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FLUID MECHANICS

REFLECTION OF PLANE WAVES IN A ROTATING TRANSVERSLY ISOTROPIC MAGNETO-THERMOELASTIC SOLID HALF-SPACE

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Abstract. The governing equations of a rotating transversely isotropic magneto-thermoelastic medium are solved to obtain the velocity equation, which indicates the existence of three quasi plane waves. Reflection of these plane waves from a stress-free thermally insulated surface is studied to obtain the reflection coefficients of various reflected waves. The effects of anisotropy, rotation, thermal and magnetic fields are shown graphically on these coefficients.
Key words: Anisotropic generalized thermoelasticity, relaxation time, rotation, magnetic field, plane waves, reflection.

1. Introduction

Thermoelasticity combines the theories of elasticity, heat transfer, and their coupled effects into a single treatment. Biot [1] explained thermoelasticity by deriving dilatation based on the thermodynamics of irreversible process and coupling it with elastic deformation. But the diffusion type heat equation used in this study predicted infinite speed for propagation of thermal signals. Lord and Shulman [2] defined the generalized theory of thermoelasticity in which a hyperbolic equation of heat conduction with a relaxation time ensured the finite speed for thermal signals. Using two relaxation times, Green and

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Lindsay [3] developed another generalized theory of thermoelasticity. A unified treatment of both Lord and Shulman and Green and Lindsay theories was presented by Ignaczak and Ostoja-Starzewski [4]. Dhaliwal and Sherief [5] extended Lord and Shulman [2] generalization of thermoelasticity for anisotropic case. Chandrasekhariah [6] presented a review of work done in the theory of thermoelasticity. The thermoelasticity has wide applications in various fields such as earthquake engineering, soil dynamics, aeronautics, astronautics, nuclear reactors, high energy particle accelerator, etc. Thermoelasticity is also used in polymer coating and to evaluate the stress redistribution in ceramic matrix composites (Mackin and Purcell [7]; Barone and Patterson [8]).

Wave propagation phenomenon in solids is important due to its relevance in composite engineering, geology, seismology, seismic exploration, control system and acoustics. The amplitudes of seismic signals are applicable not only in investigating the internal structure of the earth, but also in exploration of valuable materials, oils, water, chemicals etc. In view of the fact that most large bodies, like the earth, the moon, and other planets, have angular velocity and their own magnetic field. The study of wave propagation in a generalized thermoelastic media with additional parameters like rotation, electric, magnetic, anisotropy, porosity, viscosity, microstructure, temperature and other parameters provide vital information about existence of new or modified waves. Such information may be useful for experimental seismologists in correcting earthquake estimation.

Sinha and Sinha [9] studied the reflection of thermoelastic waves at a solid half-space with one thermal relaxation time. Sinha and Elsibai [10] studied the reflection of thermoelastic waves at a solid half-space with two thermal relaxation times. Sharma, et al. [11] studied the reflection of generalized thermoelastic waves from the boundary of a half-space for different generalized theories of thermoelasticity.

Schoenberg and Censor [12] studied the effect of rotation on plane wave propagation in an isotropic medium. They showed the propagation of three plane waves in a rotating isotropic medium. They found that the longitudinal or transverse wave can exist only if the direction of propagation and axis of rotation are either parallel or perpendicular. The effect of rotation does not increase the number of waves in a transversely isotropic medium, but it affects their speeds significantly. Chandrasekharaiah and Srinath [13, 14] studied the thermoelastic plane waves in a rotating isotropic solid. Ahmad and Khan [15] discussed the theory of thermoelastic plane waves in a rotating isotropic ma-

terial and have shown the existence of four waves in the medium. Paria [16] and Wilson [17] investigated the propagation of magneto-thermo elastic waves in a non-rotating isotropic material. Agarwal [18] studied the thermoelastic plane waves with electro-magnetic fields. Purushothama [19] studied the combined effects of uniform thermal and magnetic fields on the propagation of plane waves in a homogenous, initially unstressed electrically conducting elastic medium. Roy Chaudhuri and Debnath [20] studied the propagation of magneto-thermoelastic plane wave in a rotating isotropic material. Abd-alla, et al. [21] studied the reflection of plane waves from free surface of a magneto-thermo-viscoelastic solid half-space. Sharma and Thakur [22] studied the effect of rotation on Rayleigh-Lamb waves in magneto- thermoelastic media. Othman and Song [23] studied the reflection of magneto-thermoelastic waves from a rotating elastic half-space. Othman and Song [24] studied the effect of rotation on 2-D thermal shock problem for a generalized magneto-thermoelasticity half-space under three theories.

Wave propagation in an anisotropic solid has been studied by many researchers. For example, Keith and Crampin [25] studied the reflection and refraction of seismic waves at a plane interface between two dissimilar anisotropic media. Daley and Hron [26] studied the reflection and transmission coefficients in transversely isotropic media. Crampin [27] presented a review of wave motion in an anisotropic media. Chadwick [28] studied homogeneous plane waves in a transversely isotropic elastic media. Singh and Khurana [29] studied the propagation of plane waves in an anisotropic elastic medium possessing monoclinic symmetry. Ting [30] studied the surface waves in a rotating anisotropic elastic half-space. Chadwick and Seet [31] studied the wave propagation in a transversely isotropic heat conducting elastic material. Banerjee and Pao [32] studied the thermoelastic plane waves in unbounded anisotropic solids. Sharma and Sidhu [33] studied the propagation of plane harmonic waves in anisotropic generalised thermoelasticity. Sharma [34] studied the reflection of thermoelastic waves from the stress-free thermally insulated boundary of a transversely isotropic solid half-space. Verma [35] presented an analysis of waves in a heat-conducting N-layered plate of an arbitrary anisotropic media and obtained the dispersion relations of thermoelastic waves by invoking continuity at the interface and boundary conditions on the surfaces of layered plate. Singh [36-39] studied the propagation of plane harmonic waves in transversely isotropic and monoclinic medium. Kumar and Singh [40] studied the effects of rotation and imperfection on reflection and transmission of plane waves. Ku-

mar and Devi [41] studied the reflection of plane wave in transversely isotropic magneto-thermoelastic medium in contact with vacuum in the context of the theories of thermoelasticity of Green Naghdi theories of type-II and type III.

To the best of our knowledge, the effect of rotation and magnetic field on plane waves in transversely isotropic thermoelastic medium under the Lord and Shulman [2] theory of generalized thermoelasticity has not been studied so for. The present paper is organized as follows: Section 2 presents the list of symbols used in this paper. In section 3, the governing equations of generalized thermoelasticity of rotating transversely isotropic medium with magnetic field are solved in x-y plane and a cubic velocity equation is obtained to show the existence of three quasi plane waves in the medium. In section 4, the reflection of quasi plane waves is considered from a thermally insulated and stress-free surface, where three relations between the reflection coefficients are obtained. In section 5, some particular cases are considered. The reflection coefficients are computed for single crystal of Zinc in section 6 to observe the effects of rotation, magnetic field, transverse isotropy and relaxation time on the reflection coefficients. In the last section, the theoretical and numerical results are concluded.

2. Nomenclature

C_{ij}	elastic constants
u, v	components of displacement vector
λ, μ	Lame's constants
ρ	density of the medium
$\overrightarrow{J}$	electric current density vector
$\vec{B}$	magnetic induction vector
$\vec{H}$	total magnetic field vector
$\vec{E}$	electric field vector
σ	electric conductivity of the medium
β_1, β_2	thermal coefficients
ω	circular frequency
$\vec{\Omega}$	rotation vector
τ_0	relaxation time
T	change in temperature variable
T_0	uniform temperature
K_1, K_2	thermal conductivities

μ_e	magnetic permeability of the medium
C_E	specific heat at constant strain
θ	angle of propagation measured from normal to the half-space

3. Formulation of the problem and solution

We consider a transversely isotropic magneto- thermoelastic half-space rotating about z-axis $\vec{\Omega} = (0, 0, \Omega)$ with $\vec{u} = (u, v, 0)$. The governing field equations in absence of body forces, heat sources, the displacement current and charge density are as follows:

(i) Equations of motion

$$(1)\quad C_{11}\frac{\partial^2 u}{\partial x^2} + C_{12}\frac{\partial^2 v}{\partial x \partial y} + C^*\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y}\right) - \beta_1\frac{\partial T}{\partial x} + (\vec{J}\times\vec{B})_1 = \rho\left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega\frac{\partial v}{\partial t}\right),$$

$$(2)\quad C_{22}\frac{\partial^2 v}{\partial y^2} + C_{12}\frac{\partial^2 u}{\partial x \partial y} + C^*\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y}\right) - \beta_2\frac{\partial T}{\partial y} + (\vec{J}\times\vec{B})_2 = \rho\left(\frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega\frac{\partial u}{\partial t}\right),$$

Here, $C^* = \dfrac{1}{2}(C_{11} - C_{12})$ and $\left(\vec{J}\times\vec{B}\right)$ are the electromagnetic body force.

