

# GENERAL MECHANICS

## EQUATION OF MOTION OF A GIG WITH HYDRAULIC OPERATING

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It is well known, that in the ascending motion of a lifting machine with a hydraulic system, if the lubricous pump efficiency is constant and the lubricant losses depend on time definitely, then the motion itself can be described without being necessary to solve differential equations. Contrary to this, under jib drop the velocity depends on various regulating elements of the system and on the hydraulic resistance in the liquid consumption. In connection with this the motion law of the system is obtained only by integrating the essentially

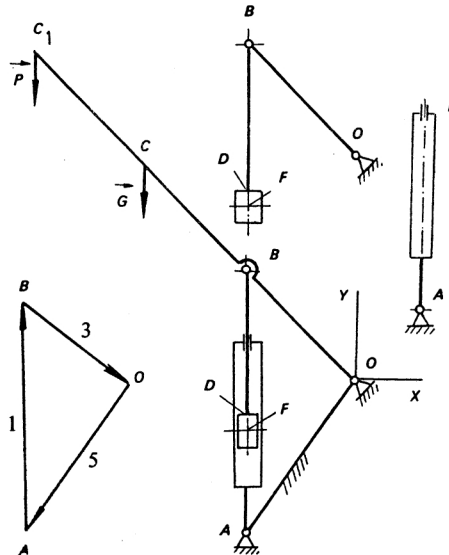


Fig. 1. Kinematic scheme of a gig

nonlinear differential equations. The nonlinearity is caused by the system geometrical properties.

A simplest scheme of a gig is shown in Fig. 1. The approach to solve the problem is the following. What is used is the fact that many of the effects on a mechanism in the sense of the formulated problem are addends. For example, the generalized force for the  $q_\alpha$  coordinate can be determined for an unique point ( $M_j$ ) from the mechanism, which mass is equal to  $m_j$  and it can be computed the corresponding generalized force  $Q_\alpha^{(j)}$ , which is relative only to this point. If it happens that the points are numerous, no matter where they are placed on the links, then the total generalized force  $Q_\alpha$  can be computed by constituting the sum

$$(1) \quad Q_\alpha = \sum_{j=1}^N Q_\alpha^{(j)}.$$

The same concerns the kinetic energy, which is defined by the sum

$$(2) \quad T = \sum_{j=1}^N T^{(j)},$$

where  $T^{(j)}$  denotes the kinetic energy of ( $M_j$ ).

In the present paper it used the following expression of the total kinetic energy of a complex mechanism

$$(3) \quad T = \sum_{\alpha} \sum_{\beta} T_{\alpha\beta} \dot{q}_\alpha \dot{q}_\beta,$$

where  $\alpha$  and  $\beta$  are items of the mechanism generalized coordinates and  $T_{\alpha\beta}$  are coefficient which are obtained by the same summation method as in the case of the generalized forces above<sup>1</sup>. On the other hand  $T_{\alpha\beta}$  are dynamical characteristics, which can be computed using the formulae

$$(4) \quad T_{\alpha\beta} = \begin{cases} \frac{1}{2} J_\alpha & \text{if } \alpha = \beta \\ \vec{S}_\alpha \cdot \vec{l}_\beta & \text{if } \alpha < \beta \\ 0 & \text{if } \alpha > \beta. \end{cases}$$

In these formulae  $J_\alpha$  and  $\vec{S}_\alpha$  are related to a determined link with item  $\alpha$ .  $I_\alpha$  is its inertial moment with respect to the joint to which all masses are

<sup>1</sup> This method is recommended when working with computers.

transferred mentally, while  $\vec{S}_\alpha$  is the static moment corresponding to the same joint. Usually, it is assumed that the link is solid and the quantities  $\vec{S}_\alpha$  and  $J_\alpha$  are defined laboratorially.

In open chains, which are used as fictitious auxiliary systems, the generalized velocities  $\dot{q}_\alpha$  and  $\dot{q}_\beta$  are considered to be independent velocities [1,2]. In such a way, from the kinetic energy expression it can be reached the set of the adjusted inertial moments and simultaneously the possibility to compose in an adequate form the motion equations of a complex mechanism. In the case of the mechanism under consideration we have an unique adjusted inertial moment. To this point it has been assumed that the whole mechanism has broken in material points. Sometime, the mechanism elements can be separate bodies with simple geometric form. In these cases some of the items refer to such numbers. The bodies themselves can be of complex form as well while their characteristics have been determined experimentally.

