{6\times 6}$;

$(c_{ij}^{(O)} = c_{ij}^{(OA)}$, when $c_{A^*} = c_*$ and $l_{A^*} = l'_{A^*} = l_*, s = 1,4)$;

$c_{ij}^{(O)} = c_{ij}^{(O)'} + c_{ij}^{(A)'}$, $(c_{ij}^{(A)'} = c_{ij}^{(OA)}$, when $l_{A^*} = l'_{A^*} = l'_{A^*}, s = 1,N)$;

$C_A = [c_{ij}^{(A)}]_{6\times 6}$ $(c_{ij}^{(A)} = c_{ij}^{(OA)}$, when $l_{A^*} = l'_{A^*} = l_{A^*}$, $s = 1,N)$;

$C_Q = \mathrm{diag}(c_C, c_T)_{2\times 2}$; $C_T = [c_{ij}^{(T)}]_{4\times 4}$;

$c_{11}^{(T)} = c_{wL} + c_{12}^{(T)}$; $c_{22}^{(T)} = c_{wR} + c_{12}^{(T)}$; $c_{33}^{(T)} = c_d + c_e u_g^2$; $c_{44}^{(T)} = c_e$;

$c_{12}^{(T)} = c_d(u_d^2/4)$; $c_{13}^{(T)} = c_{23}^{(T)} = -c_d(u_d/2)$; $c_{34}^{(T)} = -c_e u_g$;

$C_R = [c_{ij}^{(R)}]_{6\times 4}$; $c_{11}^{(R)} = -c_{wL}/R_{wL}$; $c_{22}^{(R)} = -c_{wR}/R_{wR}$;

$c_{51}^{(R)} = -c_{wL}l_{z3}/R_{wL}$; $c_{52}^{(R)} = -c_{wR}l_{z4}/R_{wR}$;

$c_{61}^{(R)} = c_{wL}l_{y3}/R_{wL}$; $c_{62}^{(R)} = c_{wR}l_{y4}/R_{wR}$; when $c_{wL} = c_{x3}R_{wL}^2$;

$c_{wR} = c_{x4}R_{wR}^2$.

The matrix B expresses the dissipation properties of the system, has analogous structure as the matrix C, and its elements can be obtained by replacing c_{**} with b_{**}.

The non-linear properties in case of small oscillations are described by the matrix S and the vectors $\dot{\tilde{\psi}}$, and $R(\psi, \dot{\psi})$, as follows:

$$\dot{\tilde{\psi}} = (\dot{\theta}_x^2, \dot{\theta}_x\dot{\theta}_y, \dot{\theta}_x\dot{\theta}_z, \dot{\theta}_y^2, \dot{\theta}_y\dot{\theta}_z, \dot{\theta}_z^2, \dot{\theta}_x\dot{\varphi}_C\dot{\phi}_C, \dot{\theta}_y\dot{\varphi}_C\dot{\phi}_C, \dot{\theta}_z\dot{\varphi}_C\dot{\phi}_C, \dot{\theta}_x\dot{q}_T, \dot{\theta}_y\dot{q}_T, \dot{\theta}_z\dot{q}_T, \dot{\theta}_x\dot{q}_B, \dot{\theta}_y\dot{q}_B, \dot{\theta}_z\dot{q}_B)_{15\times 1}^T; \tag{8}$$

$$S = [S_O^T, O_{10\times 15}^T]_{19\times 15}^T; \qquad O_{10\times 15} \text{ zero matrix}; \tag{9}$$

$S_O = [s_{ij}^{(O)}]_{9\times 15}$; $s_{22}^{(O)} = s_{33}^{(O)} = -2s_{14}^{(O)} = -2s_{16}^{(O)} = -(\sum_{i=1}^{2} 2m_i e_{ix}^*)$;

$s_{21}^{(O)} = s_{26}^{(O)} = -2s_{35}^{(O)} = -(\sum_{i=1}^{2} m_i e_{iy}^*)$; $s_{31}^{(O)} = s_{34}^{(O)} = -(\sum_{i=1}^{2} 2m_i e_{iz}^*)$;

$s_{29}^{(O)} = -s_{38}^{(O)} = (\sum_{i=1}^{2} 2m_i e'_{iz})$;

$s_{111}^{(O)} = -s_{210}^{(O)} = 2m_2\alpha$; $s_{212}^{(O)} = -s_{311}^{(O)} = 2m_2\alpha^*$;

$s_{114}^{(O)} = -s_{213}^{(O)} = 2m_3\gamma$; $s_{215}^{(O)} = -s_{314}^{(O)} = 2m_3\gamma^*$;