(ii) Heat conduction equation

$$(3)\quad K_1\frac{\partial^2 T}{\partial x^2} + K_2\frac{\partial^2 T}{\partial y^2} = \rho C_E\left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 T}{\partial t^2}\right) + \beta_1 T_0\left(\frac{\partial^2 u}{\partial t \partial x} + \tau_0\frac{\partial^3 u}{\partial t^2 \partial x}\right) + \beta_2 T_0\left(\frac{\partial^2 v}{\partial y \partial t} + \tau_0\frac{\partial^3 v}{\partial t^2 \partial y}\right),$$

(iii) Maxwell equations

$$(4)\qquad Curl\ \vec{H} = \vec{J},\quad Curl\ \vec{E} = -\frac{\partial \vec{B}}{\partial t},\quad div\ \vec{B} = 0,\quad \vec{B} = \mu_e \vec{H},$$

The generalized Ohm's law in deformable continua is

$$\vec{J} = \sigma[\vec{E} + (\dot{\vec{u}} \times \vec{B})], \tag{5}$$

where the small effect of temperature gradient on the conduction current $\vec{J}$ is neglected. We set $\vec{H} = \vec{H}_0 + \vec{h}$, where $\vec{H}_0 = (0, 0, H_0)$. The perturbed magnetic field $\vec{h}$ is so small that the product of $\vec{h}$, $\vec{u}$ and their derivatives can be neglected while linearizing the field equations. With the help of equations (4) and (5), the equations (1) and (2) become

$$C_{11}\frac{\partial^2 u}{\partial x^2} + C_{12}\frac{\partial^2 v}{\partial x \partial y} + C^*\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y}\right) - \beta_1 \frac{\partial T}{\partial x} + \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y}\right) = \rho\left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega \frac{\partial v}{\partial t}\right), \tag{6}$$

$$C_{22}\frac{\partial^2 v}{\partial y^2} + C_{12}\frac{\partial^2 u}{\partial x \partial y} + C^*\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y}\right) - \beta_2 \frac{\partial T}{\partial y} + \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2}\right) = \rho\left(\frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega \frac{\partial u}{\partial t}\right), \tag{7}$$

To solve the equations (3), (6) and (7), we assume

$$\{u, v, T\} = \{A, B, C\} \exp\{ik(x \sin\theta + y \cos\theta - ct)\}, \tag{8}$$

With the help of equation (8), the equations (3), (6) and (7) result into the following cubic equation in ζ:

$$A_0\zeta^3 + A_1\zeta^2 + A_2\zeta + A_3 = 0, \tag{9}$$

where,

$$\zeta = \rho c^2, \quad A_0 = 4\left(\frac{\Omega}{\omega}\right)^2 - \Omega^{*2},$$

$$A_1 = \Omega^*(D_1 + D_3 + \varepsilon \sin^2\theta + \bar{\beta}^2 \varepsilon \cos^2\theta) + \left(\Omega^{*2} - 4\left(\frac{\Omega}{\omega}\right)^2\right) D_5,$$

$$A_2 = D_2^2 - D_1 D_3 - \Omega^*(D_1 D_5 + D_3 D_5) + \varepsilon\left(2\bar{\beta} D_2 \sin\theta \cos\theta - \bar{\beta}^2 D_1 \cos^2\theta - D_3 \sin^2\theta\right),$$

$$A_3 = \left(D_1 D_3 - D_2^2\right) D_5,$$

$$D_1 = C_{11} \sin^2\theta + C^* \cos^2\theta + \mu_e H_0^2 \sin^2\theta,$$

$$D_2 = C_{12} \sin\theta\cos\theta + C^* \sin\theta\cos\theta + \mu_e H_0^2 \sin\theta\cos\theta,$$

$$D_3 = C^* \sin^2\theta + C_{22} \cos^2\theta + \mu_e H_0^2 \cos^2\theta,$$

$$D_4 = K_1 \sin^2\theta + K_2 \cos^2\theta, \quad D_5 = \frac{D_4}{\tau^* C_E},$$

$$\tau^* = \left(\tau_0 + \frac{i}{\omega}\right), \quad \Omega^* = \left(1 + \left(\frac{\Omega}{\omega}\right)^2\right), \quad \overline{\beta} = \frac{\beta_2}{\beta_1}, \quad \omega = kc, \quad \varepsilon = \frac{\beta_1^2 T_0}{\rho C_E}.$$

Here, the three roots $\zeta_j = \rho c_j^2$, $(j = 1, 2, 3)$ of equation (9) correspond to the complex phase velocities c_j $(j = 1, 2, 3)$ of qP, qSV and qT waves, respectively. If we write $c_j^{-1} = V_j^{-1} - i\omega^{-1} q_j$, $(j = 1, 2, 3)$, then clearly V_j and q_j are the speeds of propagation and the attenuation coefficients of the qP, qSV and qT waves.

4. Reflection from free surface

A thermoelastic solid half-space $(y \geq 0)$ is considered with thermally insulated and stress-free surface $y = 0$. The positive y-axis is taken into the half-space. Reflection of qP or qT or qSV wave will result into three reflected qP, qT and qSV waves in the half-space. The complete geometry showing the incident and reflected waves is depicted in Fig. 1 and can be given here. The appropriate displacement and temperature fields in the half space are:

$$\begin{aligned}(10)\quad u = {} & A_0 \exp\left\{ik_0 \left(x \sin\theta_0 - y \cos\theta_0 - c_0 t\right)\right\} \\ & + A_1 \exp\left\{ik_1 \left(x \sin\theta_1 + y \cos\theta_1 - c_1 t\right)\right\} \\ & + A_2 \exp\left\{ik_2 \left(x \sin\theta_2 + y \cos\theta_2 - c_2 t\right)\right\} \\ & + A_3 \exp\left\{ik_3 \left(x \sin\theta_3 + y \cos\theta_3 - c_3 t\right)\right\},\end{aligned}$$

$$\begin{aligned}(11)\quad v = {} & F_0 A_0 \exp\left\{ik_0 \left(x \sin\theta_0 - y \cos\theta_0 - c_0 t\right)\right\} \\ & + F_1 A_1 \exp\left\{ik_1 \left(x \sin\theta_1 + y \cos\theta_1 - c_1 t\right)\right\} \\ & + F_2 A_2 \exp\left\{ik_2 \left(x \sin\theta_2 + y \cos\theta_2 - c_2 t\right)\right\}\end{aligned}$$

$$+ F_3 A_3 \exp\left\{ik_3\left(x\sin\theta_3 + y\cos\theta_3 - c_3 t\right)\right\},$$

$$\begin{aligned}(12)\quad T = {} & G_0 A_0 \exp\left\{ik_0\left(x\sin\theta_0 - y\cos\theta_0 - c_0 t\right)\right\} \\ & + G_1 A_1 \exp\left\{ik_1\left(x\sin\theta_1 + y\cos\theta_1 - c_1 t\right)\right\} \\ & + G_2 A_2 \exp\left\{ik_2\left(x\sin\theta_2 + y\cos\theta_2 - c_2 t\right)\right\} \\ & + G_3 A_3 \exp\left\{ik_3\left(x\sin\theta_3 + y\cos\theta_3 - c_3 t\right)\right\},\end{aligned}$$

where c_0 is the velocity of the incident qP or qT or qSV wave, k_q, $(q = 0, 1, 2, 3)$ are complex wave numbers, $c_l^2 = \zeta_l/\rho$ $(l = 0, 1, 2, 3)$ and,

$$F_0 = -\frac{\beta_1 \sin\theta_0 \left(D_{20} - 2i\frac{\Omega}{\omega}\zeta_0\right) + \beta_2 \cos\theta_0 \left(D_{10} - \Omega^*\zeta_0\right)}{\beta_2 \cos\theta_0 \left(D_{20} + 2i\frac{\Omega}{\omega}\zeta_0\right) + \beta_1 \sin\theta_0 \left(D_{30} - \Omega^*\zeta_0\right)},$$