For the mechanism under consideration, first it is introduced a vector contour, which has been obtained from the triangle  $AOB$ . It is consisted of the vectors  $\vec{a}_1 = \vec{AB}$ ,  $\vec{a}_3 = \vec{BO}$  and  $\vec{a}_5 = \vec{OA}$ . For the generalized coordinate it is introduced  $q_2 = \ln(a_1/a_{10})$ , where  $a_1 = |\vec{a}_1|$ ,  $a_{10}$  is the initial value of  $a_1$ . The generalized velocity  $\dot{q}_2 = \dot{a}_1/a_1$  is the dillatational velocity of the vector  $\vec{a}_1$ . By the motion of the triangle  $AOB$  it is described the kinematic velocity of the motion, which is expressed by the formulae

$$(5a, b) \quad \dot{q}_1 = \dot{q}_2 p_{231}, \quad \dot{q}_3 = \dot{q}_2 p_{213}.$$

According to (3) the kinetic energy is determined by two formulae

$$(6a) \quad T' = T_{11}\dot{q}_1^2 + T_{13}\dot{q}_1\dot{q}_3 + T_{33}\dot{q}_3^2.$$

It refer to the open chain which is consisted of the links  $FB$  and  $BO$ , when in the inertial moment of  $BO$  it is included the inertial moment of the jib and

$$(6b) \quad T'' = T_{11}''\dot{q}_1^2,$$

which is resulted by the rotation of the cylinder  $AI$ . It represents a separate open chain – pendulum of the simplest type. The total kinetic energy is  $T = T' + T''$ . After using the formulae (5) it is reached the following expression for the kinetic energy

$$(7) \quad T = \frac{1}{2}\dot{q}_2^2(p_{231}^2 I_{11} + 2I_{13}p_{231}p_{213} + p_{213}^2 I_{33}).$$

In this formula  $I_{11} = m_b a_9^2 + (1/3)m_p a_7^2 + (1/3)m_z a_{13}^2$ ,  $m_p$  is the mass of the piston rod,  $m_b$  is the piston mass,  $m_z$  is the cylinder mass;  $\vec{a}_9 = \vec{FB}$ ,  $\vec{a}_7 = \vec{DB}$  and  $\vec{a}_3 = \vec{AI}$ .

$$I_{13} = m_b \vec{a}_9 \cdot \vec{a}_3 + \frac{1}{2} m_p \vec{a}_7 \cdot \vec{a}_3,$$

$$I_{33} = m_b \vec{a}_3^2 + m_p \vec{a}_3^2 + \frac{1}{3} m_s \vec{a}_{15}^2,$$

where  $m_s$  is the jib mass;  $\vec{a}_{15} = \vec{OC}_1$ . For  $I_{13}$  it is obtained

$$I_{13} = \frac{1}{a_1} \left( m_b a_9 + \frac{1}{2} m_p a_7 \right) \vec{a}_1 \cdot \vec{a}_3.$$

The expression in the brackets of (7) is adjusted inertial moment. It will be denoted by  $J_p$ .

Thus, the left hand side of the fundamental equation turns out to be

$$(8) \quad \ddot{q}_2 J_p + \frac{1}{2} \cdot \frac{\partial J_p}{\partial q_2} \dot{q}_2^2 = Q_2.$$

We must note here, that due to the small piston velocity with respect to the cylinder,  $\dot{q}_2$  is comparatively small quantity and the second addend  $b$  is also a small quantity. It can be neglected.

The generalized force in the right-hand side depends mainly on the jib weight and on the resistance of the flowing liquid. From the jib we have

$$Q_2^s = \vec{v} \cdot (\vec{a}_{17} \times \vec{G}) p_{213}.$$

The liquid resistance leads to the generalized force

$$Q_2^R = F(3,48 \cdot 10^5 \cdot |r| + 1,1 \cdot 10^6 \cdot |v|^{1,715}) \cdot H,$$

where  $V = \dot{a}_1$ ,  $F$  is the piston area,  $H$  is the distance of the cylinder axis from the point  $O$ . After substituting in (8) it is derived the final differential equation

$$\ddot{q}_2 J_p = \vec{v} \cdot (\vec{a}_{17} \times \vec{G}) p_{213} - S \cdot H - M_F.$$

By  $M_F$  it is denoted the friction resistance in the axes of the rotating bodies. After integrating a result is obtained, which is placed in the table below

|                 |       |       |         |         |        |        |         |         |
|-----------------|-------|-------|---------|---------|--------|--------|---------|---------|
| $t$ in s        | 0     | 0,5   | 1,0     | 1,5     | 2      | 2,5    | 3       | 3,5     |
| $\alpha$ in deg | 0     | 9,6   | 26      | 40      | 52     | 62     | 72      | 81      |
| $v$ in m/s      | 0     | 0,22  | 0,23    | 0,254   | 0,239  | 0,225  | 0,212   | 0,20    |
| $\ddot{\alpha}$ | 1,730 | 0,413 | -0,2297 | -0,1493 | -0,089 | -0,059 | -0,0373 | -1,8645 |

The method of transfer functions described above can be used for solving dynamic problems of wide class of planar mechanisms.

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