$s_{41}^{(O)} = -(\sum_{i=1}^{2} m_i e_{iy}^* e_{iz}^*)$; $s_{42}^{(O)} = 2s_{54}^{(O)} + 2(\sum_{i=1}^{3} J_{xz}^{(i)}) - 2J_{xz}$;

$$s_{43}^{(O)} = s_{59}^{(O)} - 2J_{xy};\quad s_{44}^{(O)} = J_{yz};\quad s_{46}^{(O)} = s_{41}^{(O)} + s_{44}^{(O)};$$

$$s_{45}^{(O)} = \sum_{i=1}^{3}(J_x^{(i)} - J_y^{(i)} + J_z^{(i)}) + (J_{xx} - J_{yy} + J_{zz});\quad s_{47}^{(O)} = \sum_{i=1}^{2} m_i e'_{iz} e^*_{iz};$$

$$s_{49}^{(O)} = \sum_{i=1}^{2} m_i e'_{ix} e^*_{iz};\quad 2s_{48}^{(O)} = -2s_{57}^{(O)} = s_{72}^{(O)} = s_{75}^{(O)} = -(\sum_{i=1}^{2} 2m_i e'_{iz} e^*_{iy});$$

$$s_{410}^{(O)} = s_{412}^{(O)} = m_2 \alpha e^*_{2z};\quad s_{413}^{(O)} = s_{415}^{(O)} = m_3 \gamma^* e^*_{3z};\quad s_{51}^{(O)} = \sum_{i=1}^{3} J_{xz}^{(i)} + J_{xz};$$

$$s_{52}^{(O)} = s_{59}^{(O)} + \sum_{i=1}^{2} J_y^{(i)};\quad s_{53} = \sum_{i=1}^{3}(J_x^{(i)} - J_z^{(i)}) + (J_{xx} - J_{yy} - J_{zz});$$

$$s_{54}^{(O)} = s_{69}^{(O)} = -(\sum_{i=1}^{2} m_i e^*_{ix} e^*_{iz});\quad s_{56}^{(O)} = s_{54}^{(O)} - 2J_{xz};\quad s_{55}^{(O)} = 2s_{48}^{(O)} - 2J_{xy};$$

$$s_{59}^{(O)} = \sum_{i=1}^{2} m_i e^*_{ix} e^*_{iz};\quad s_{510}^{(O)} = -s_{610}^{(O)} = 2s_{83}^{(O)} = m_2 \alpha^* e^*_y;$$

$$s_{512}^{(O)} = -s_{612}^{(O)} = -s_{81}^{(O)} = s_{85}^{(O)} = m_2 \alpha e^*_y;\quad s_{61}^{(O)} = -s_{59}^{(O)} - J_{xy};$$

$$s_{64}^{(O)} = s_{59}^{(O)} - J_{xy};\quad s_{62}^{(O)} = \sum_{i=1}^{3}(J_z^{(i)} - J_x^{(i)}) + \sum_{i=1}^{2} m_i e_{ix}^2;$$

$$s_{67}^{(O)} = s_{74}^{(O)} = \sum_{i=1}^{2} 2m_i e'_{ix} e^*_{ix};\quad s_{71}^{(O)} = s_{47}^{(O)} + \sum_{i=1}^{2} J_{xz}^{(i)};$$

$$s_{73}^{(O)} = s_{74}^{(O)} = -J_y;\quad s_{76}^{(O)} = \sum_{i=1}^{2} 2m_i e^*_{ix} e'_{iz};\quad s_{84}^{(O)} = 2s_{86}^{(O)} = -2m_2 \alpha^* e^*_{2x};$$

(10) $$R(\psi, \dot{\psi}) = [r_j]_{19\times 19};\quad r_7 = f_{cC}(\varphi_C) + f_{bC}(\dot{\varphi}_C);$$
$$r_8 = f_{cT}(q_T) + f_{bT}(\dot{q}_T);\quad r_9 = f_{cB}(q_B) + f_{bB}(\dot{q}_B).$$

The vibrations of the mechanical system are excited basically by force disturbance from the engine (the vector Q_F), and by kinematic disturbance from the road (the vector Q_k):

$$Q_F = \mathrm{Re}\{\sum_{k=0}^{n} Q^*_{Fk} e^{jk\omega_e}\};$$

(11) $$Q^*_{Fk} = (O_9^T, \kappa_H H_k^{*T}, \kappa_M M_k^{*T}, \kappa_{wL} M^*_{wL}, \kappa_{wR} M^*_{wR}, 0, \kappa_m M_A^{(e)*})^T;$$
$$Q_k = \mathrm{Re}\{\sum_{l=0}^{n} Q^*_{kl} e^{jp^*_k t}\};\qquad Q^*_{kl} = (((jp_k B^{(l)} + C^{(l)})\mu^*_{kl})^T)^T;$$

where $B(l)$ and $C(l)$ are obtained from the matrices B and C, and O_9 and O_3 are zero vectors.

The system of differential equations Eq.(4) gives an opportunity to make analysis of the non-linear motorcar vibrations.

The results can be used as a base for dynamical synthesis and design of motorcars.

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