$$G_0 = -ik_0 \left[\frac{\left(4\left(\frac{\Omega}{\omega}\right)^2 - \Omega^{*2}\right)\zeta_0^2 + \Omega^*\left(D_{10} + D_{30}\right)\zeta_0 + D_{20}^2 - D_{10}D_{30}}{\beta_2 \cos\theta_0 \left(D_{20} + 2i\frac{\Omega}{\omega}\zeta_0\right) + \beta_1 \sin\theta_0 \left(D_{30} - \Omega^*\zeta_0\right)}\right],$$

$$D_{10} = \left(C_{11} + \mu_e H_0^2\right)\sin^2\theta_0 + C^*\cos^2\theta_0,$$

$$D_{20} = -\left(C_{12} + C^* + \mu_e H_0^2\right)\sin\theta_0\cos\theta_0,$$

$$D_{30} = C^*\sin^2\theta_0 + \left(C_{22} + \mu_e H_0^2\right)\cos^2\theta_0,.$$

$$F_l = \frac{\beta_1 \sin\theta_l \left(D_{2l} - 2i\frac{\Omega}{\omega}\zeta_l\right) - \beta_2 \cos\theta_l \left(D_{1l} - \Omega^*\zeta_l\right)}{\beta_2 \cos\theta_l \left(D_{2l} + 2i\frac{\Omega}{\omega}\zeta_l\right) - \beta_1 \sin\theta_l \left(D_{3l} - \Omega^*\zeta_l\right)}, (l = 1, 2, 3),$$

$$G_l = ik_l \left[\frac{\left(4\left(\frac{\Omega}{\omega}\right)^2 - \Omega^{*2}\right)\zeta_l^2 + \Omega^*\left(D_{1l} + D_{3l}\right)\zeta_l + D_{2l}^2 - D_{1l}D_{3l}}{\beta_2 \cos\theta_l \left(D_{2l} + 2i\frac{\Omega}{\omega}\zeta_l\right) - \beta_1 \sin\theta_l \left(D_{3l} - \Omega^*\zeta_l\right)}\right],$$

$$D_{1l} = \left(C_{11} + \mu_e H_0^2\right)\sin^2\theta_l + C^*\cos^2\theta_l,$$

$$D_{2l} = \left(C_{12} + C^* + \mu_e H_0^2\right)\sin\theta_l\cos\theta_l,$$

$$D_{3l} = C^*.\sin^2\theta_l + \left(C_{22} + \mu_e H_0^2\right)\cos^2\theta_l.$$

The required boundary conditions at the free surface y = 0 are vanishing of normal stresses, tangential stresses and normal component of the heat

flux vector, i.e.,

$$\sigma_{yy} + \bar{\sigma}_{yy} = 0, \quad \sigma_{yx} + \bar{\sigma}_{yx} = 0, \quad \frac{\partial T}{\partial y} = 0, \quad \text{on} \quad y = 0, \tag{13}$$

where,

$$\sigma_{yy} = C_{22}\frac{\partial v}{\partial y} + C_{12}\frac{\partial u}{\partial x} - \beta_2 T, \quad \sigma_{yx} = C_{44}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),$$

$$\bar{\sigma}_{yy} = -\mu_e H_0^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right), \qquad \bar{\sigma}_{yx} = 0.$$

The potentials (10) to (12) corresponding to incident and reflected waves must satisfy the boundary conditions (13). Since the phases of the waves must be same for each value of x and $c_j^{-1} = V_j^{-1} - i\omega^{-1}q_j$, $(j = 0, 1, 2, 3)$, then we obtain the following Snell's law for the present problem:

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\theta_3}{V_3}, \tag{14}$$

and, we also obtain the following system of three non-homogeneous:

$$\sum_{j=1}^{3} a_{ij} Z_j = b_i, \quad (i = 1, 2, 3), \tag{15}$$

where,

$$a_{11} = F_1\left(C_{22} - \mu_e H_0^2\right)\sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0} + \left(C_{12} - \mu_e H_0^2\right)\sin\theta_0 + \frac{i\beta_2 G_1}{k_0},$$

$$a_{12} = F_2\left(C_{22} - \mu_e H_0^2\right)\sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0} + \left(C_{12} - \mu_e H_0^2\right)\sin\theta_0 + \frac{i\beta_2 G_2}{k_0},$$

$$a_{13} = F_3\left(C_{22} - \mu_e H_0^2\right)\sqrt{\left(\frac{V_0}{V_3}\right)^2 - \sin^2\theta_0} + \left(C_{12} - \mu_e H_0^2\right)\sin\theta_0 + \frac{i\beta_2 G_3}{k_0},$$

$$a_{21} = \sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0} + F_1\sin\theta_0, \quad a_{22} = \sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0} + F_2\sin\theta_0,$$

$$a_{23} = \sqrt{\left(\frac{V_0}{V_3}\right)^2 - \sin^2\theta_0} + F_3 \sin\theta_0, \quad a_{31} = G_1\sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0},$$

$$a_{32} = G_2\sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0}, \qquad a_{33} = G_3\sqrt{\left(\frac{V_0}{V_3}\right)^2 - \sin^2\theta_0},$$

$$b_1 = F_0\cos\theta_0\left(C_{22} - \mu_e H_0^2\right) - \left(C_{12} - \mu_e H_0^2\right)\sin\theta_0 - \frac{i\beta_2 G_0}{k_0},$$

$$b_2 = \cos\theta_0 - F_0\sin\theta_0, \quad b_3 = G_0\cos\theta_0,$$

$$Z_1 = \frac{A_1}{A_0}, \quad Z_2 = \frac{A_2}{A_0}, \quad Z_3 = \frac{A_3}{A_0},$$

where Z_1, Z_2 and Z_3 are reflection coefficient of reflected qP, qSV and qT waves, respectively.

5. Particular cases

(a) Transversely isotropic case

In absence of thermal, magnetic and rotational parameters, we have

$$\varepsilon = 0, \quad \beta_1 = \beta_2 = \beta \to 0, \quad \frac{\beta_2}{\beta_1} = 1, \quad K_1 = K_2 = K \to 0, \quad D_5 = 0,$$

$$H = 0, \quad \Omega = 0, \quad \Omega^* = 1.$$

Then, the equation (15) reduces to

$$\sum_{j=1}^{2} a_{ij}^a Z_j = b_i^a, , \quad (i = 1, 2)\,, \tag{16}$$

where,

$$a_{11}^a = F_1^a C_{22}\sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0} + C_{12}\sin\theta_0,$$

$$a_{12}^a = F_2^a C_{22}\sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0} + C_{12}\sin\theta_0,$$

$$a_{21}^a = \sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0} + F_1^a \sin\theta_0,$$

$$a_{22}^a = \sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0} + F_2^a \sin\theta_0,$$

$$b_1^a = F_0^a C_{22}\cos\theta_0 - C_{12}\sin\theta_0, \quad b_2^a = \cos\theta_0 - F_0^a \sin\theta_0,$$

$$F_0^a = -\frac{D_{20}^a \sin\theta_0 + (D_{10}^a - \zeta_0)\cos\theta_0}{D_{20}^a \cos\theta_0 + (D_{30}^a - \zeta_0)\sin\theta_0},$$

$$F_l^a = \frac{D_{2l}^a \sin\theta_l - (D_{1l}^a - \zeta_l)\cos\theta_l}{D_{2l}^a \cos\theta_l - (D_{3l}^a - \zeta_l)\sin\theta_l}, \quad l = (1,2),$$

$$D_{10}^a = C_{11}\sin^2\theta_0 + C^*\cos^2\theta_0,$$

$$D_{20}^a = -\left(C_{12} + C^*\right)\sin\theta_0\cos\theta_0,$$

$$D_{30}^a = C^*\sin^2\theta_0 + C_{22}\cos^2\theta_0,$$

$$D_{1l}^a = C_{11}\sin^2\theta_l + C^*\cos^2\theta_l,$$

$$D_{2l}^a = (C_{12} + C^*)\sin\theta_l\cos\theta_l,$$

$$D_{3l}^a = C^*.\sin^2\theta_l + C_{22}\cos^2\theta_l,$$

and $Z_1 = \dfrac{A_1}{A_0}$ and $Z_2 = \dfrac{A_2}{A_0}$ are the reflection coefficients of the reflected qP and qSV waves, respectively in a transversely isotropic media. Here, the equation (9) reduces to:

$$L\zeta^2 + M\zeta + N = 0, \tag{17}$$

where,

$$L = 1, \quad M = -\left(D_1^a + D_3^a\right), \quad N = D_1^a D_3^a - D_2^{a2},$$

$$D_1^a = C_{11}\sin^2\theta + C^*\cos^2\theta, \quad D_2^a = (C_{12} + C^*)\sin\theta\cos\theta,$$

$$D_3^a = C^*\sin^2\theta + C_{22}\cos^2\theta.$$

The real parts of the roots of equation (17) represent the speeds V_1 and V_2 of qP and qSV waves, respectively in a transversely isotropic media.

(b) Transversely isotropic magneto-thermoelastic case

In absence of rotational parameters, we have $\Omega = 0$ and $\Omega^* = 1$. Then, the equation (15) reduces to:

$$\sum_{j=1}^{3} a_{ij}^b Z_j = b_i^b, \quad (i = 1, 2, 3)\,, \tag{18}$$

where,

$$a_{11}^b = F_1^b \left(C_{22} - \mu_e H_0^2\right) \sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0} + \left(C_{12} - \mu_e H_0^2\right) \sin\theta_0 + \frac{i\beta_2 G_1^b}{k_0},$$

$$a_{12}^b = F_2^b \left(C_{22} - \mu_e H_0^2\right) \sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0} + \left(C_{12} - \mu_e H_0^2\right) \sin\theta_0 + \frac{i\beta_2 G_2^b}{k_0},$$

$$a_{13}^b = F_3^b \left(C_{22} - \mu_e H_0^2\right) \sqrt{\left(\frac{V_0}{V_3}\right)^2 - \sin^2\theta_0} + \left(C_{12} - \mu_e H_0^2\right) \sin\theta_0 + \frac{i\beta_2 G_3^b}{k_0},$$

$$a_{21}^b = \sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0} + F_1^b \sin\theta_0, \quad a_{22}^b = \sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0} + F_2^b \sin\theta_0,$$

$$a_{23}^b = \sqrt{\left(\frac{V_0}{V_3}\right)^2 - \sin^2\theta_0} + F_3^b \sin\theta_0, \quad a_{31}^b = G_1^b \sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0},$$

$$a_{32}^b = G_2^b \sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0}, \quad a_{33}^b = G_3^b \sqrt{\left(\frac{V_0}{V_3}\right)^2 - \sin^2\theta_0},$$

$$b_1^b = F_0^b \cos\theta_0 \left(C_{22} - \mu_e H_0^2\right) - \left(C_{12} - \mu_e H_0^2\right) \sin\theta_0 - \frac{i\beta_2 G_0^b}{k_0},$$

$$b_2^b = \cos\theta_0 - F_0^b \sin\theta_0, \quad b_3^b = G_0^b \cos\theta_0,$$

$$F_0^b = -\frac{\beta_1 D_{20}^b \sin\theta_0 + \beta_2 (D_{10}^b - \zeta_0) \cos\theta_0}{\beta_2 D_{20}^b \cos\theta_0 + \beta_1 (D_{30}^b - \zeta_0) \sin\theta_0},$$

$$G_0^b = ik_0 \left[\frac{\zeta_0^2 - (D_{10}^b + D_{30}^b)\zeta_0 - D_{20}^{b2} + D_{10}^b D_{30}^b}{\beta_2 D_{20}^b \cos\theta_0 + \beta_1 (D_{30}^b - \zeta_0) \sin\theta_0} \right],$$

$$F_l^b = \frac{\beta_1 D_{2l}^b \sin\theta_l - \beta_2 (D_{1l}^b - \zeta_l) \cos\theta_l}{\beta_2 D_{2l}^b \cos\theta_l - \beta_1 (D_{3l}^b - \zeta_l) \sin\theta_l},$$

$$G_l^b = -ik_l \left[\frac{\zeta_l^2 - (D_{1l}^b + D_{3l}^b)\zeta_l - D_{2l}^{b2} + D_{1l}^b D_{3l}^b}{\beta_2 D_{2l}^b \cos\theta_l - \beta_1 (D_{3l}^b - \zeta_l) \sin\theta_l} \right],$$

$$D_{10}^b = \left(C_{11} + \mu_e H_0^2\right) \sin^2\theta_0 + C^* \cos^2\theta_0,$$

$$D_{20}^b = -\left(C_{12} + C^* + \mu_e H_0^2\right) \sin\theta_0 \cos\theta_0,$$

$$D_{30}^b = C^* \sin^2\theta_0 + \left(C_{22} + \mu_e H_0^2\right) \cos^2\theta_0,$$

$$D_{1l}^b = \left(C_{11} + \mu_e H_0^2\right) \sin^2\theta_l + C^* \cos^2\theta_l,$$

$$D_{2l}^b = \left(C_{12} + C^* + \mu_e H_0^2\right) \sin\theta_l \cos\theta_l,$$

$$D_{3l}^b = C^* \sin^2\theta_l + \left(C_{22} + \mu_e H_0^2\right) \cos^2\theta_l, \quad (l = 1, 2, 3).$$

and $Z_1 = \dfrac{A_1}{A_0}$, $Z_2 = \dfrac{A_2}{A_0}$ and $Z_3 = \dfrac{A_3}{A_0}$ are reflection coefficient of reflected qP, qSV and qT waves, respectively in a transversely isotropic magneto-thermoelastic medium. Here, the equation (9) reduces to

$$\zeta^3 + G_1\zeta^2 + G_2\zeta + G_3 = 0, \tag{19}$$

where,

$$G_1 = -\left(D_1 + D_3 + D_5 + \varepsilon \sin^2\theta + \bar{\beta}^2 \varepsilon \cos^2\theta\right),$$

$$G_2 = D_1 D_3 + D_1 D_5 + D_3 D_5 - D_2^2 + \varepsilon\left(\bar{\beta}^2 D_1 \cos^2\theta + D_3 \sin^2\theta - 2\bar{\beta} D_2 \sin\theta \cos\theta\right),$$

$$G_3 = \left(D_2^2 - D_1 D_3\right) D_5.$$

The real parts of the roots of equation (19) represent the speeds V_1, V_2 and V_3 of qP, qSV and qT waves, respectively in a transversely isotropic magneto-thermoelastic medium.

(c) Rotating isotropic thermoelastic case

Neglecting anisotropy and electro-magnetic parameters, we have:

$$\beta_1 = \beta_2 = \beta, \quad \overline{\beta} = 1, \quad K_1 = K_2 = K, \quad C_{11} = \lambda + 2\mu, \quad C_{12} = \lambda,$$

$$C^* = \mu, \quad C_{22} = \lambda + 2\mu, \quad H = 0.$$

Then, the equation (15) reduces to:

$$\sum_{j=1}^{3} a_{ij}^c Z_j = b_i^c, \quad (i = 1, 2, 3)\,, \tag{20}$$

where,

$$a_{11}^c = F_1^c(\lambda + 2\mu)\sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0} + \lambda \sin\theta_0 + \frac{i\beta G_1^c}{k_0},$$

$$a_{12}^c = F_2^c(\lambda + 2\mu)\sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0} + \lambda \sin\theta_0 + \frac{i\beta G_2^c}{k_0},$$

$$a_{13}^c = F_3^c(\lambda + 2\mu)\sqrt{\left(\frac{V_0}{V_3}\right)^2 - \sin^2\theta_0} + \lambda \sin\theta_0 + \frac{i\beta G_3^c}{k_0},$$

$$a_{21}^c = \sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0} + F_1^c \sin\theta_0, \quad a_{22}^c = \sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0} + F_2^c \sin\theta_0,$$

$$a_{23}^c = \sqrt{\left(\frac{V_0}{V_3}\right)^2 - \sin^2\theta_0} + F_3^c \sin\theta_0, \quad a_{31}^c = G_1^c\sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0},$$

$$a_{32}^c = G_2^c\sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0}, \quad a_{33}^c = G_3^c\sqrt{\left(\frac{V_0}{V_3}\right)^2 - \sin^2\theta_0},$$

$$b_1^c = F_0^c \cos\theta_0(\lambda + 2\mu) - \lambda \sin\theta_0 - \frac{i\beta G_0^c}{k_0},$$

$$b_2^c = \cos\theta_0 - F_0^c \sin\theta_0, \quad b_3^c = G_0^c \cos\theta_0,$$

$$F_0^c = -\frac{\left(D_{20}^c - 2i\frac{\Omega}{\omega}\zeta_0\right)\sin\theta_0 + (D_{10}^c - \Omega^*\zeta_0)\cos\theta_0}{\left(D_{20}^c + 2i\frac{\Omega}{\omega}\zeta_0\right)\cos\theta_0 + (D_{30}^c - \Omega^*\zeta_0)\sin\theta_0},$$

$$G_0^c = -ik_0\left[\frac{\left(4\left(\frac{\Omega}{\omega}\right)^2 - \Omega^{*2}\right)\zeta_0^2 + \Omega^*\left(D_{10}^c + D_{30}^c\right)\zeta_0 + D_{20}^{c2} - D_{10}^c D_{30}^c}{\beta\left(D_{20}^c + 2i\frac{\Omega}{\omega}\zeta_0\right)\cos\theta_0 + \beta(D_{30}^c - \Omega^*\zeta_0)\sin\theta_0}\right],$$

$$F_l^c = \frac{\left(D_{2l}^c - 2i\frac{\Omega}{\omega}\zeta_l\right)\sin\theta_l - (D_{1l}^c - \Omega^*\zeta_l)\cos\theta_l}{\left(D_{2l}^c + 2i\frac{\Omega}{\omega}\zeta_l\right)\cos\theta_l - (D_{3l}^c - \Omega^*\zeta_l)\sin\theta_l}, \quad (l = 1, 2, 3),$$

$$G_l^c = ik_l\left[\frac{\left(4\left(\frac{\Omega}{\omega}\right)^2 - \Omega^{*2}\right)\zeta_l^2 + \Omega^*\left(D_{1l}^c + D_{3l}^c\right)\zeta_l + D_{2l}^{c2} - D_{1l}^c D_{3l}^c}{\beta\left(D_{2l}^c + 2i\frac{\Omega}{\omega}\zeta_l\right)\cos\theta_l - \beta\left(D_{3l}^c - \Omega^*\zeta_l\right)\sin\theta_l}\right],$$

$$D_{10}^c = (\lambda + 2\mu)\sin^2\theta_0 + \mu\cos^2\theta_0, \quad D_{20}^c = -(\lambda + \mu)\sin\theta_0\cos\theta_0,$$

$$D_{30}^c = \mu\sin^2\theta_0 + (\lambda + 2\mu)\cos^2\theta_0, \quad D_{1l}^c = (\lambda + 2\mu)\sin^2\theta_l + \mu\cos^2\theta_l,$$

$$D_{2l}^c = (\lambda + \mu)\sin\theta_l\cos\theta_l, \quad D_{3l}^c = \mu\sin^2\theta_l + (\lambda + 2\mu)\cos^2\theta_l.$$

and $Z_1 = \frac{A_1}{A_0}$, $Z_2 = \frac{A_2}{A_0}$ and $Z_3 = \frac{A_3}{A_0}$ are reflection coefficient of reflected P, SV and T waves, respectively in a rotating isotropic thermoelastic medium. Here, the equation (9) reduces to

$$P\zeta^3 + Q\zeta^2 + R\zeta + S = 0, \tag{21}$$

where,

$$P = 4\left(\frac{\Omega}{\omega}\right)^2 - \Omega^{*2}, \quad Q = \Omega^*(D_1^c + D_3^c) + \Omega^*\varepsilon + \left(\Omega^{*2} - 4\left(\frac{\Omega}{\omega}\right)^2\right)D_5^c,$$

$$R = (D_2^{c2} - D_1^c D_3^c) - \Omega^*\left(D_1^c + D_3^c\right)D_5^c + \left(D_1^c D_3^c - D_2^{c2}\right)D_5^c$$

$$-\varepsilon\left(D_1^c\cos^2\theta - 2D_2^c\sin\theta\cos\theta + D_3^c\sin^2\theta\right),$$

$$S = \left(D_1^cD_3^c - D_2^{c2}\right)D_5^c.$$

$$D_1^c = (\lambda+2\mu)\sin^2\theta + \mu\cos^2\theta, \quad D_2^c = (\lambda+\mu)\sin\theta\cos\theta,$$

$$D_3^c = \mu\sin^2\theta + (\lambda+2\mu)\cos^2\theta, \quad D_4^c = K, \quad D_5^c = \frac{K}{\tau^*C_E}, \quad \varepsilon = \frac{\beta^2T_0}{\rho C_E}.$$

The real parts of the roots of equation (21) represent the speeds V_1, V_2 and V_3 of P, SV and T waves, respectively in a rotating isotropic thermoelastic medium. The speeds and reflection coefficients in this case agree with those obtained by Othman and Song [23], after making certain correspondences between the symbols used in these studies.

(d) Isotropic thermoelastic case

Neglecting transverse isotropy, electro-magnetic and rotational parameters, we have:

$$\beta_1 = \beta_2 = \beta, \quad \overline{\beta} = 1, K_1 = K_2 = K, \quad C_{11} = \lambda+2\mu, C_{12} = \lambda, \quad C^* = \mu,$$

$$C_{22} = \lambda+2\mu, \quad \Omega = 0, \quad \Omega^* = 1, \quad H = 0.$$

Then, the equation (15) reduces to:

$$\sum_{j=1}^{3} a_{ij}^d Z_j = b_i^d, \quad (i = 1,2,3), \tag{22}$$

where,

$$a_{11}^d = F_1^d(\lambda+2\mu)\sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0} + \lambda\sin\theta_0 + \frac{i\beta G_1^d}{k_0},$$

$$a_{12}^d = F_2^d(\lambda+2\mu)\sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0} + \lambda\sin\theta_0 + \frac{i\beta G_2^d}{k_0},$$

$$a_{13}^d = F_3^d(\lambda+2\mu)\sqrt{\left(\frac{V_0}{V_3}\right)^2 - \sin^2\theta_0} + \lambda\sin\theta_0 + \frac{i\beta G_3^d}{k_0},$$

$$a_{21}^d = \sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0} + F_1^d \sin\theta_0, \quad a_{22}^d = \sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0} + F_2^d \sin\theta_0,$$

$$a_{23}^d = \sqrt{\left(\frac{V_0}{V_3}\right)^2 - \sin^2\theta_0} + F_3^d \sin\theta_0, \quad a_{31}^d = G_1^d \sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0},$$

$$a_{32}^d = G_2^d \sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0}, \quad a_{33}^d = G_3^d \sqrt{\left(\frac{V_0}{V_3}\right)^2 - \sin^2\theta_0},$$

$$b_1^d = F_0^d \cos\theta_0(\lambda + 2\mu) - \lambda \sin\theta_0 - \frac{i\beta G_0^d}{k_0}, \quad b_2^d = \cos\theta_0 - F_0^d \sin\theta_0,$$

$$b_3^d = G_0^d \cos\theta_0,$$

$$F_0^d = -\frac{D_{20}^d \sin\theta_0 + \cos\theta_0 \left(D_{10}^d - \zeta_0\right)}{D_{20}^d \cos\theta_0 + \sin\theta_0 \left(D_{30}^d - \zeta_0\right)},$$

$$G_0^d = ik_0 \left[\frac{\zeta_0^2 - \left(D_{10}^d + D_{30}^d\right)\zeta_0 - D_{20}^{d2} + D_{10}^d D_{30}^d}{\beta D_{20}^d \cos\theta_0 + \beta \sin\theta_0 (D_{30}^d - \zeta_0)}\right],$$

$$F_l^d = \frac{D_{2l}^d \sin\theta_l - \cos\theta_l \left(D_{1l}^d - \zeta_l\right)}{D_{2l}^d \cos\theta_l - \sin\theta_l \left(D_{3l}^d - \zeta_l\right)},$$

$$G_l^d = -ik_l \left[\frac{\zeta_l^2 - \left(D_{1l}^d + D_{3l}^d\right)\zeta_l - D_{2l}^{d2} + D_{1l}^d D_{3l}^d}{\beta D_{2l}^d \cos\theta_l - \beta \sin\theta_l \left(D_{3l}^d - \zeta_l\right)}\right],$$

$$D_{10}^d = (\lambda + 2\mu)\sin^2\theta_0 + \mu\cos^2\theta_0, \quad D_{20}^d = -(\lambda + \mu)\sin\theta_0 \cos\theta_0$$

$$D_{30}^d = \mu\sin^2\theta_0 + (\lambda + 2\mu)\cos^2\theta_0, \quad D_{1l}^d = (\lambda + 2\mu)\sin^2\theta_l + \mu\cos^2\theta_l,$$

$$D_{2l}^d = (\lambda + \mu)\sin\theta_l \cos\theta_l, \quad D_{3l}^d = \mu\sin^2\theta_l + (\lambda + 2\mu)\cos^2\theta_l.$$

and $Z_1 = \dfrac{A_1}{A_0}$, $Z_2 = \dfrac{A_2}{A_0}$ and $Z_3 = \dfrac{A_3}{A_0}$ are reflection coefficient of reflected P, SV and T waves, respectively in an isotropic thermoelastic medium. Then, the equation (9) reduces to:

$$\zeta^3 \quad + P_1\zeta^2 + Q_1\zeta + R_1 = 0, \tag{23}$$

where,

$$P_1 = -(D_1^d + D_3^d + \varepsilon + D_5^d),$$

$$Q_1 = (D_1^d D_3^d - D_2^{d2}) + \left(D_1^d + D_3^d\right) D_5^d + \left(D_2^{d2} - D_1^d D_3^d\right) D_5^d + \varepsilon \left(D_1^d \cos^2\theta - 2D_2^d \sin\theta\cos\theta + D_3^d \sin^2\theta\right),$$

$$R_1 = \left(D_2^{d2} - D_1^d D_3^d\right) D_5^d.$$

$$D_1^d = (\lambda + 2\mu)\sin^2\theta + \mu\cos^2\theta, \quad D_2^d = (\lambda + \mu)\sin\theta\cos\theta,$$

$$D_3^d = \mu\sin^2\theta + (\lambda + 2\mu)\cos^2\theta, \quad D_4^d = K, \quad D_5^d = \frac{K}{\tau^* C_E}, \quad \varepsilon = \frac{\beta^2 T_0}{\rho C_E}.$$

The real parts of the roots of equation (23) represent the speeds V_1, V_2 and V_3 of P, SV and T waves, respectively in an isotropic thermoelastic medium. The speeds and reflection coefficients in this case agree with those obtained by Sinha and Sinha [9], after making certain correspondences between the symbols used in these studies.

(e) Isotropic elastic case

Neglecting the transverse isotropy, thermal, magnetic and rotational parameters, we have:

$$\beta_1 = \beta_2 = \beta \to 0, \quad \frac{\beta_2}{\beta_1} = 1, \quad K_1 = K_2 = K \to 0, \quad C_{11} = \lambda + 2\mu, \quad C_{12} = \lambda,$$

$$C^* = \mu, \quad C_{22} = \lambda + 2\mu, \quad H_0 = 0, \quad \Omega = 0, \quad \Omega^* = 1.$$

Then, the equation (15) reduces to:

$$\sum_{j=1}^{2} a_{ij}^e Z_j = b_i^e, \quad (i = 1, 2), \tag{24}$$

where

$$a_{11}^e = F_1^e(\lambda + 2\mu)\sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0} + \lambda\sin\theta_0,$$

$$a_{12}^e = F_2^e(\lambda+2\mu)\sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0} + \lambda\sin\theta_0,$$

$$a_{21}^e = \sqrt{\left(\frac{V_0}{V_1}\right)^2 - \sin^2\theta_0} + F_1^e\sin\theta_0,$$

$$a_{22}^e = \sqrt{\left(\frac{V_0}{V_2}\right)^2 - \sin^2\theta_0} + F_2^e\sin\theta_0,$$

$$b_1^e = F_0^e(\lambda+2\mu)\cos\theta_0 - \lambda\sin\theta_0, \quad b_2^e = \cos\theta_0 - F_0^e\sin\theta_0,$$

$$F_0^e = -Cot\,\theta_0, \quad F_l^e = Cot\,\theta_l, \quad l = (1,2)$$

and $Z_1 = \dfrac{A_1}{A_0}$ and $Z_2 = \dfrac{A_2}{A_0}$ are reflection coefficient of the reflected P and SV waves, respectively in an isotropic elastic medium. Here, the equation (9) reduces to:

$$U\zeta^2 + V\zeta + W = 0, \tag{25}$$

where,

$$U = 1, \quad V = -\left(D_1^e + D_3^e\right), \quad W = D_1^e D_3^e - D_2^{e2}$$

$$D_1^e = (\lambda+2\mu)\sin^2\theta + \mu\cos^2\theta, \quad D_2^e = (\lambda+\mu)\sin\theta\cos\theta,$$

$$D_3^e = \mu\sin^2\theta + (\lambda+2\mu)\cos^2\theta,$$

The real parts of the roots of equation (25) represent the speeds V_1 and V_2 of P and SV waves, respectively in an isotropic elastic medium. We obtain from equation (24), the reflection coefficients of reflected P and SV waves in closed from as:

(a) for incident P wave

$$\frac{A_1}{A_{0P}} = \frac{\sin 2\theta_1 \sin 2\theta_2 - (V_1/V_2)^2\cos^2 2\theta_2}{\sin 2\theta_1 \sin 2\theta_2 + (V_1/V_2)^2\cos^2 2\theta_2},$$

$$\frac{A_2}{A_{0P}} = \frac{-2(V_1/V_2)\sin 2\theta_1 \cos 2\theta_2}{\sin 2\theta_1 \sin 2\theta_2 + (V_1/V_2)^2\cos^2 2\theta_2}, \tag{26}$$

(b) for incident SV wave

$$\frac{A_1}{A_{0SV}} = \frac{(V_1/V_2)\sin 4\theta_2}{\sin 2\theta_1 \sin 2\theta_2 + (V_1/V_2)^2 \cos^2 2\theta_2},$$

$$\frac{A_2}{A_{0SV}} = \frac{\sin 2\theta_1 \sin 2\theta_2 - (V_1/V_2)^2 \cos^2 2\theta_2}{\sin 2\theta_1 \sin 2\theta_2 + (V_1/V_2)^2 \cos^2 2\theta_2}. \tag{27}$$

The reflection coefficients given by equations (26) and (27) agree with those obtained by Ben-Menahem and Singh [42].

6. Numerical results and discussion

In order to compute the complex modulus of the reflection coefficients of plane waves, the following relevant elastic and thermal constants of single crystal of Zinc are considered [31]:

$$C_{11} = 1.628 \times 10^9 \text{ Nm}^{-2},\ C_{22} = 1.562 \times 10^9 \text{ Nm}^{-2},\ C_{12} = 0.385 \times 10^9 \text{Nm}^{-2},$$

$$\beta_1 = 5.75 \times 10^6 \text{ Nm}^{-2}\,\text{deg}^{-1}, \quad \beta_2 = 5.17 \times 10^6 \text{ Nm}^{-2}\,\text{deg}^{-1},$$

$$K_1 = 1.24 \times 10^2 \text{ Wm}^{-1}\,\text{deg}^{-1}, \quad K_2 = 1.34 \times 10^2 \text{ Wm}^{-1}\,\text{deg}^{-1},$$

$$C_E = 3.9 \times 10^2 \text{ JKg}^{-1}\,\text{deg}^{-1}, \quad T_0 = 296 \text{ K}, \quad \rho = 7.14 \times 10^3 \text{ Kgm}^{-3}.$$

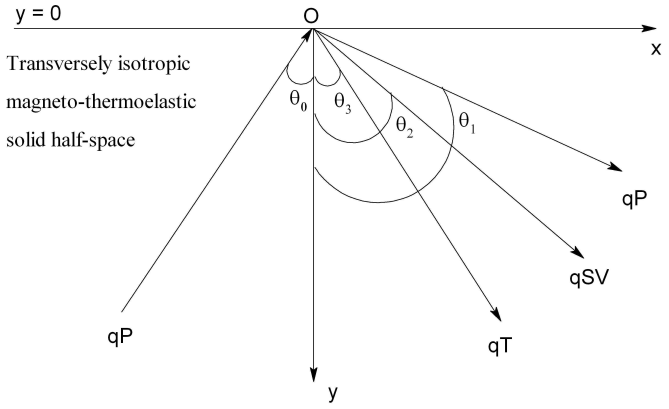


Fig. 1. Geometry of the problem showing incident and reflected waves

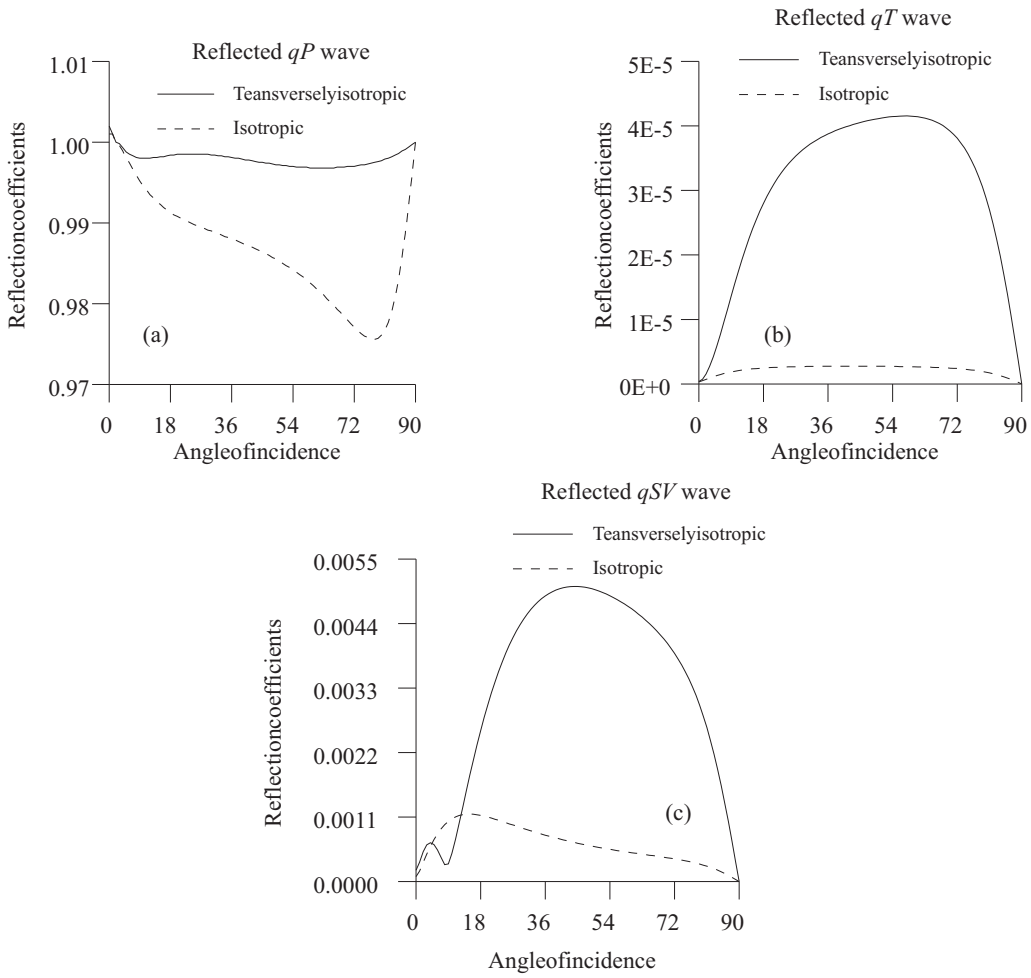


Fig. 2. Variations of the reflection coefficients of reflected waves against the angle of incidence when $H = 10$, $\dfrac{\Omega}{\omega} = 5$ and $\tau_0 = 0.05$ s

The reflection coefficients of various quasi plane waves are computed with the help of a FORTRAN program against angle of incidence of qP wave for different thermal, magnetic and rotation parameters. These variations of the reflection coefficients of the reflected waves are shown graphically in Figs 2–5.

The reflection coefficient for transversely isotropic case of reflected qP wave oscillates near the value one for the range $0 \leq \theta_0 \leq 90^\circ$ of angle of

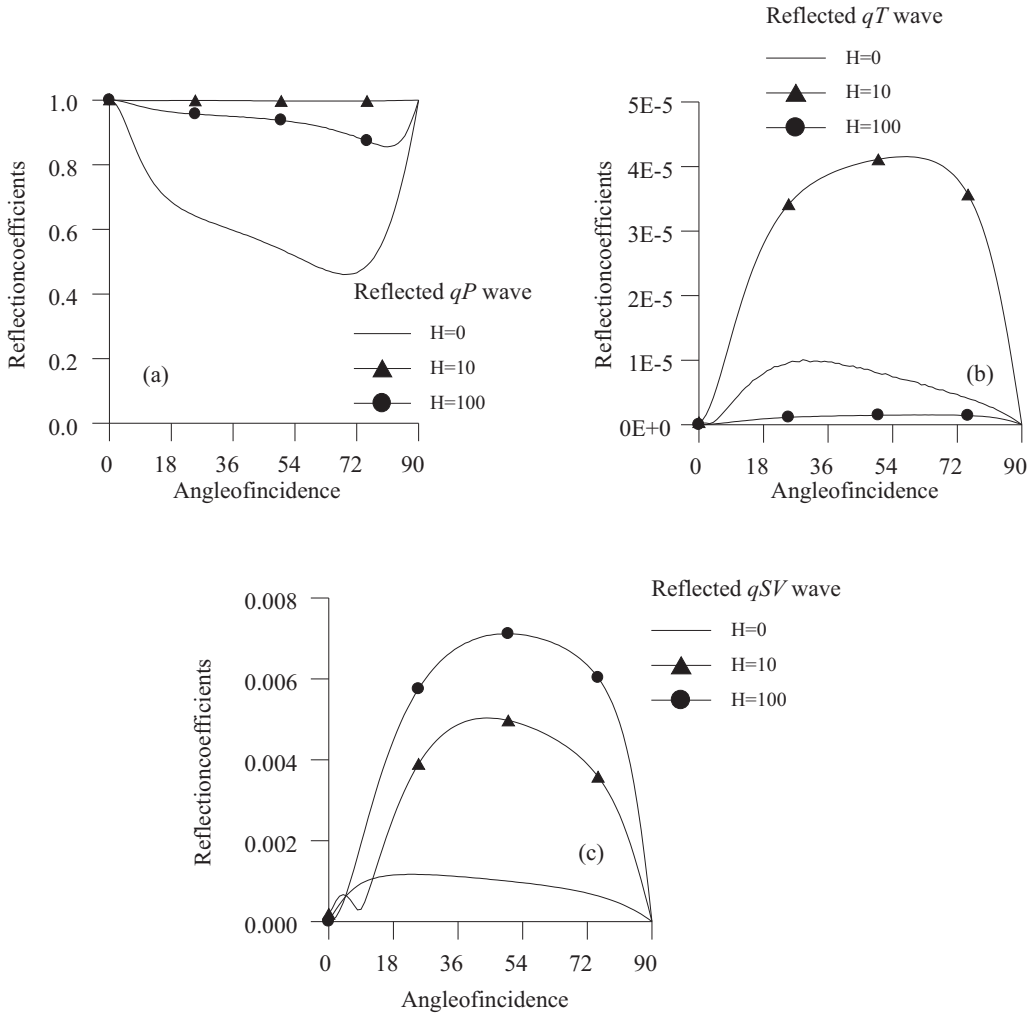


Fig. 3. Variations of the reflection coefficients of reflected waves against the angle of incidence when $H = 0,\ 10,\ 100$; $\dfrac{\Omega}{\omega} = 5$ and $\tau_0 = 0.05$ s

incidence of qP wave, whereas for isotropic case, it decreases at each angle of incidence except at normal and grazing incidences. The maximum effect of transverse isotropy is observed neat angle $\theta_0 = 77°$. These variations of the reflection coefficients of the reflected qP waves are shown graphically in Fig. 2(a). The reflection coefficient for the transversely isotropic case of reflected qT wave attains to its maximum value 0.4153e-04 near angle $\theta_0 = 59°$ and

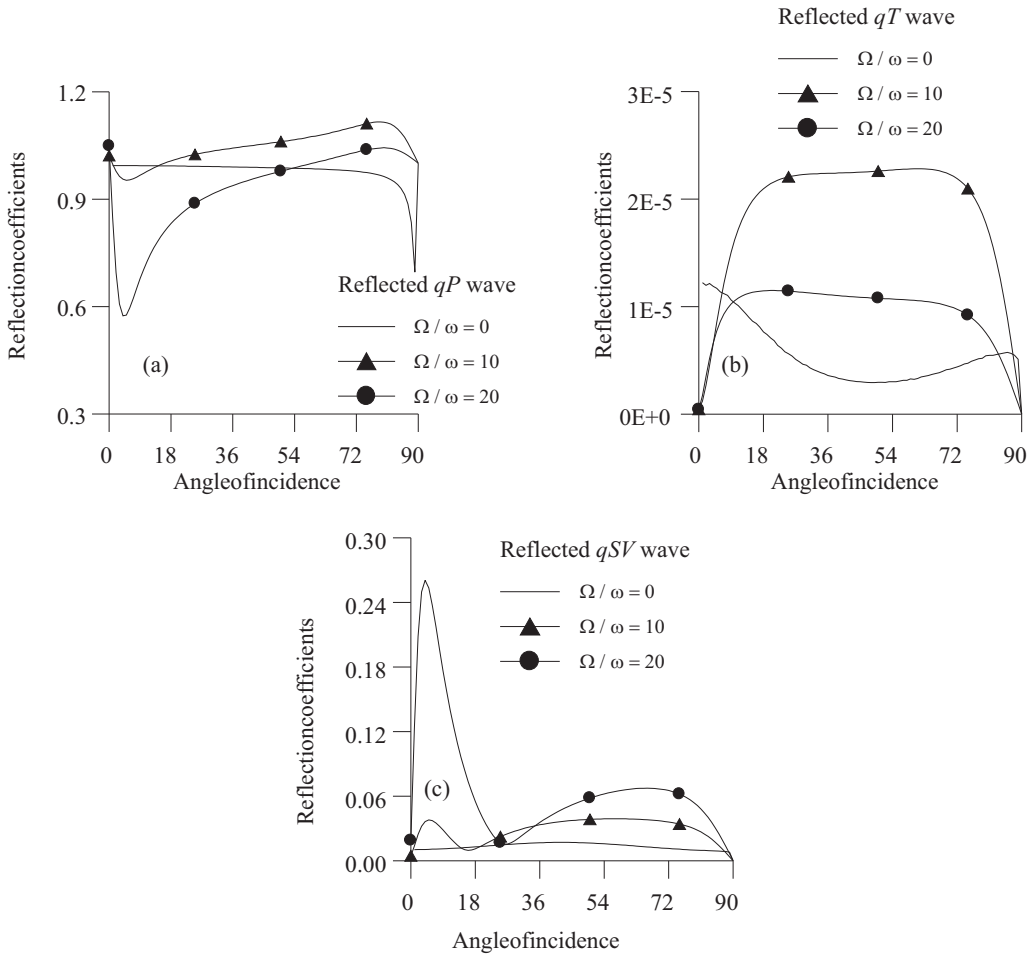


Fig. 4. Variations of the reflection coefficients of reflected waves against the angle of incidence when $H = 10$; $\dfrac{\Omega}{\omega} = 0$, 10, 20 and $\tau_0 = 0.05$ s

its minimum value zero is at normal and grazing incidence. It has similar variation for isotropic case where the reflection coefficients decrease at each angle of incidence except at normal and grazing incidences. These variations of the reflection coefficients of reflected qT waves are shown graphically in Fig. 2(b). The reflection coefficient for transversely isotropic case of reflected qSV wave attains to its maximum value 0.5032e-02 near angle $\theta_0 = 44^\circ$ and its minimum value is zero at normal and grazing incidence. The reflection

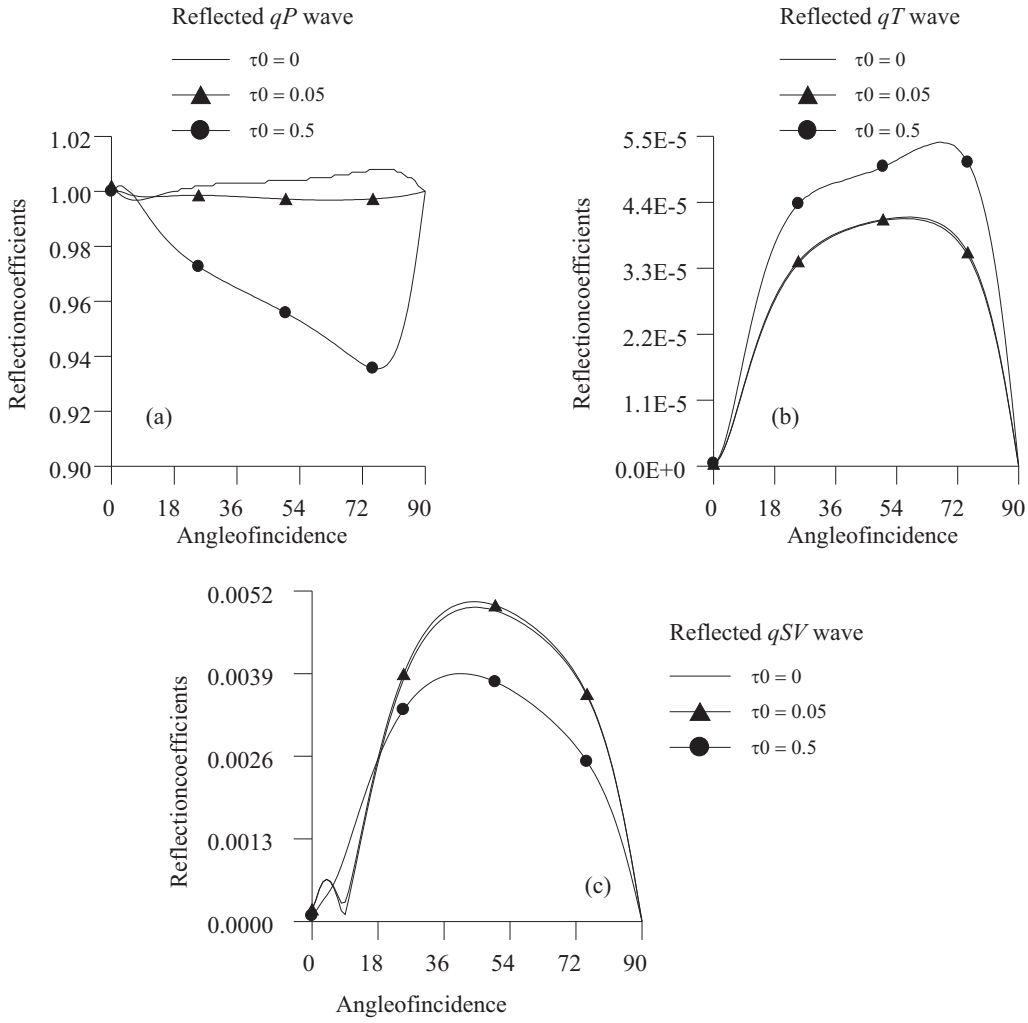


Fig. 5. Variations of the reflection coefficients of reflected waves against the angle of incidence when $H = 10$; $\dfrac{\Omega}{\omega} = 5$ and $\tau_0 = 0,\ 0.05,\ 0.5$ s

coefficients for isotropic case decrease at each angle of incidence except some angles near normal incidence and at grazing incidence. These variations of the reflection coefficients of reflected qT waves are shown graphically in Fig. 2(c).

The reflection coefficients of qP, qT and qSV waves are shown graphically to observe the effect of magnetic field parameter against the angle of incidence in Figs 3(a)–(c) for $H = 0$, 10 and 100. The comparison of solid

line with solid lines for $H = 0$ with the solid lines for $H = 10$ and 100, shows that the reflection coefficients of reflected waves change at each angle of incidence except the normal and grazing incidences. The change is different for each reflected wave, for example, the reflection coefficients of reflected qSV wave increase with the increase in magnetic field. The reflection coefficients of reflected qP and qT waves are larger at $H = 10$. The reflection coefficients of qP wave for $H = 100$ are larger than for $H = 0$ but less than for $H = 10$. The reflection coefficients for $H = 100$ are smaller than for $H = 0$ in case of reflected qT wave.

The reflection coefficients of qP, qT and qSV waves to observe the effect of rotation parameter are shown graphically against the angle of incidence in Figs 4(a–c) for $\dfrac{\Omega}{\omega} = 0$, 10 and 20. The comparison of the solid lines in these figures show the effect of rotation parameter on the reflection coefficients of reflected qP, qT and qSV waves. This effect is observed maximum near the normal incidence.

The reflection coefficients of qP, qT and qSV waves to observe the effect of relaxation time are shown graphically against the angle of incidence in Figs 5(a–c) for $\tau_0 = 0,\ 0.005 \times 10^{-12}$ s and 0.05×10^{-12} s. The comparison of the solid lines in these figures show the effect of relaxation time on the reflection coefficients of reflected qP, qT and qSV waves. This effect becomes prominent for $\tau_0 = 0.05 \times 10^{-12}$ s.

7. Conclusion

The required boundary conditions at a thermally insulated stress-free surface of a transversely isotropic rotating magneto-thermoelastic solid half-space are satisfied by the appropriate particular solutions in the half-space to obtain the relations between the reflection coefficients of various reflected waves for incidence of qP wave. These coefficients are computed numerically for a relevant material and it is observed that these coefficients are affected significantly by transverse isotropy, magnetic field, rotation and relaxation times.

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